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Dissipation effect on MHD mixed convection flow over a stretching sheet through porous medium with non-uniform heat source/sink

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KEYWORDS

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Abstract Dissipative effect on magnetohydrodynamic (MHD) mixed convective unsteady flow of an electrically conducting fluid over a stretching sheet embedded in a porous medium subject to transverse magnetic field in the presence of non-uniform heat source/sink has been investigated in this paper. The method of solution involves similarity transformation. The coupled nonlinear partial differential equations governing flow, heat and mass transfer phenomena are reduced into set of nonlinear ordinary differential equations. The transformed equations are solved numerically by using Runge–Kutta fourth order method associated with shooting technique. The numerical computation of skin friction, Nusselt number and Sherwood number is presented in tables. The work of previous authors is compared with the present work as particular cases in the absence of unsteady parameter, solutal buoyancy, Darcy dissipation and chemical reaction. The results of steady and unsteady cases are also discussed. The important findings are as follows: effect of electric field enhances the skin friction contributing to flow instability. Higher Prandtl number is suitable for the reduction of coefficient of skin friction which is desirable.

1. Introduction

The production of sheeting material, which includes both metal and polymer sheets, arises in a number of industrial manufacturing processes. The fluid flow due to a stretching surface has important applications in many engineering processes. In recent past, studies on boundary layer flows of viscous fluids due to a uniformly stretching sheet have been carried out by many authors. For instance, Nazar et al. [1],

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Nomenclature

$\bar{J}$	Joule current
T	temperature of the fluid
T_w	stretching sheet temperature
C_b	drag coefficient
C_p	specific heat at constant temperature
C_w	concentration at wall
C_∞	ambient concentration
E	electric field
Ec	Eckert number
E_0	uniform electric field
b	parameter of temperature distribution
b^*	mass distribution parameter
A^*	space dependent heat source
B^*	temperature dependent heat source
B	transverse magnetic field
B_0	uniform transverse magnetic field
k	permeability of porous medium
k_1	porous parameter
k_T	thermal diffusion ratio
K	mean absorption coefficient
a	stretching parameter
g	acceleration due to gravity
(x, y)	flow directional coordinate
u	velocity of the fluid in x-direction
Pr	Prandtl number
Pr_∞	ambient Prandtl number
Sc	Schmidt number
Sr	Soret number
Sh_x	local Sherwood number
Gr_x	Grashof number
F^*	local inertia coefficient
Ha	Hartmann number
Re_x	local Reynolds number
q'''	non-uniform heat source or sink
T_m	mean fluid temperature
T_∞	ambient temperature
C	concentration of species
E_1	local electromagnetic parameter

Greek symbols

θ	non-dimensional temperature
θ_r	variable viscosity constant
ρ	density of the fluid
σ	magnetic permeability
λ	buoyancy parameter
μ	fluid viscosity
υ	kinematic viscosity
η	similarity variable
κ	thermal conductivity
β_T	coefficient of thermal expansion
β_c	coefficient of solutal expansion
Δ	unsteady parameter

Kumari et al. [2], Hayat et al. [3], and many others have worked in this area of interest.

Heat transfer on a continuously moving surface has many applications in industrial manufacturing processes. The flow of fluids through porous media in a rotating system is of interest for instance to the petroleum engineering movement of oil and gas through the reservoir; and to the hydrologist who is interested in the study of migration of underground water. Study of flows through porous media in a rotating system also finds applications in geothermal energy systems, oil and gas recovery, and in the spread of pollutants in groundwater. Research on flows through porous media has lately been applied in the manufacture of industrial machinery and computer disk drives (Herrero et al. [4]). In the field of energy conservation, attention has been focused on the use of saturated porous materials for insulation in storage tanks so as to control the rate of heat transfer. Insulating underground water pipes prevents the water in the pipes from freezing during winter.

The stretching problems for steady flow have been used in various engineering and industrial processes, such as non-Newtonian fluid flows through porous medium. The boundary layer flow over a stretching sheet was first studied by Sakiadis [5,6]. Later, Crane [7] extended this idea for the two dimensional flow over a stretching sheet problem. Gupta and Gupta [8], Carragher and Crane [9], and Dutta et al. [10] studied the heat transfer in the flow over a stretching surface considering different aspects of the problem. Problem of flow and heat transfer through a porous medium over a stretching surface is considered by Cortell [11], Chauhan and Agrawal [12], and Chauhan and Rastogi [13]. Abel et al. [14] studied heat transfer in MHD slip flow of a second grade fluid through a porous medium past a stretching sheet with non-uniform heat source or sink.

Dutta et al. [10] discussed the flow over a stretching sheet with uniform heat flux. In this study, the viscous dissipation was considered in the energy equation. Further, Xu [15] studied an explicit analytic solution for convective heat transfer in an electrically conducting fluid at a stretching surface with uniform free stream.

