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Vector F-implicit complementarity problems in topological vector spaces

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Abstract

Recently, Huang and Li [J. Li, N.J. Huang, Vector F-implicit complementarity problems in Banach spaces, Appl. Math. Lett. 19 (2006) 464–471] introduced and studied a new class of vector F-implicit complementarity problems and vector F-implicit variational inequality problems in Banach spaces. In this work, we study this class in topological vector spaces and drive some existence theorems for the vector F-implicit variational inequality and vector F-implicit complementarity problem. Also, their equivalence is presented under certain conditions.

Keywords: Vector F-implicit complementarity problems; Vector F-implicit variational inequalities; KKM-map; Positively homogeneous map; Topological vector space

1. Introduction and preliminaries

Vector variational inequalities were first introduced and studied by Giannessi [4] in the setting of finite-dimensional Euclidean spaces. There are generalizations of scalar variational to the vector case. Vector variational inequalities have many applications in vector optimization, approximate vector optimization, and other areas (see [5]).

In 2001, Yin et al. [12] introduced a class of F-complementarity problems (F-CP), which consist in finding $x \in K$ such that

$$\langle Tx, x\rangle + F(x) = 0, \qquad \langle Tx, y\rangle + F(y) \geq 0, \quad \forall y \in K,$$

where X is a Banach space with topological dual X^*, and $\langle \cdot, \cdot\rangle$ duality pairing between them, K a closed convex cone of X, and $T : K \to X^*$, $F : K \to \mathbb{R}$. They obtained an existence theorem for solving (F-CP) and also proved that if F is positively homogeneous (i.e. $F(tx) = tF(x)$ for all $t > 0$ and $x \in K$) and convex, the problem (F-CP) is equivalent to the following generalized variational inequality problem (GVIP) which consists in finding $x \in K$ such that

$$\langle Tx, y - x\rangle + F(y) - F(x) \geq 0, \quad \forall y \in K.$$

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In 2003, Fang and Huang [3] introduced a new class of vector F-complementarity problems with demipseudomonotone mappings in Banach spaces. They presented the solvability of this class of vector F-complementary problems with demipseudomonotone mappings and finite-dimensional continuous mappings in reflexive Banach spaces. Later, Huang and Li [6] introduced and studied a new class of (scalar) F-implicit complementarity problems and F-implicit variational inequality problems in Banach spaces. They obtained some existence theorems for F-implicit complementarity and F-variational problems. Also, under special assumptions, they established the equivalence between F-implicit complementarity and F-variational problems. Recently, in [7], they extended those problems to a vector valued setting.

In this work our aim is to generalize some results of [7] to topological vector spaces under certain weaker conditions. We first consider the following vector F-implicit variational inequality (in short, VF-IVIP). Find $x \in K$ such that

$$\text{(VF-IVIP)} \quad \langle Tx, y - x\rangle + F(y) - F(x) \in C(x), \quad \forall y \in K,$$

and the second problem which we study, is called vector F-implicit complementarity problem (in short, VF-ICP) which consists of finding $x \in K$ such that

$$\text{(VF-ICP)} \quad \langle Tx, x\rangle = 0, \qquad \langle Tx, y\rangle + F(y) \in C(x), \quad \forall y \in K,$$

where X, Y are topological vector spaces, K is a nonempty convex subset of X, $C : K \to 2^Y$ a multi-valued map with convex cone values, $T : K \to L(X, Y)$, and $F : K \to Y$.

In the rest of this section, we recall some definitions and preliminary results which are used in next sections.

We shall denote by 2^A the family of all subsets of A and by $\mathcal{F}(A)$ the family of all nonempty finite subsets of A. Let X be a real Hausdorff topological vector space (in short, t.v.s.). A nonempty subset P of X is called convex cone if (i) $P + P = P$, (ii) $\lambda P \subset P$, for all $\lambda \geq 0$. Let Y be a t.v.s. and $P \subset Y$ be a cone. The cone P induces an order in Y (in this case the pair (Y, P) is called an ordered t.v.s.) which is defined as follows:

$$x \leq y \Leftrightarrow y - x \in P.$$

This ordering is anti-symmetrical if P is pointed. Let X and Y be two t.v.s., K a nonempty subset of X, and $C : K \to 2^Y$ a multi-valued map with nonempty convex cone values.

