Physics Letters B 749 (2015) 376–382

Contents lists available at ScienceDirect

Physics Letters B

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Condensation for non-relativistic matter in Hořava–Lifshitz gravity

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A R T I C L E I N F O

Article history:
Received 25 June 2015
Received in revised form 1 August 2015
Accepted 3 August 2015
Available online 6 August 2015
Editor: N. Lambert

Keywords:
Holographic superconductors
Non-relativistic matter
Hořava–Lifshitz gravity

A B S T R A C T

We study condensation for non-relativistic matter in a Hořava–Lifshitz black hole without the condition of the detailed balance. We show that, for the fixed non-relativistic parameter α_2 (or the detailed balance parameter ϵ), it is easier for the scalar hair to form as the parameter ϵ (or α_2) becomes larger, but the condensation is not affected by the non-relativistic parameter β_2. We also find that the ratio of the gap frequency in conductivity to the critical temperature decreases with the increase of ϵ and α_2, but increases with the increase of β_2. The ratio can reduce to the Horowitz–Roberts relation $\omega_g/T_c \approx 8$ obtained in the Einstein gravity and Cai's result $\omega_g/T_c \approx 13$ found in a Hořava–Lifshitz gravity with the condition of the detailed balance for the relativistic matter. Especially, we note that the ratio can arrive at the value of the BCS theory $\omega_g/T_c \approx 3.5$ by taking proper values of the parameters.

1. Introduction

The AdS/CFT correspondence [1–3] relates a weak coupling gravity theory in an anti-de Sitter space to a strong coupling conformal field theory in one less dimensions. Recently it has been applied to condensed matter physics and in particular to superconductivity [4,5]. In the pioneering papers Gubser [4,5] suggested that near the horizon of a charged black hole there is in operation a geometrical mechanism parameterized by a charged scalar field of breaking a local $U(1)$ gauge symmetry. Then, the gravitational dual of the transition from normal to superconducting states in the boundary theory was constructed. This dual consists of a system with a black hole and a charged scalar field, in which the black hole admits scalar hair at temperature lower than a critical temperature, but does not possess scalar hair at higher temperatures [6]. In this system a scalar condensate can take place through the coupling of the scalar field with the Maxwell field of the background. Much attention has been focused on the application of AdS/CFT correspondence to condensed matter physics since then [7–19].

Hořava [20,21] proposed a new class of quantum gravity. The key property of this theory is the three-dimensional general covariance and time re-parameterization invariance. It is this anisotropic rescaling that makes Hořava's theory power-counting renormalizable. Therefore, many authors pay their attention to this gravity theory and its cosmological and astrophysical applications, and found many interesting results [22–35]. These investigations imply that there exists the distinct difference between the Hořava–Lifshitz theory and Einstein's gravity.

In the Hořava–Lifshitz gravity, Kiritsis and Kofinas [36], Kimpton and Padilla [37] proposed the non-relativistic matter. They constructed the most general action of matter coupled to gravity with the foliation-preserving diffeomorphism. The action obeys the usual power-counting renormalizability conditions used in Hořava–Lifshitz gravity and assuming the temporal derivatives are as in the relativistic theory.

Recently, in order to see what difference will appear for the holographic superconductivity in the Hořava–Lifshitz theory, comparing with the case of the relativistic general relativity, Cai et al. [38] studied the phase transition of planar black holes in the Hořava–Lifshitz gravity with the condition of the detailed balance in which the metric function is described by $f(r) = x^2 - \sqrt{c_0 x}$. They argued that the holographic superconductivity is a robust phenomenon associated with asymptotic AdS black holes. And they also got a relation connecting the gap frequency in conductivity with the critical temperature, which is given by $\frac{\omega_g}{T_c} \approx 13$, with the accuracy more than 93% for a range of scalar masses. More recently, Lin, Abdalla and Wang [39] generalized the investigation to the holographic superconductors related to the non-relativistic matter in the Schwarzschild black hole in the low energy limit of Hořava–Lifshitz spacetime.

Note that the Hořava–Lifshitz black hole without the condition of the detailed balance has rich physics [40–42], i.e., changing the

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http://dx.doi.org/10.1016/j.physletb.2015.08.009

parameter of the detailed balance ϵ from 0 to 1 it can produce the different black holes for the Hořava–Lifshitz theory and Einstein's gravity, and the non-relativistic matter in Hořava–Lifshitz gravity has new properties. In this paper we will extend the study to case of the non-relativistic matter in a Hořava–Lifshitz black hole without the condition of the detailed balance, and investigate how the parameter of the detailed balance and non-relativistic parameters influence on the scalar condensation formation, the electrical conductivity, and the ratio ω_g/T_c which connects the gap frequency in conductivity with the critical temperature.