When temperature difference exists between the solid–fluid interface and the fluid in the free stream, a thermal boundary layer is formed. The fluid particles in contact with the solid–fluid interface acquire the temperature of the interface. If the temperature of the interface is higher than that of the ambient fluid, the kinetic energy of the molecules of the adjacent fluid particles increases. These particles in turn exchange the acquired kinetic energy with those fluid particles in the adjacent fluid layers further away from the interface. This process continues in the adjacent fluid layers and temperature gradients develop in the fluid. Lai and Kulacki [16] analyzed the effects of variable viscosity on mixed convection heat transfer along a vertical surface in a saturated porous medium considering Newtonian fluid. Later, Kafoussias and Williams [17] investigated the effects of temperature-dependent viscosity on free-forced convective boundary layer flow past a vertical

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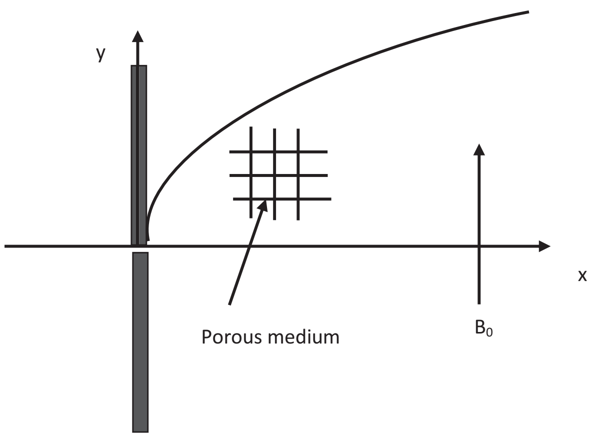

Figure 1 Flow geometry.

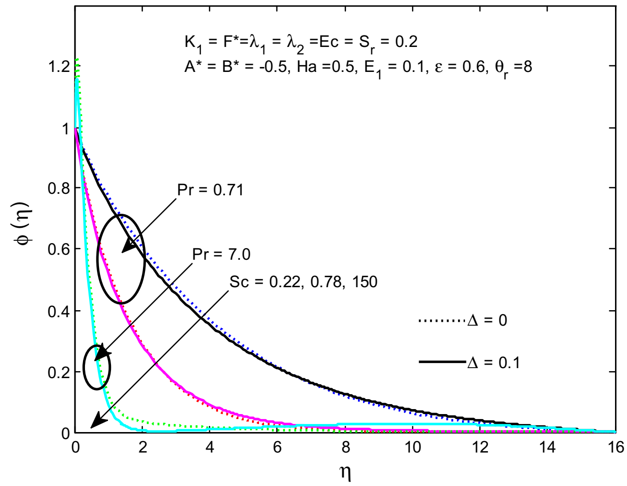

Figure 4 Variation of *Sc*, *Pr* and *Δ* on concentration profile.

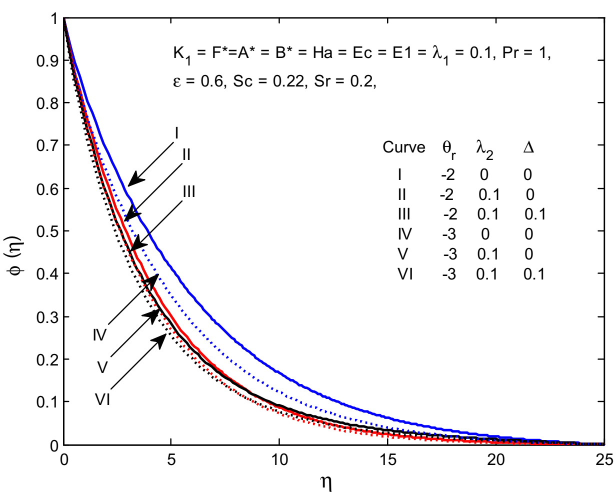

Figure 2 Variation of θ_r, λ_2 and Δ on concentration profile.

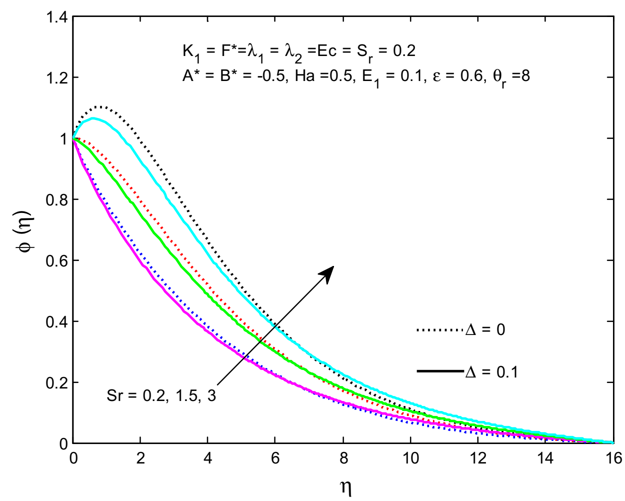

Figure 5 Variation of *Sr* and *Δ* on concentration profile.