We say that $f : K \times K \to Y$ is vector C-upper semicontinuous (C-u.s.c.) in the first variable, if the set $\{x \in K : f(x, y) \in C(x)\}$ is closed in K, for every $y \in K$. This definition reduces to vector 0-u.s.c., if $C(x) = P$ for every $x \in K$, where P is a constant convex cone.

Let X be a nonempty set, Y a topological space, and $\Gamma : X \to 2^Y$ a multi-valued map. Then, Γ is called transfer closed-valued if, for every $y \notin \Gamma(x)$, there exists $x' \in X$ such that $y \notin \mathrm{cl}\Gamma(x')$, where cl denotes the closure of a set. It is clear that, $\Gamma : X \to 2^Y$ is transfer closed-valued if and only if

$$\bigcap_{x\in X} \Gamma(x) = \bigcap_{x\in X} \mathrm{cl}\Gamma(x).$$

If $B \subseteq Y$ and $A \subseteq X$, then $\Gamma : A \to 2^B$ is called transfer closed-valued if the multi-valued mapping $x \to \Gamma(x) \cap B$ is transfer closed-valued. In this case where $X = Y$ and $A = B$, Γ is called transfer closed-valued on A.

Let K be a nonempty convex subset of a t.v.s. X and let K_0 be a subset of K. A multi-valued map $\Gamma : K_0 \to 2^K$ is said to be a KKM map if

$$\mathrm{co}A \subseteq \bigcup_{x\in A} \Gamma(x), \quad \forall A \in \mathcal{F}(K_0),$$

where co denotes the convex hull.

In the next section, we need the following theorem.

Theorem 1.1 ([2]). *Let X be a t.v.s. and K be a nonempty convex subset of X. Suppose that $\Gamma, \widehat{\Gamma} : K \to 2^K$ are two multivalued mappings such that:*

(i) $\widehat{\Gamma}(x) \subseteq \Gamma(x), \forall x \in K$;
(ii) $\widehat{\Gamma}$ *is a KKM map;*

(iii) *for each* $A \in \mathcal{F}(K)$, Γ *is transfer closed-valued on* $\mathrm{co}A$*;*
(iv) *for each* $A \in \mathcal{F}(K)$, $\mathrm{cl}_K(\bigcap_{x\in \mathrm{co}A}\Gamma(x))\bigcap \mathrm{co}A = (\bigcap_{x\in \mathrm{co}A}\Gamma(x))\bigcap \mathrm{co}A$*;*
(v) *there is a nonempty compact convex set* $B \subseteq K$ *such that* $\mathrm{cl}_K(\bigcap_{x\in B}\Gamma(x))$ *is compact.*

Then, $\bigcap_{x\in K}\Gamma(x) \neq \emptyset$.

2. Main results

Throughout this section, let X and Y be real Hausdorff t.v.s. and K be a nonempty convex subset of X. Denote by $L(X, Y)$ the space of all continuous linear mappings from X into Y, and $\langle t, x\rangle$ be the value of the linear continuous mapping $t \in L(X, Y)$ at x. Suppose that $C : K \to 2^Y$ is a multivalued map with nonempty convex cone values, $f : K \to L(X, Y)$, $g : K \to K$ and $F : K \to Y$. We consider the following vector F-implicit complementarity problem (VF-ICP).

Find $x \in K$ such that

$$\langle f(x), g(x)\rangle + F(g(x)) = 0 \quad \text{and} \quad \langle f(x), y\rangle + F(y) \in C(x), \quad \forall y \in K.$$

The above problem reduces to vector F-implicit complementarity problem considered in [7] for the case $C(x) = P$, where (Y, P) is an ordered t.v.s. and P is a convex cone subset of K.