The paper is organized as follows. In Section 2 we present black hole with hyperbolic horizons in Hořava–Lifshitz gravity in which the action without the condition of the detailed balance. In Section 3 we explore the condensation of the relativistic matter in the Hořava–Lifshitz black hole background by numerical approach. In Section 4 we study the electrical conductivity and find ratio of the gap frequency in conductivity to the critical temperature. We summarize and discuss our conclusions in the last section.

2. Black hole with hyperbolic horizon in $z = 3$ Hořava–Lifshitz gravity

In non-relativistic field theory, space and time have different scalings, which is called anisotropic scaling, $x^i \to bx^i$, $t \to b^z t$, $i = 1, 2, 3$, where z is called *dynamical critical exponent*. In order for a theory to be power counting renormalizable, the critical exponent has at least $z = 3$ in four spacetime dimensions. For $z = 3$, the action without the condition of the detailed balance for the Hořava–Lifshitz theory can be expressed as [40,41]

$$I = \int dt\, d^3x[\mathcal{L}_0 + (1-\epsilon^2)\mathcal{L}_1], \tag{2.1}$$

with

$$\mathcal{L}_0 = \sqrt{g}N\left[\frac{2}{\kappa^2}(K_{ij}K^{ij} - \lambda K^2) + \frac{\kappa^2\mu^2(\Lambda R - 3\Lambda^2)}{8(1-3\lambda)}\right],$$

$$\mathcal{L}_1 = \sqrt{g}N\left[\frac{\kappa^2\mu^2(1-4\lambda)}{32(1-3\lambda)}R^2 - \frac{\kappa^2}{2\omega^4}\left(C_{ij} - \frac{\mu\omega^2}{2}R_{ij}\right)\left(C^{ij} - \frac{\mu\omega^2}{2}R^{ij}\right)\right],$$

$$K_{ij} = \frac{1}{2N}(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i),$$

$$C^{ij} = \epsilon^{ikl}\nabla_k\left(R^j_l - \frac{1}{4}R\delta^j_l\right) = \epsilon^{ikl}\nabla_k R^j_l - \frac{1}{4}\epsilon^{ikj}\partial_k R,$$

where κ^2, μ, Λ, and ω are constant parameters, ϵ is parameter of the detailed balance $(0 < \epsilon \le 1)$, N^i is the shift vector, K_{ij} is the extrinsic curvature and C_{ij} the Cotten tensor. It is interesting to note that the action (2.1) reduces to the action in Ref. [41] if $\epsilon = 0$, and it becomes the action for the Einstein's gravity if $\epsilon = 1$.

From the action (2.1), Cai et al. [42] found a static black hole with hyperbolic horizon whose horizon has an arbitrary constant scalar curvature $2k$ with $\lambda = 1$. The line element of the black hole can be expressed as

$$ds^2 = -N^2(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_k^2, \tag{2.2}$$

with

$$N^2 = f = k + \frac{x^2}{1-\epsilon^2} - \frac{\sqrt{\epsilon^2 x^4 + (1-\epsilon^2)c_0 x}}{1-\epsilon^2}, \tag{2.3}$$

where $x = \sqrt{-\Lambda}\ r$, $k = -1,\ 0,\ 1$, and $c_0 = [x_+^4 + 2kx_+ + (1 - \varepsilon^2)k^2]/x_+$ in which x_+ is the horizon radius of the black hole, i.e., the largest root of $f(r) = 0$. Comparing with the standard AdS_4 spacetime, we may set $\frac{-\Lambda}{1+\epsilon} = \frac{1}{L^2_{AdS}}$, where L_{AdS} is the radius of AdS_4. The authors in Ref. [42] also found that the solution has a finite mass $M = \kappa^2\mu^2\Omega_k\sqrt{-\Lambda}c_0/16$. For $\epsilon = 0$, the solution goes back to the solution in Ref. [41].

The Hawking temperature of the black hole is

$$T = \frac{\sqrt{-\Lambda}}{8\pi}\frac{3x_+^4 + 2kx_+^2 - (1-\epsilon^2)k^2}{x_+[x_+^2 + (1-\epsilon^2)k]}, \tag{2.4}$$

which is always a monotonically increasing function of horizon radius x_+ in the physical regime. This implies that the black holes with hyperbolic horizons in the Hořava–Lifshitz theory are thermodynamically stable.