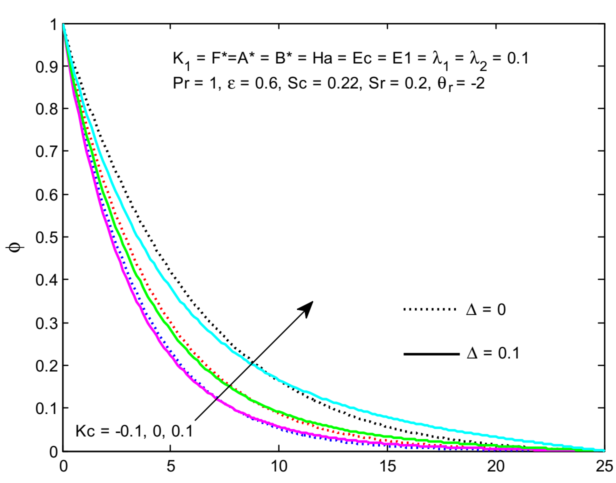

Figure 3 Variation of *Kc* and *Δ* on concentration profile.

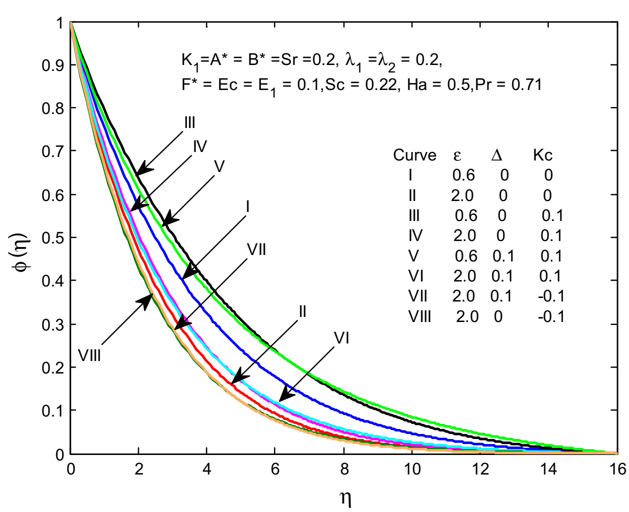

Figure 6 Variation of ε, *Δ* and *Kc* on concentration profile.

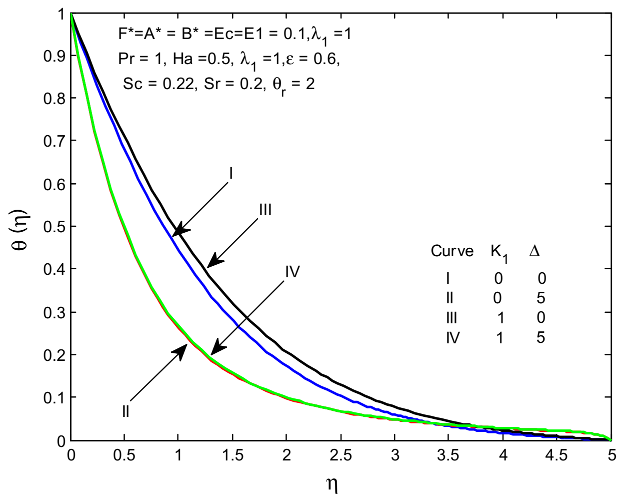

Figure 7 Variation of Δ and K_1 on temperature profile.

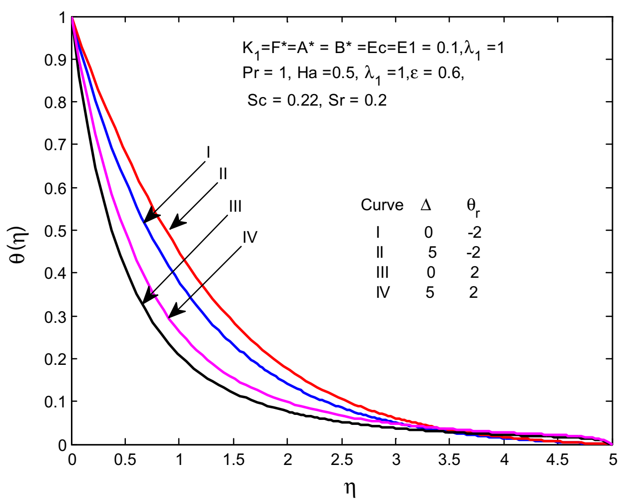

Figure 8 Variation of Δ and θ_r on temperature profile.