Examples of (VF-ICP) in t.v.s.
(1) If g is an identity mapping on K, then (VF-ICP) reduces to the vector F-complementary problem (in short VF-CP) which consists in finding $x \in K$ such that:

$$\langle f(x), x\rangle + F(x) = 0 \quad \text{and} \quad \langle f(x), y\rangle + F(y) \in C(x), \quad \forall y \in K.$$

(2) If $F = 0$, then (VF-CP) reduces to the vector complementary problem (in short, VCP) which consists in finding $x \in K$ such that:

$$\langle f(x), x\rangle = 0 \quad \text{and} \quad \langle f(x), y\rangle \in C(x), \quad \forall y \in K,$$

which has been studied by Chen and Yang [1], and Yang [11] in particular case $C(x) = P, \forall x \in K$.
(3) If $L(X, Y) = X^*$ and $F : K \to \mathbb{R}$, then (VF-ICP) reduces to the F-implicit complementary problems (in short, F-ICP) which consists of finding $x \in K$ such that:

$$\langle f(x), g(x)\rangle + F(g(x)) = 0 \quad \text{and} \quad \langle f(x), y\rangle + F(y) \in C(x), \quad \forall y \in K$$

which were considered by Huang and Li [6] in the particular case, where $C(x) = P, \forall x \in K$.
(4) If g is the identity mapping, then (F-ICP) reduces to the F-complementary problem (in short, F-CP) which consists in finding $x \in K$ such that:

$$\langle f(x), x\rangle + F(x) = 0 \quad \text{and} \quad \langle f(x), y\rangle + F(y) \in C(x), \quad \forall y \in K,$$

which was studied by Yin et al. [12] in the particular case, where $C(x) = P, \forall x$.
(5) If $F = 0$, then (F-ICP) reduces to the implicit complementary problem (in short ICP) which consists in finding $x \in K$ such that:

$$\langle f(x), g(x)\rangle = 0 \quad \text{and} \quad \langle f(x), y\rangle \in C(x), \quad \forall y \in K,$$

which has been studied by Isac [9,10].
(6) If g is the identity mapping and $F = 0$, then (F-ICP) reduces to the complementary problem (in short, CP) which consists in finding $x \in K$ such that:

$$\langle f(x), x\rangle = 0 \quad \text{and} \quad \langle f(x), y\rangle \in C(x), \quad \forall y \in K,$$

which has been studied by many authors, (for instance, see [10]). If $X = X^* = \mathbb{R}^n$, then (CP) becomes the classical complementary problem.

We also introduce the following vector F-implicit variational inequality problem (in short VF-IVIP) which consists in finding $x \in K$ such that

$$\langle f(x), y - g(x)\rangle + F(y) - F(g(x)) \in C(x), \quad \forall y \in K.$$

This problem is a generalization of the problem (VF-IVIP) introduced in [7] in a Banach space setting.

Remark 2.1. Any solution of (VF-ICP) is a solution of (VF-IVIP). The following theorem says that the converse holds if F is positively homogeneous; the proof is similar to Theorem 3.1 in [7] and thus will be omitted.

Theorem 2.2. *If $F : K \to Y$ is positively homogeneous, then* (VF-IVIP) *and* (VF-ICP) *are equivalent.*

The following example shows that if F is not positively homogenous, the conclusion of Theorem 2.2 may be incorrect:

Example 2.3. Let $X = Y = \mathbb{R}$, $K = [0, +\infty)$, $g(x) = 0$, $F(x) = 1$, and $C(x) = [0, +\infty)$, for all $x \in K$. Define $f : K \to \mathbb{R}$ (note that $L(\mathbb{R}, \mathbb{R}) \equiv \mathbb{R}$) by

$$f(x) = \begin{cases} 0 & \text{if } x = 0, \\ -1 & \text{otherwise.} \end{cases}$$

Obviously, $x = 0$ is a solution of (VF-IVIP) but is not a solution of (VF-ICP).