3. Condensation for non-relativistic matter in Hořava–Lifshitz gravity

We now study the condensation for non-relativistic matter in the Hořava–Lifshitz gravity. For the Arnowitt–Deser–Misner metric

$$ds^2 = -N^2 dt^2 + \gamma_{ij}\left(dx^i - N^i dt\right)\left(dx^j - N^j dt\right), \tag{3.1}$$

the Lagrangian of complex scalar and electromagnetic fields for the non-relativistic matter in the Hořava–Lifshitz gravity can be expressed as [36]

$$\mathcal{L}_H^E = \frac{2}{N^2}\gamma^{ij}\left(F_{0i} - F_{ki}N^k\right)\left(F_{0j} - F_{lj}N^l\right) - F_{ij}F^{ij} - \beta_0 - \beta_1 a_i B^i - \beta_2 B_i B^i - \mathcal{G}_E, \tag{3.2}$$

$$\mathcal{L}_H^S = \frac{1}{2N^2}\left|\partial_t\Psi - N^i\partial_i\Psi\right|^2 - \frac{1}{2}|\partial\Psi|^2 - \frac{1}{2}V(|\Psi|) + \alpha_2|\partial\Psi|^2 - \mathcal{H}_S, \tag{3.3}$$

with

$$\mathcal{G}_E = \beta_3\left(B_iB^i\right)^2 + \beta_4\left(B_iB^i\right)^3 + \beta_5\left(\nabla_i B_j\right)\left(\nabla^i B^j\right) + \beta_6\left(B_iB^i\right)\left(\nabla_k B_j\right)\left(\nabla^k B^j\right) + \beta_7\left(\nabla_i B_j\right)\left(\nabla^i B^k\right)\left(\nabla^j B_j\right) + \beta_8\left(\nabla_i\nabla_j B_k\right)\left(\nabla^i\nabla^j B^k\right),$$

$$\mathcal{H}_S = \alpha_3(\Psi\Delta\Psi)^2 + \alpha_4(\Psi\Delta\Psi)^3 + \alpha_5\Psi\Delta^2\Psi + \alpha_6(\Psi\Delta\Psi)\left(\Psi\Delta^2\Psi\right) + \alpha_7\Psi\Delta^3\Psi, \tag{3.4}$$

where $\mathcal{G}_E$ and $\mathcal{H}_S$ are the Hořava–Lifshitz higher order corrections, α_i and β_i can be taken as constants, $F_{ij} = \partial_j A_i - \partial_i A_j$, $V(|\Psi|) = m^2|\Psi|^2$, and $B^i = \frac{1}{2}\frac{\epsilon^{ijk}}{\sqrt{\gamma}}F_{jk}$ with the Levi-Civita symbol ϵ^{ijk}. In this paper, we just consider the lower order terms of above equations, i.e., the higher order terms $\mathcal{G}_E$ and $\mathcal{H}_S$ are ignored.

The coupling between electromagnetic field and scalar field can be constructed and then the Lagrangian $\mathcal{L}_H^S$ should be rewritten as [39]

$$\tilde{\mathcal{L}}_H^S = \frac{1}{2N^2}\left|\partial_t\Psi - iqA_0\Psi - N^i(\partial_i\Psi - iqA_i\Psi)\right|^2 - \left(\frac{1}{2} - \alpha_2\right)|\partial\Psi - iqA_i\Psi|^2 - \frac{1}{2}V(|\Psi|) - \tilde{\mathcal{H}}_S, \tag{3.5}$$

where $\mathcal{H}_S$ is replaced by $\tilde{\mathcal{H}}_S$ with $\partial_i \to \partial_i - iqA_i$. Therefore, the action of coupling between complex scalar and electromagnetic fields for the non-relativistic matter in the Hořava–Lifshitz gravity can be taken as