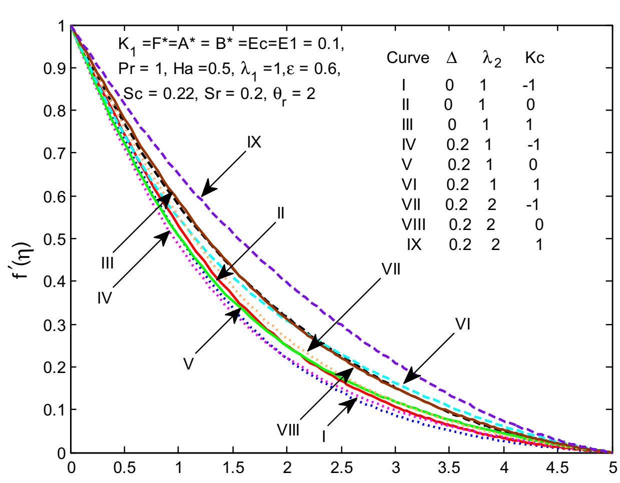

Figure 9 Variation of Δ, λ_2 and Kc on velocity profile.

Table 1 Comparison of local Nusselt number ($-\theta'(0)$) and Sherwood ($-\phi'(0)$) for various values of Ec, Pr, A^* and B^* for $Ha = \lambda_2 = 0$.

Ec	Pr	A^*	B^*	Abel [28]	Pal and Mondal [34]	Present
0.02	4	0.3	0.3	2.68986	2.694002	2.695353

isothermal flat plate in Newtonian fluid. Pantokratoras [18] made a theoretical study to investigate the effect of variable.

Concentration is a measure of how much of a given species is dissolved in another substance per unit volume. Concentration boundary layer manifests itself when species concentration difference exists between the solid–fluid interface and the free stream region of the fluid. The region in which the species concentration gradient exists is known as the concentration boundary layer. The species transfer takes place through the process of diffusion and convection, and is governed by the properties of the concentration boundary layer.

Many researchers [19,20] investigated the steady boundary layer flow of an incompressible viscous fluid over a linearly stretching plate and gave an exact similarity solution in a closed analytical form under various physical conditions. Bhukta et al. [21] have studied heat and mass transfer on MHD flow of a viscoelastic fluid through porous media over a shrinking sheet and Cortell [22,23], investigated the boundary layer flows over a non-linear stretching sheet. Van Gorder and Vajravelu [24] have given the flow geometries and the similarity solutions of the boundary layer equations for a non-linearly stretching sheet. Mahapatra et al. [25] investigated heat transfer due to magnetohydrodynamic stagnation-point flow of a power-law fluid toward a stretching surface in the presence of thermal radiation and suction/injection. Dessie and Kishan [26] have studied the MHD effects on heat transfer over stretching sheet embedded in porous medium with variable viscosity, viscous dissipation and heat source/sink.

Water is widely used as the cooling additives. In the rate of cooling, porous medium also plays a vital role. Eldabe and Mohamed [27] have obtained the solution for both heat and mass transfer in a MHD flow of a non-Newtonian fluid with a heat source over an accelerating surface through a porous medium. The rate of cooling also depends on the physical properties of the cooling medium but practical situation demands for physical properties with variable characteristics. Thermal conductivity is one of such properties, which is assumed to vary linearly with temperature. Some researchers [28–30] have studied the effect of variable thermal conductivity with temperature dependent heat source/sink.

In the present study we have considered mass transfer along with heat transfer on MHD mixed convective unsteady flow of an electrically conducting fluid over a stretching sheet embedded in a porous medium. In many industrial applications thermal diffusion is associated with mass diffusion if there is a difference in concentration of diffusive species. Many authors including Saville and Churchil [31,32] may be considered as the origin of the modern research on the effect of mass transfer on free convection flow. Further, Gebhart and Pera [33] studied the laminar flows which arise in the flow due to the concentration of the gravity force and density differences caused by simultaneous difference of thermal energy and chemical species neglecting the thermal diffusion and diffusion-thermo (Soret

Table 2 Skin friction coefficient, Nusselt number and Sherwood number for $\varepsilon = 0.6$, $\lambda_1 = \lambda_2 = 0.2$, $E_1 = 0.1 E_1 = 0.1 \theta_r = -1.2$, $Ha = 0.5$, $Sr = 0.1$, $Sc = 0.22$, $K_1 = 0.1$, $F^* = 0.1$.

Ec	Pr	A^*	B^*	Δ	E_1	Kc	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.005	3	0.3	0.3	0	0	0	−1.815	2.250425	−0.37117
0.005	3	0.3	0.3	0.3	0	0	−2.039	2.452314	−0.24258
0.01	3	0.3	0.3	0	0	0	−1.810	2.244578	−0.37001
0.01	3	0.3	0.3	0.3	0	0	−2.039	2.445284	−0.24128
0.02	4	0.3	0.3	0.3	0	0	−1.837	2.850788	0.324167
0.02	0.71	0.3	0.3	0.3	0	0	−1.927	1.069058	0.378021
0.02	0.71	0.3	0.3	0.3	0	1	−1.748	1.145011	−1.77905
0.02	0.71	0.5	0.3	0.3	0	1	−1.748	1.14666	−1.77966
0.02	0.71	0.3	0.5	0.3	0	1	−1.748	1.146375	−1.77909
0.02	0.71	0.3	0.3	0.3	0	−1	−1.944	1.062051	0.623843
0.02	0.71	−0.3	−0.3	0.3	0	1	−1.747	1.135315	−1.77705
0.005	3	0.3	0.3	0	0	1	−1.472	2.371753	−3.43853
0.005	3	0.3	0.3	0.3	0	1	−1.835	2.520743	−2.53824
0.01	3	0.3	0.3	0	0	1	−1.472	2.368435	−3.43761
0.01	3	0.3	0.3	0.3	0	1	−1.835	2.515896	−2.53717
0.01	3	0.3	0.3	0.3	2	1	−1.707	2.518621	−0.51339