In Theorem 2.2, if g is the identity mapping, then we have the following corollary:

Corollary 2.4. *Let $F : K \to Y$ be positively homogeneous. Then any solution of* (VF-VIP) *is a solution for* (VF-CP).

The following theorem provides an existence result for the (VF-IVIP) in t.v.s. which improves Theorem 3.2. in [7].

Theorem 2.5. *Assume that:*

(a) *the function $G : \text{co}A \times \text{co}A \to Y$ where,*

$$G(x, y) = \langle f(x), y - g(x)\rangle + F(y) - F(g(x))$$

is C-u.s.c. in the first variable, $\forall A \in \mathcal{F}(K)$;

(b) *let $A \in \mathcal{F}(K)$, $x, y \in \text{co}A$. If (x_α) is any net on K converging to x then,*

$$\langle f(x_\alpha), tx + (1 - t)y - g(x_\alpha)\rangle + F(tx + (1 - t)y) - F(g(x_\alpha)) \in C(x_\alpha), \quad \forall t \in [0, 1]$$

implies

$$\langle f(x), y - g(x)\rangle + F(y) - F(g(x)) \in C(x).$$

(c) *There exists a mapping $h : K \times K \to Y$ such that:*
  (i) *$h(x, x) \in C(x), \forall x \in K$;*
  (ii) *$\langle f(x), y - g(x)\rangle + F(y) - F(g(x)) - h(x, y) \in C(x), \forall x \in K, \forall y \in K$;*
  (iii) *the set $\{y \in K : h(x, y) \notin C(x)\}$ is convex, $\forall x \in K$;*

(d) *there exist a nonempty compact subset B and a nonempty convex compact subset D of K such that, for each $x \in K \setminus B$, there exists $y \in D$ such that $\langle f(x), y - g(x)\rangle + F(y) - F(g(x)) \notin C(x)$.*

Then (VF-IVIP) *has a solution. Moreover, the solution set of* (VF-IVIP) *is compact.*

Proof. We define $\Gamma, \widehat{\Gamma} : K \to 2^K$ as follows:

$$\Gamma(y) = \{x \in K : \langle f(x), y - g(x)\rangle + F(y) - F(g(x)) \in C(x)\},$$
$$\widehat{\Gamma}(y) = \{x \in K : h(x, y) \in C(x)\}.$$

We show that $\Gamma, \widehat{\Gamma}$ satisfy conditions of Theorem 1.1. From assumption (ii) of (c), $\widehat{\Gamma}(y) \subseteq \Gamma(y)$, for all $y \in K$. If $A = \{x_1, x_2, \ldots, x_n\} \subseteq K, z \in \text{co}A$ and $z \notin \cup_{i \in \{1,2,\ldots,n\}} \widehat{\Gamma}(x_i)$, then $h(z, x_i) \notin C(z)$ for $i = 1, 2, 3, \ldots, n$. It follows by (c)(iii) that, $h(z, z) \notin C(z)$ contradicting (c)(i). So $\widehat{\Gamma}$ is a KKM map. Let $A \in \mathcal{F}(K)$, $x \in \text{co}A$ and

$(x_\alpha) \in \Gamma(x) \cap \mathrm{co}A$ converges to z. Then, $\langle f(x_\alpha), x - g(x_\alpha)\rangle + F(y) - F(g(x_\alpha)) \in C(x_\alpha)$. By (a), we conclude that $z \in \Gamma(x) \cap \mathrm{co}A$. Since x is an arbitrary element of co$A$, we obtain:

$$\bigcap_{x\in \mathrm{co}A} \Gamma(x) \cap \mathrm{co}A = \bigcap_{x\in \mathrm{co}A} \mathrm{cl}(\Gamma(x) \cap \mathrm{co}A).$$

Similarly, using (b) we get:

$$\bigcap_{x\in \mathrm{co}A} \Gamma(x) \cap \mathrm{co}A = \mathrm{cl}_K\left(\bigcap_{x\in \mathrm{co}A} \Gamma(x)\right) \cap \mathrm{co}A, \quad A \in \mathcal{F}(K).$$