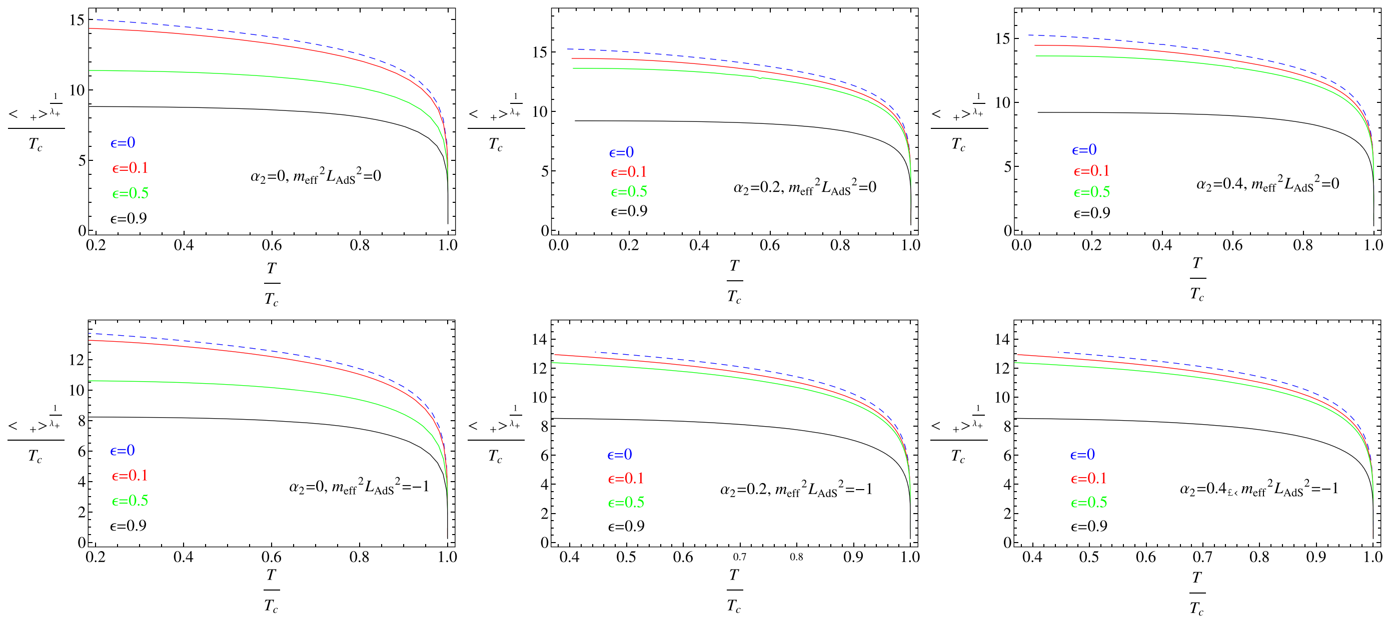

Fig. 1. (Color online.) The condensate as a function of the temperature with fixed values $m_{eff}^2 L_{AdS}^2 = 0, -1$ and arbitrary β_2. The four lines from top to bottom correspond to increasing ϵ, i.e., 0 (blue), 0.1 (red), 0.5 (green) and 0.9 (black), respectively. It is shown that the condensation gap becomes smaller as ϵ (or α_2) increases for the same m_{eff}^2.

$$S_H = \int dt d^3x N\sqrt{\gamma}\left(\frac{1}{4}\mathcal{L}_H^E + 2\tilde{\mathcal{L}}_H^S\right), \tag{3.6}$$

which will reduce into the model in general relativity when $\alpha_i = \beta_i = 0$.

In the background of the black hole described by Eq. (2.3) with $k = 0$, we focus our attention on the case that these fields are weakly coupled to gravity, i.e., they do not backreact on the metric of the spacetime. Thus, we can take the ansatz

$$A_\mu = (\phi(r), 0, 0, 0),$$
$$\psi = \psi(r). \tag{3.7}$$

This ansatz implies that the phase factor of the complex scalar field is a constant. Therefore, we may take ψ to be real. In the background of the black hole described by Eqs. (2.2) and (2.3) with $k = 0$, the equations of the scalar field $\psi(r)$ and the scalar potential $\phi(r)$ are given by

$$\psi'' + \left(\frac{f'}{f} + \frac{2}{r}\right)\psi' + \frac{1}{1-2\alpha_2}\left(\frac{\phi^2}{f^2} - \frac{m^2}{f}\right)\psi = 0, \tag{3.8}$$

$$\phi'' + \frac{2}{r}\phi' - \frac{2\psi^2}{f}\phi = 0\,, \tag{3.9}$$

where a prime denotes the derivative with respect to r.