and Dufour) because the level of species concentration is very low. Porous media is very widely used to insulate a heated body to maintain its temperature. They are considered to be useful in diminishing the natural free convection which would otherwise occur intensely on the heated surface. Further, the effect of free convection on the flow through porous medium plays an important role in agricultural engineering and petroleum industries in extracting pure petrol from the crude.

In practice the steady state condition is rarely experienced. Therefore, it is more realistic to include unsteadiness into the governing equations. Ishak et al. [34] studied the heat transfer associated with unsteady stretching permeable surface with prescribed wall temperature. Sharidan et al. [35] studied the unsteady boundary layer over a stretching sheet with a similarity analysis. Recently, Pal [36] analyzed the combined effect of thermal radiation and non-uniform heat source/sink on unsteady boundary layer flow of a viscous liquid and heat transfer over a permeable vertical surface.

Now, regarding the inclusion of Darcy dissipation which has not been considered as Pal and Mondal [37] it has been shown by Gebhart [38] that the viscous dissipative heat in the natural convective flow is important when the flow field is of extreme size or at extremely low temperature or in high gravity field. In such situation when we consider the flow through porous medium the Darcy dissipation term cannot be neglected in the energy equation because it is of the same order of magnitude with viscous dissipation term.

Further, the present work considers also the unsteady case of Pal and Mondal [37] in which they have considered the steady MHD non-Darcy mixed convective diffusion of species over a stretching sheet embedded in a porous medium with variable viscosity.

2. Mathematical formulation

Two-dimensional unsteady, incompressible electrically conducting viscous fluid over a stretching sheet embedded in a saturated non-Darcian porous medium on the plane $y = 0$ of a coordinate system is shown in flow geometry (Fig. 1). The flow is being confined to $y > 0$. Two equal and opposite forces are applied along x-axis so that the surface is stretched keeping the origin fixed. The fluid properties are assumed to be isotropic and constant, except the viscosity μ which varies as an inverse linear function of temperature.

We assumed that magnetic Reynolds number of the fluid is small so that induced magnetic field and Hall effect may be neglected. We take into account magnetic field effect as well as electric field in momentum. The fluid properties are assumed to be isotopic and constant, except for the fluid viscosity μ which is assumed to vary as an inverse linear function of temperature T. Following Lai and Kulachi [16] the temperature is in the form of

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}[1 + \gamma(T - T_\infty)] = a(T - T_r) \tag{1}$$

where $a = \frac{\gamma}{\mu_\infty}$ and $T_r = T_\infty - \frac{1}{\gamma}$. Both a and T_r are constant and their values depend on the reference state and the thermal property of the fluid, i.e. γ.

In general, $a > 0$ for liquids and $a < 0$ for gases, θ_r is a constant which is defined by

$$\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} = \frac{1}{\gamma(T_w - T_\infty)}$$

It is noteworthy that for $\gamma \to 0$ i.e. $\mu = \mu_\infty$ (constant) $\theta_r \to \infty$. It is also important to note that θ_r is negative for liquids and positive for gases.

Moreover, the flow domain is subject to uniform transverse magnetic fields $\vec{B_0} = (0, B_0, 0)$ and uniform electric field $\vec{E} = (0, 0, -E_0)$ (Fig. 1). Studies of MHD boundary layer flow over flat plates and in the saturated region of bodies with transverse magnetic field demonstrate that the magnetic field reduces skin friction and heat transfer and increase the shock detachment distances (Cramer and Pai [39]).

Now, applying $\nabla . \vec{B} = 0$ and $\nabla \times \vec{E} = 0$ and $\vec{J} = \sigma(\vec{E} + \vec{q} \times \vec{B})$ (Ohm's Law) provided magnetic field is not so strong (for validity of Ohm's law), and following Pal and Mondal [34] the governing boundary equations of momentum, energy and concentration for mixed convection under Boussinesq's approximation are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \quad (2)$$

$$\rho_\infty \frac{1}{\varepsilon^2}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\frac{1}{\varepsilon}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+\sigma(E_0B_0-B_0^2u) - \frac{\mu}{k}u-\left(\frac{C_b}{\sqrt{k}}\right)u^2+\rho_\infty g\beta_T(T-T_\infty)+\rho_\infty g\beta_c(C-C_\infty) \quad (3)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho_\infty C_P}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho_\infty C_P}\left(\frac{\partial u}{\partial y}\right)\rho^2 + \frac{\sigma}{\rho_\infty C_P}(uB_0-E_0)^2+\frac{1}{\rho_\infty C_P}q'''+\frac{\mu}{\rho_\infty C_p}\left(\frac{u^2}{\kappa'}\right) \quad (4)$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}+\frac{D_mk_T}{T_m}\frac{\partial^2 T}{\partial y^2}-K_c'(C-C_\infty) \quad (5)$$

