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Cosmology of nonlinear oscillations

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Abstract

The nonlinear oscillations of a scalar field are shown to have cosmological equations of state with $w = p/\rho$ ranging from $-1 < w < 1$. We investigate the possibility that the dark energy is due to such oscillations.

Astrophysical data support the existence of dark energy [1,2]. Since many proposed solutions of the cosmological constant problem lead to exactly zero vacuum energy for empty space, it is natural to consider so-called quintessence models in which the dark energy is comprised of some scalar field which is slowly evolving towards its minimum [2]. The main objections to these models are their typically unnatural potentials, and that they require the suppression of higher dimension operators likely to be induced by quantum gravity [3].

In this Letter we investigate a qualitatively different idea: that the dark energy is due to (possibly rapid) nonlinear oscillations rather than slow evolution on cosmological timescales. We consider oscillations in scalar models

$$\mathcal{L} = \frac{1}{2}(\partial_\mu z)^2 - V(z), \tag{1}$$

where the potential $V(z) = a|z|^l$ near its minimum. Potentials with $l < 2$ are particularly interesting, as we will see below that they yield an equation of state $w = p/\rho < 0$. This form of $V(z)$ may appear odd, but a change of variables to, e.g., $\phi = |z|^{l/2}$ yields

$$\mathcal{L} = \frac{1}{2}K(\phi)(\partial_\mu \phi)^2 - a\phi^2, \tag{2}$$

with $K(\phi) = (2/l)^2\phi^{(4-2l)/l}$. A kinetic term of this type can be obtained from the Kähler potential in supersymmetric models. We focus here on the classical behavior of z, but its quantization when $l < 2$ warrants further investigation. It has the unusual property that there are no perturbative degrees of freedom—that is, small oscillations about $z = 0$ have infinite frequency, since $V''(z = 0)$ does not exist. Only large (non-infinitesimal) z oscillations can have finite energy density. This may lead to a number of interesting features: one might expect that z decay and production rates, as well as radiative corrections, only arise from nonperturbative effects and are exponentially small.

The redshift of a field undergoing nonlinear oscillations can be calculated through its average equation of state and depends on the ratio $w = p/\rho$. From the scalar field equations in a Robertson–Walker universe,

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one obtains

$$\rho(t) = \rho_0 \left(\frac{R_0}{R(t)} \right)^{3(1+w)}, \quad (3)$$

where R is the Robertson–Walker scale factor. The scalar field energy redshifts like radiation when $w = 1/3$, like matter when $w = 0$, etc.

It remains to calculate the equation of state for nonlinear oscillations. We note that $p = T - V$ and $\rho = T + V$, where T is the kinetic energy density and V is the potential energy density. We calculate the relation between T and V averaged over an oscillation period, which is much smaller than the cosmological timescale except during the very first oscillations, which begin when the age of the universe is of order $V''(z)^{-1/2}$.

We define

$$\langle T \rangle = \frac{1}{2} \int dt\, \dot{z}^2 \quad (4)$$

and

$$\langle V \rangle = \int dt\, z^l, \quad (5)$$

where each integral is taken over the same period with boundary conditions $\dot{z}(0) = \dot{z}(\tau) = 0$ (or equivalently $z = 0$ at the endpoints) and we have adopted units in which the overall scale of the potential is unity. Since

$$\frac{d}{dt}(z\dot{z}) = \dot{z}^2 + z\ddot{z}, \quad (6)$$

and the equation of motion is $\ddot{z} = -l\, z^{l-1}$, we can rewrite the average potential energy as

$$\langle V \rangle = \frac{1}{l} \int dt\, \dot{z}^2 = \frac{2}{l} \langle T \rangle. \quad (7)$$

This yields

$$w = \frac{(l-2)}{(l+2)}, \quad (8)$$

with $-1 < w < 1$. In potentials with $l < 2$, the average potential energy dominates over the kinetic part, and the pressure is negative. In the limit $l \to 0$ these oscillations behave like a cosmological constant. In higher order potentials ($l > 2$), the situation is reversed, leading asymptotically to $w = 1$ as $l \to \infty$, or $\rho \sim 1/R^6$. These oscillations redshift away rapidly, although it was noted in [4] that the large l behavior of $w = 1$ can never be achieved, due to an instability to a nonoscillatory scaling solution.

Given a periodic solution to the z equations of motion, one can obtain a rescaled solution via

$$z(t) \to a^{2/(l-2)} z(at). \quad (9)$$

For $l < 2$, the frequency of oscillation goes to infinity as the amplitude goes to zero. Note, however, that the average energy density goes to zero in this limit.

An advantage of nonlinear oscillation models of quintessence is that the potential $V(z)$ need not be characterized by the size of the current dark energy density $\sim (10^{-3}\ \text{eV})^4$, nor be fine-tuned to be flat (i.e., have curvature of order the inverse horizon size squared $\sim (10^{-33}\ \text{eV})^2$). Rather, the potential can be characterized by a larger energy scale more familiar to particle physics, with no small dimensionless parameters. The smallness of the energy density today relative to this scale could be explained by a small z oscillation amplitude.

One scenario that would result in a small oscillation amplitude is if the original energy density in the z field were diluted away by inflation, and the subsequent reheat temperature insufficient to repopulate it. This is quite plausible if the couplings between the inflaton and ordinary matter fields to the z field are small, for example suppressed by the Planck scale if z is a hidden sector field.[1] The current z energy density is dependent on the number of e-foldings N during inflation

$$\rho_{\text{today}} \sim \rho_i \left[2 \times 10^{-5} e^{-N} \left(\frac{T_{\text{md}}}{T_{\text{rh}}} \right) \right]^{\nu}, \quad (10)$$

where $\nu = 3(1+w)$, $T_{\text{md}} \sim 5$ eV is the temperature at which the universe becomes matter dominated and T_{rh} is the reheat temperature after inflation. For example, using $\nu = 1/2$ (or $w = -5/6$, consistent with the WMAP limit of $w < -0.78$ [1]), $T_{\text{rh}} = 5 \times 10^{10}$ GeV and ρ_i the Planck energy density, we find that $N \sim 510$ in order that $\rho_{\text{today}} \sim (10^{-3}\ \text{eV})^4$. For smaller $\rho_i \sim (10^{11}\ \text{GeV})^4$, appropriate for intermediate scale inflation, we find $N \sim 370$. Note, while the scenario described here explains the small dark energy density

[1] Some reheating of the z field is inevitable, even if its couplings to the inflaton are very small. However, the thermal z bosons produced do not necessarily contribute to the coherent oscillations studied here—their energy redshifts away more rapidly.

today, it does not address the question of coincidence: why is ρ_z of order ρ_{critical} today?

If the energy scale characterizing $V(z)$ is small (within several orders of magnitude of an electron volt) little dilution is necessary, and it could be provided by the expansion of the universe after inflation. If z originates from the scalar component of a chiral superfield Φ, the energy scale of $V(z)$ (i.e., the parameter a in (2)) is protected by supersymmetry from radiative corrections. A Yukawa coupling between z and its superpartner can be excluded by imposing $\Phi \to -\Phi$ symmetry in the superpotential, thereby stabilizing z.

To conclude, we find that realistic quintessence models based on nonlinear oscillations can be constructed without any fine-tuning of fundamental parameters. These models will be disfavored if future data show that $w = -1$.

Note added

After this work was completed, we learned that the result (8) for the equation of state was previously derived by Turner in [5]. Also, models based on a hyperbolic cosine potential raised to a fractional power have been considered in [6], which at late times exhibit oscillations of the type considered here.

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