At the event horizon $r = r_+$, because $f(r_+) = 0$, we must have

$$\psi(r_+) = -\frac{3(1-2\alpha_2)r_+\psi'(r_+)}{2m^2L^2},$$
$$\phi(r_+) = 0, \tag{3.10}$$

because their norms are required to be finite, where $L^2 = L_{AdS}^2/(1+\epsilon)$. And at the asymptotic region ($r \to \infty$), because $f(r) \to r^2$, we can get the solutions behave like

$$\psi = \frac{\psi_-}{r^{\lambda_-}} + \frac{\psi_+}{r^{\lambda_+}},$$
$$\phi = \mu - \frac{\rho}{r}, \tag{3.11}$$

with

$$\lambda_\pm = \frac{1}{2}\left(3 \pm \sqrt{9 + \frac{4m^2L_{AdS}^2}{1-2\alpha_2}}\right), \tag{3.12}$$

where μ and ρ are interpreted as the chemical potential and charge density in the dual field theory, respectively. Because the boundary is a $(2+1)$-dimensional field theory, μ is of mass dimension one and ρ is of mass dimension two. We can read off the expectation values of operator $\mathcal{O}$ dual to the field ψ. From Ref. [43], we know that for ψ, both of these falloffs are normalizable, and in order to keep the theory stable, we should either impose

$$\psi_- = 0, \quad \text{and} \quad \langle\mathcal{O}_+\rangle = \psi_+, \tag{3.13}$$

or

$$\psi_+ = 0, \quad \text{and} \quad \langle\mathcal{O}_-\rangle = \psi_-. \tag{3.14}$$

Note that the dimension of temperature T is of mass dimension one, the ratio T^2/ρ is dimensionless. Therefore increasing ρ while T is fixed, is equivalent to decrease T while ρ is fixed. In our calculation, we find that when $\rho > \rho_c$, the operator condensate will appear; this means when $T < T_c$ there will be an operator condensate, that is to say, the superconducting phase occurs. We will impose boundary condition $\psi_- = 0$ in the following discussion.

Eqs. (3.8) and (3.9) can be solved numerically by doing integration from the horizon out to the infinity with the boundary conditions mentioned above. Changing the values of the balance parameter ϵ and non-relativistic parameter α_2, we present in Fig. 1 the influence of the parameters ϵ and α_2 on the condensation with fixed values $m_{eff}^2 L_{AdS}^2 = 0, -1$ (here and hereafter $m_{eff}^2 = \frac{m^2}{1-2\alpha_2}$) and arbitrary β_2, and in Fig. 2 the critical temperature as a function of the balance parameter and non-relativistic parameter α_2 with fixed values $m_{eff}^2 L_{AdS}^2 = 0, -1$ and arbitrary β_2. In Table 1 we present the critical temperature obtained by the numerical method. We know from the figures and the table that as the parameter of the detailed balance increases with fixed non-relativistic parameter α_2 and effective mass of the scalar field,

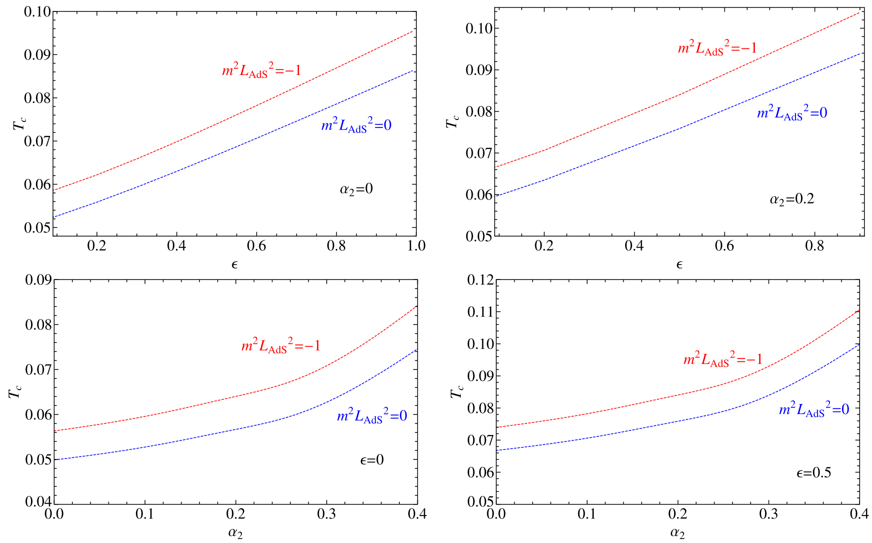

Fig. 2. (Color online.) The critical temperature as a function of the balance parameter (or non-relativistic parameter) with fixed values $m^2_{eff}L^2_{AdS}$ and arbitrary β_2. The two lines from top to bottom correspond to $m^2_{eff}L^2_{AdS} = -1$ (red) and 0 (blue), respectively.