The corresponding boundary conditions are

$$\left.\begin{array}{llll} u=U_w(x,t)=\frac{ax}{1-ct}, \; v=0, & T=T_w=T_\infty+\frac{bx}{1-ct}, & C=C_w=C_\infty+\frac{b^*x}{1-ct}, & \text{at } y=0 \\ u=0, & T\to T_\infty, & C\to C_\infty, & \text{as } y\to\infty \end{array}\right\} \quad (6)$$

To solve the governing boundary layer Eqs. (2)–(5), the following stream function and similarity transformations are introduced ([38]):

$$\psi(x,y,t)=\sqrt{\frac{a\upsilon_\infty}{1-ct}}xf(\eta),\;\; \eta=\sqrt{\frac{a}{\upsilon_\infty(1-ct)}}y,$$

$$u=\frac{ax}{1-ct}f'(\eta),\;\; v=\sqrt{\frac{a\upsilon_\infty}{1-ct}}f(\eta). \quad (7)$$

and $T_w(x,y,t)=T_\infty+\frac{bx}{1-ct},\;\; \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},$

$$C_w(x,y,t)=C_\infty+\frac{b^*x}{1-ct},\;\; \phi(\eta)=\frac{C-C_w}{C_w-C_\infty},$$

where b and b^* are constant with $b, b^* \geqslant 0$.

The non-uniform heat source/sink, q''' ([38]) is modeled as

$$q'''=\frac{ku_w(x)}{x\upsilon_\infty}[A^*(T_w-T_\infty)f'(\eta)+(T-T_\infty)B^*], \quad (8)$$

where A^* and B^* are the coefficients of space and temperature dependent heat source/sink respectively. Here we make a note that the case $A^*>0, B^*>0$ corresponds to internal heat generation and that $A^*<0, B^*<0$ corresponds to internal heat absorption.

The flow is caused by the stretching of the sheet which moves in its own plane with the surface velocity $U_w(x,t)=\frac{ax}{1-ct}$ where a (stretching rate) and c are positive constants having dimension time^{-1} (with $ct<1, c\geqslant 0$). It is noted that the stretching rate $\frac{a}{1-ct}$ increases with time since $a>0$. The surface temperature and concentration of the sheet vary with distance x from the slot and time t in the form $T_w(x,t)=T_\infty+\frac{bx}{1-ct}$ and $C_w(x,t)=C_\infty+\frac{b^*x}{1-ct}$.

Substituting (7) and (8) into the governing Eqs. (3)–(5) and using the above relations we finally obtain a system of non-linear ordinary differential equations with appropriate boundary conditions

$$\frac{f'''}{\varepsilon}+\left(1-\frac{\theta}{\theta_r}\right)\left(\frac{ff''}{\varepsilon^2}-\Delta\left(f'+\frac{\eta}{2}f''\right)\right)+\frac{1}{\theta_r-\theta}\frac{\theta'f''}{\varepsilon} + \left(1-\frac{\theta}{\theta_r}\right)Ha^2(E_1-f')=\left(1-\frac{\theta}{\theta_r}\right)\left(\frac{f'^2}{\varepsilon^2}+F^*f'^2-\lambda_1\theta-\lambda_2\phi\right) + K_1f' \quad (9)$$

$$\theta''-\left(1-\frac{\theta}{\theta_r}\right)Pr\left[(f'\theta-f\theta')-\Delta\left(\theta+\frac{\eta}{2}\theta'\right)-Ha^2E_c(E_1-f')^2\right] = -E_cPr((f'')^2+K_1(f')^2-(A^*e^{-\eta}+B^*\theta)) \quad (10)$$

$$\phi''+Sc\left((f\phi'-f'\phi)-\Delta\left(\phi+\frac{\eta}{2}\phi'\right)\right)=-ScSr\theta''+KcSc\phi \quad (11)$$