From (d) we deduce that $\mathrm{cl}(\bigcap_{x\in D} \Gamma(x)) \subseteq B$. Hence, $\Gamma, \widehat{\Gamma}$ satisfy the conditions of Theorem 1.1. Then

$$\bigcap_{x\in K} \Gamma(x) \neq \emptyset,$$

which shows that the problem (VF-IVIP) has a solution. Now, let (x_α) be a net of solutions of (VF-IVIP) which converges to x. Then, for all $y \in K$ and all $t \in [0, 1]$, we have

$$\langle f(x_\alpha), tx + (1-t)y - g(x_\alpha)\rangle + F(tx + (1-t)y) - F(g(x_\alpha)) \in C(x_\alpha).$$

Thus, from assumption (b) we obtain

$$\langle f(x), y - g(x)\rangle + F(y) - F(g(x)) \in C(x).$$

Therefore, the solution set of (VF-IVIP) is closed and thanks to (d), it is a subset of B and consequently is compact. Thus the proof is completed. □

Remark 2.6. Let us endow $L(X, Y)$ with the following topology. We say that a net $F_\alpha \in L(X, Y)$ converges to $F \in L(X, Y)$ if, for each convergent net $x_\alpha \to x$ we have $\langle F_\alpha, x_\alpha\rangle \to \langle F, x\rangle$. Now if, f, g, F are continuous and C is a map with the closed graph then, the assumptions (a) and (b) are satisfied. Also, if K is compact then, the condition (d) trivially holds.

Corollary 2.7. *Assume that:*

(a) *the function* $G : \mathrm{co}A \times \mathrm{co}A \to Y$ *where,*

$$G(x, y) = \langle f(x), y - x\rangle + F(y) - F(x)$$

is C*-u.s.c. in the first variable,* $\forall A \in \mathcal{F}(K)$*;*

(b) *Let* $A \in \mathcal{F}(K)$, $x, y \in \mathrm{co}A$. *If* (x_α) *be any net on* K *converging to* x *then*

$$\langle f(x_\alpha), tx + (1-t)y - g(x_\alpha)\rangle + F(tx + (1-t)y) - F(g(x_\alpha)) \in C(x_\alpha), \quad \forall t \in [0, 1]$$

implies

$$\langle f(x), y - g(x)\rangle + F(y) - F(g(x)) \in C(x).$$

(c) *there exists a mapping* $h : K \times K \to Y$ *such that*
   (i) $h(x, x) \in C(x), \forall x \in K$*;*
   (ii) $\langle f(x), y - x\rangle + F(y) - F(x) - h(x, y) \in C(x), \forall x \in K, \forall y \in K$*;*
   (iii) *the set* $\{y \in K : h(x, y) \notin C(x)\}$ *is convex,* $\forall x \in K$*;*

(d) *there exist a nonempty compact subset* B *and a nonempty convex compact subset* D *of* K *such that, for each* $x \in K \setminus B$, *there exists* $y \in D$ *such that* $\langle f(x), y - x\rangle + F(y) - F(x) \notin C(x)$.

Then, (VF-VIP) *has a solution. Moreover, the solution set of* (VF-VIP) *is compact.*

By slight modifications of the proof of Corollary 2.4, we can obtain the following existence theorems.

Theorem 2.8. *Assume that:*

(a) *the function* $G : \mathrm{co}A \times \mathrm{co}A \to Y$ *where*

$$G(x, y) = \langle f(x), y - g(x)\rangle + F(y) - F(g(x))$$

is C*-u.s.c. in the first variable,* $\forall A \in \mathcal{F}(K)$*;*

(b) *Let* $A \in \mathcal{F}(K)$*,* $x, y \in \mathrm{co}A$*. If* (x_α) *be any net on* K *converging to* x *then, for all* $t \in [0, 1]$ *the following implication holds:*

$$\langle f(x_\alpha), tx + (1-t)y - g(x_\alpha)\rangle + F(tx + (1-t)y) - F(g(x_\alpha)) \in C(x_\alpha)$$
$$\text{then } \langle f(x), y - g(x)\rangle + F(y) - F(g(x)) \in C(x).$$