The boundary condition (6) becomes

$$\left.\begin{array}{llll} f(0), f'(0)=1, & \theta(0)=1, & \phi(0)=1, & \text{at } \eta=0 \\ f'(\infty)=0, & \theta(\infty)=0, & \phi(\infty)=0 & \text{as } \eta\to\infty \end{array}\right\} \quad (12)$$

$$\left.\begin{array}{llll} K_1=\frac{\upsilon_\infty(1-ct)}{ak}, & Ha^2=\frac{\sigma B_0^2(1-ct)}{a\rho_\infty}, & E_1=\frac{E_0(1-ct)}{B_0ax}, & F^*=\frac{C_b}{\sqrt{k}}x, \\ \lambda_1=\frac{g\beta_Tb(1-ct)}{a^2}, & Kc=\frac{Kc^*(1-ct)}{a}, & \lambda_2=\frac{g\beta_cb^*(1-ct)}{a^2}, & Pr=(1-\frac{\theta}{\theta_r})^{-1}Pr_\infty \\ Pr_\infty=\frac{\rho_\infty\upsilon_\infty C_p}{k}, & E_c=\frac{a^2x}{C_pb(1-ct)}, & Sc=\frac{\upsilon_\infty}{D_m}, & Sr=\frac{D_mk_T}{\upsilon_\infty T_m} \end{array}\right\}$$

The physical quantities of interest are skin friction C_f, Nusselt number Nu_x and Sherwood number Sh_x, which are defined as

$$C_f=\frac{\tau}{\rho U_w^2/2},\;\; Nu_x=\frac{xq_w}{k(T_w-T_\infty)},\;\; Sh_x=\frac{xq_m}{D_m(C_w-C_\infty)} \quad (13)$$

where surface shear stress, surface heat and mass flux are defined as

$$\tau=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0},\;\; q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0},\;\; q_m=-D_m\left(\frac{\partial C}{\partial y}\right)_{y=0}. \quad (14)$$

Using the non-dimensional variables (7), we get from Eqs. (13) and (14) as

$$\frac{C_f}{Re_x^{1/2}}=f''(0),\;\; \frac{Nu_x}{Re_x^{1/2}}=-\theta'(0),\;\; \frac{Sh_x}{Re_x^{1/2}}=-\phi'(0), \quad (15)$$

where $Re_x=\frac{xU_w(x)}{\upsilon_\infty}$.

In the case when $\varepsilon=1, \theta_r\to\infty, \Delta=Ha=\lambda=K_1=F^*=0$, then upon substitution, the governing equations and the boundary conditions reduced to

$$f'''(\eta)+f(\eta)f''(\eta)-(f'(\eta))^2=0 \quad (16)$$

$$f(0)=0, f'(0)=1 \text{ and } f'(\eta)\to 0 \text{ as } \eta\to\infty. \quad (17)$$

The exact solution of (16) and (17) is (see [25])

$$f(\eta)=1-e^{-\eta} \quad (18)$$

The existence and uniqueness of a more general form of Eq. (16) i.e.

$$f'''(\eta)+f(\eta)f''(\eta)-\rho(f'(\eta))^2=0, \quad (19)$$

is associated with boundary condition (17), when $\rho=\frac{2n}{(n+1)}$; which holds for all values of n, namely $n\in\left(-\frac{1}{3},\infty\right)$. It is to note that when $n=-\frac{1}{3}$, the exact solution, has the form

$$f(\eta) = \sqrt{2}\ \tan h\left(\frac{1}{\sqrt{2}}\eta\right).$$

However, for the governing Eqs. (9)–(11) associated boundary condition has no exact solution. Hence these equations are solved numerically.

3. Results and discussion

Main objective of the following discussion is to bring out the effects of additional parameters introduced in the present study such as unsteadiness parameter (Δ), mass buoyancy (λ_2), Darcy dissipation (K_1) and chemical reaction (K_c) parameters besides the other parameters appear in the convective diffusion problem. The case of Pal and Mondal [37] can be retrieved when $\Delta = \lambda_2 = k_1 = K_c = 0$.

The unsteady solutal concentration profiles for liquid medium ($\theta_r < 0$) are shown in Fig. 2. The solutal concentration decreases in the presence of mass buoyancy ($\lambda_2 = 0.1$) and it is further reduced by unsteady parameter (Δ). From curve VI, it is clear that concentration level of the diffusing species decreases with decrease in thermal property (θ_r), mass buoyancy (λ_2) and unsteady parameter (Δ).

Fig. 3 shows the effect of chemical parameter (Kc). It is seen that the solutal concentration increases in the presence of destructive reaction ($Kc > 0$) but decreases in the presence of constructive reaction ($Kc < 0$). The unsteady parameter slightly decreases it.

Fig. 4 shows three layers distribution depending upon the value of Sc. While there is an increase in Sc i.e. in case of heavier diffusing species, depletion of concentration level becomes faster and hence sharp fall of concentration is indicated ($Sc = 150$).

An increase in Soret number increases the unsteady concentration level in all the layers. One interesting point is to note that for moderately high value of Sr, a hike in concentration level is marked in the layers close to the plate (see Fig. 5).

Fig. 6 shows the effect of porosity parameter, ε on concentration distribution. It is seen that as the porosity of the medium increases the concentration level decreases in both steady (Curves I and II) and unsteady cases (Curves V and VI). Further, it is remarked that destructive reaction increases the steady state concentration level. From the curves VI and VII it is reported that in the presence of constructive reaction ($Kc < 0$), concentration level decreases at all the points and further decrease is measured in the corresponding steady case.