(c) $\langle f(x), x - g(x)\rangle + F(x) - F(g(x)) \in C(x), \forall x \in K$*;*
(d) *the set* $\{y \in K : \langle f(x), y - g(x)\rangle + F(y) - F(g(x)) \notin C(x)\}$ *is convex,* $\forall x \in K$*;*
(e) *there exist a nonempty compact set* $B \subseteq K$ *and a nonempty convex compact subset* D *of* K *such that, for each* $x \in K \setminus B$*, there exists* $y \in D$ *such that* $\langle f(x), y - g(x)\rangle + F(y) - F(g(x)) \notin C(x)$*.*

Then, (VF-IVIP) *has a solution. Moreover, the solution set of* (VF-IVIP) *is compact.*

Theorem 2.9. *Suppose that:*

(a) *the function* h *is* C*-u.s.c. in the first variable on* $\mathrm{co}A, \forall A \in \mathcal{F}(K)$*;*
(b) *for each* $A \in \mathcal{F}(K)$*, let* $x, y \in \mathrm{co}A$ *and* (x_α) *be a net on* K *converging to* x*, then, the following implication holds,*

$$\text{if } h(x_\alpha, tx + (1-t)y) \in C(x_\alpha), \quad \text{for all } t \in [0, 1], \text{ then } h(x, y) \in C(x);$$

(c) $h(x, x) \in C(x), \forall x \in K$*;*
(d) *the set* $\{y \in K : h(x, y) \notin C(x)\}$ *is convex,* $\forall x \in K$*;*
(e) *there exist a nonempty compact subset* B *and a nonempty convex compact subset* D *of* K *such that, for each* $x \in K \setminus B$*, there exists* $y \in D$ *such that* $h(x, y) \notin C(x)$*.*

If, for every $y \in K$*, the following implication holds:*

$$\langle f(x), y - g(x)\rangle + F(y) - F(g(x)) - h(x, y) \in C(x), \quad \forall x \in K.$$

Then, (VF-IVIP) *has a solution. Moreover, the solution set of* (VF-IVIP) *is compact.*

The following theorem improves Theorem 3.3. in [7].

Theorem 2.10. *Suppose that all assumptions of one of the Theorems* 2.5 *and* 2.8 *or* 2.9 *are satisfied. If* F *is positively homogeneous, then,* (VF-ICP) *has a solution. Moreover, the solution set of* (VF-ICP) *is compact.*

Proof. The result follows by Theorems 2.2 and 2.5. □

Remark 2.11. Consider the following vector F-implicit complementarity problems in t.v.s. which was studied in the special case $F(x) = 0$ and $g(x) = x$ in [8].

(Weak) vector F*-implicit complementarity problem (W-VF-ICP)*: Find $x \in K$ such that:

$$\langle f(x), g(x)\rangle + F(g(x)) \notin \mathrm{int}C(x), \qquad \langle f(x), y\rangle + F(g(x)) \notin -\mathrm{int}C(x), \quad \forall y \in K.$$

(Positive) vector F*-implicit complementarity problem (P-VF-ICP)*: Find $x \in K$ such that:

$$\langle f(x), g(x)\rangle + F(g(x)) \notin \mathrm{int}C(x), \qquad \langle f(x), y\rangle + F(g(x)) \in C(x), \quad \forall y \in K.$$

It is clear that the solution set of (VF-ICP), is a subset of the solution sets of (P-VF-ICP) and (W-VF-ICP). Thus, Theorems 2.5, 2.8 and 2.9 provide existence results for (W-VF-ICP) and (P-VF-ICP). If we take $F = 0$, which is obviously positively homogenous, then Theorem 2.8 gives a solution for the problems considered in [8].

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