Fig. 7 displays the temperature variation in respect of Darcy dissipation K_1 and unsteady parameter Δ. It is clearly seen that unsteadiness decreases the temperature resulting in a thinner boundary layer but the reverse effect is observed in case of Darcy dissipation.

Fig. 8 shows the effect of variable viscosity for liquid ($\theta_r < 0$) and gaseous medium ($\theta_r > 0$). It is interesting to note that variable viscosity for liquid reduces the temperature in both steady and unsteady cases i.e. gives rise to thinner boundary layer than the gaseous medium.

Fig. 9 shows the velocity variation for both steady and unsteady cases, in the presence of mass bouncy. The constructive reaction ($Kc < 0$) decreases the velocity for both steady (Curves I and II) and unsteady (Curves IV and V) cases, whereas destructive reaction (Curves V and VI) and mass buoyancy (Curves IV and VII) increase the velocity. It is also remarked that an increase in time parameter decreases the velocity at all points (Curves III and VI). Further, $\lambda_2 > 0$, i.e. ($C_w > C_\infty$) represents the case of mass diffusion from the plate surface to the ambient state. It is interesting to note that when $\lambda_2 > 0$, a convection current sets into increase the velocity from the state of without mass buoyancy ($\lambda_2 = 0$).

Table 1 compares the case of Abel [28], Pal and Mondal [37] with the present study for specific values of Ec, Pr, A^* and B^* for single case due to the paucity of the available data. This shows a good agreement.

Table 2 shows the effect of various parameters affecting the rates of heat transfer and mass transfer at the plate. The values of $Ec = 0.01$ and 0.005 are the representative of the liquid metals. $Pr = 3.0$ represents saturated liquid Freon at 273.3 K (Cramer and Pai [39]). From the numerical values it is observed that unsteadiness of the flow enhances the rate of heat and mass transfer at the plate. Further, it is seen that for both steady and unsteady flows an increase in Ec decreases the rate of heat transfer (Nusselt number) as the increasing Ec leads to more heat energy is stored in the fluid due to frictional heating thereby decreasing the rate of heat transfer at the plate but increasing slightly rate of mass transfer (Sherwood number) irrespective of the absence/presence of chemical reaction. It is of the interest to note that effect of increasing Pr is to increase the Nusselt number since an increase in Pr leads to slow rate of diffusion but the reverse effect is observed in case of Sherwood number. Another interesting point is that for constructive reaction ($Kc < 0$) the rate of mass transfer assumes positive value whereas in case of destructive reaction it is negative. The reverse trend is due to exothermic/endothermic reaction. Thus, it is inferred that nature of chemical reaction plays a crucial role in the mass transfer at the plate. Th effects of space (A^*) and temperature (B^*) dependent heat source/sink are shown in Table 2. It shows that an increase in A^*, increases Nusselt number but increase in B^*, causes a slight decrease and effect of A^* and B^* on rate of mass transfer is insignificant. The last two lines of the table show the effect of electric field on Nusselt and Sherwood number. It is seen that presence of electric field enhances the rate of heat transfer slightly but rate of mass transfer significantly. Therefore, it is concluded that electric field regulates the rate of solutal concentration of the reacting species at the plate.

The skin friction is the measure of shear stress at the plate. It is observed that unsteadiness of the flow contributes to the reduction of coefficient of skin friction. Moreover, the destructive chemical reaction, electromagnetic field and Eckert number increase the skin friction for both steady and unsteady cases. Further, it is seen that an increase in Pr decreases it. Therefore, choice of fluid of higher Prandtl number is suitable for the reduction of coefficient of skin friction which is desirable.

4. Conclusion

The contributions of the additional parameters appeared in the governing equations are mass buoyancy (λ_2), Darcy dissipation (K_1), chemical reaction (Kc) and unsteady parameter (Δ).

The unsteadiness of the flow phenomena is being measured by Δ ($\Delta = c/a$). For steady flow, $c = 0$ and $a \neq 0 \Rightarrow \Delta = 0$. The effect of unsteady parameter reduces the concentration level, temperature and velocity at all points in comparison with its counterpart (steady flow), in the domain of respective

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boundary layers. The solutal concentration for the gaseous medium is higher than the liquid medium.

- The constructive reaction and unsteady parameter contribute to thinning of the velocity boundary layer whereas mass buoyancy and destructive reaction have a reverse effect.
- Thinning of boundary layer is affected by heavier diffusing species.
- Thinning of thermal boundary layer occurs under the influence of increasing wall concentration and variable viscosity.
- The effect of solutal convection current gives rise to mass absorption.
- Chemical reaction plays a vital role for the structure of solutal/thermal boundary layer.
- The higher Soret number with destructive chemical reaction and upstream solutal convective current is favorable for the growth of solutal boundary layer.
- Choice of fluid of higher Prandtl number is suitable for the reduction of coefficient of skin friction which is desirable.
- Effect of diffusive medium (liquid) greatly contributes to the rate of mass transfer at the stretching surface.

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