Table 1
The critical temperature T_c obtained by numerical method.

	$\epsilon = 0.0$	$\epsilon = 0.1$	$\epsilon = 0.2$	$\epsilon = 0.5$	$\epsilon = 0.9$
$\alpha_2 = 0$					
$m^2_{eff}L^2_{AdS} = 0$	0.0499	0.0502	0.0510	0.0545	0.0599
$m^2_{eff}L^2_{AdS} = -1$	0.0563	0.0589	0.0622	0.0740	0.0914
$\alpha_2 = 0.1$					
$m^2_{eff}L^2_{AdS} = 0$	0.0527	0.0557	0.0591	0.0706	0.0874
$m^2_{eff}L^2_{AdS} = -1$	0.0596	0.0623	0.0657	0.0782	0.0966
$\alpha_2 = 0.2$					
$m^2_{eff}L^2_{AdS} = 0$	0.0567	0.0598	0.0635	0.0759	0.0938
$m^2_{eff}L^2_{AdS} = -1$	0.0640	0.0669	0.0707	0.0840	0.1038
$\alpha_2 = 0.4$					
$m^2_{eff}L^2_{AdS} = 0$	0.0746	0.0788	0.0835	0.0999	0.1235
$m^2_{eff}L^2_{AdS} = -1$	0.0842	0.0881	0.0930	0.1106	0.1366

the condensation gap becomes smaller, corresponding to larger the critical temperature, which means that the scalar hair can be formed easier for the larger ϵ. Similarly, the scalar hair can be formed easier as the non-relativistic parameter α_2 becomes larger with fixed balance parameter and effective mass of the scalar field. And the figures and table also show that, for the same ϵ or α_2, the condensation gap becomes larger if m^2_{eff} becomes less negative, which means that it is harder for the scalar hair to form as the effective mass of the scalar field becomes larger. We should point out that the parameter β_2 dose not affect the condensation in this model.

4. Electrical conductivity in Hořava–Lifshitz black-hole background

In the study of $(2+1)$- and $(3+1)$-dimensional superconductors in Einstein gravity, Horowitz et al. [8] got a universal relation connecting the gap frequency in conductivity with the critical temperature T_c, which is described by

$$\frac{\omega_g}{T_c} \approx 8, \tag{4.1}$$

with deviations of less than 8%. This is roughly twice the BCS value 3.5 indicating that the holographic superconductors are strongly coupled. The authors in Refs. [16,44] found that this relation is not stable in the presence of the Gauss–Bonnet correction terms. And Cai et al. [38] got a relation

$$\frac{\omega_g}{T_c} \approx 13, \tag{4.2}$$

with the accuracy more than 93% for a planar Hořava–Lifshitz black hole with the condition of the detailed balance for the relativistic matter.

We now study this relation for the non-relativistic matter in the Hořava–Lifshitz gravity. In order to compute the electrical conductivity, we should study the electromagnetic perturbation in this Hořava–Lifshitz black hole background, and then calculate the linear response to the perturbation. In the probe approximation, the effect of the perturbation of metric can be ignored. Assuming that the perturbation of the vector potential is translational symmetric and has a time dependence as $\delta A_x = A_x(r)e^{-i\omega t}$, we find that the equation of motion for A_x in the Hořava–Lifshitz black hole background reads

$$A_x'' + \frac{f'}{f}A_x' + \frac{2}{2+\beta_2}\left[\frac{\omega^2}{f^2} - \frac{2(1-2\alpha_2)\psi^2}{f}\right]A_x = 0\,, \tag{4.3}$$

where a prime denotes the derivative with respect to r. An ingoing wave boundary condition near the horizon is given by

$$A_x(r) \sim f(r)^{-\frac{2i\omega L^2}{3r_+\sqrt{1+\beta_2/2}}}. \tag{4.4}$$

In the asymptotic AdS region ($r \to \infty$), the general behavior should be

$$A_x = A^{(0)} + \frac{A^{(1)}}{r} + \cdots\,. \tag{4.5}$$

By using AdS/CFT correspondence and the Ohm's law, we know that the conductivity can be expressed as [8]

$$\sigma = \frac{\langle J_x\rangle}{E_x} = -\frac{i\langle J_x\rangle}{\omega A_x} = \frac{A^{(1)}}{i\omega A^{(0)}}\,. \tag{4.6}$$

In Figs. 3 and 4 we plot the frequency dependent conductivity obtained by solving the equation of motion (4.3) numerically for

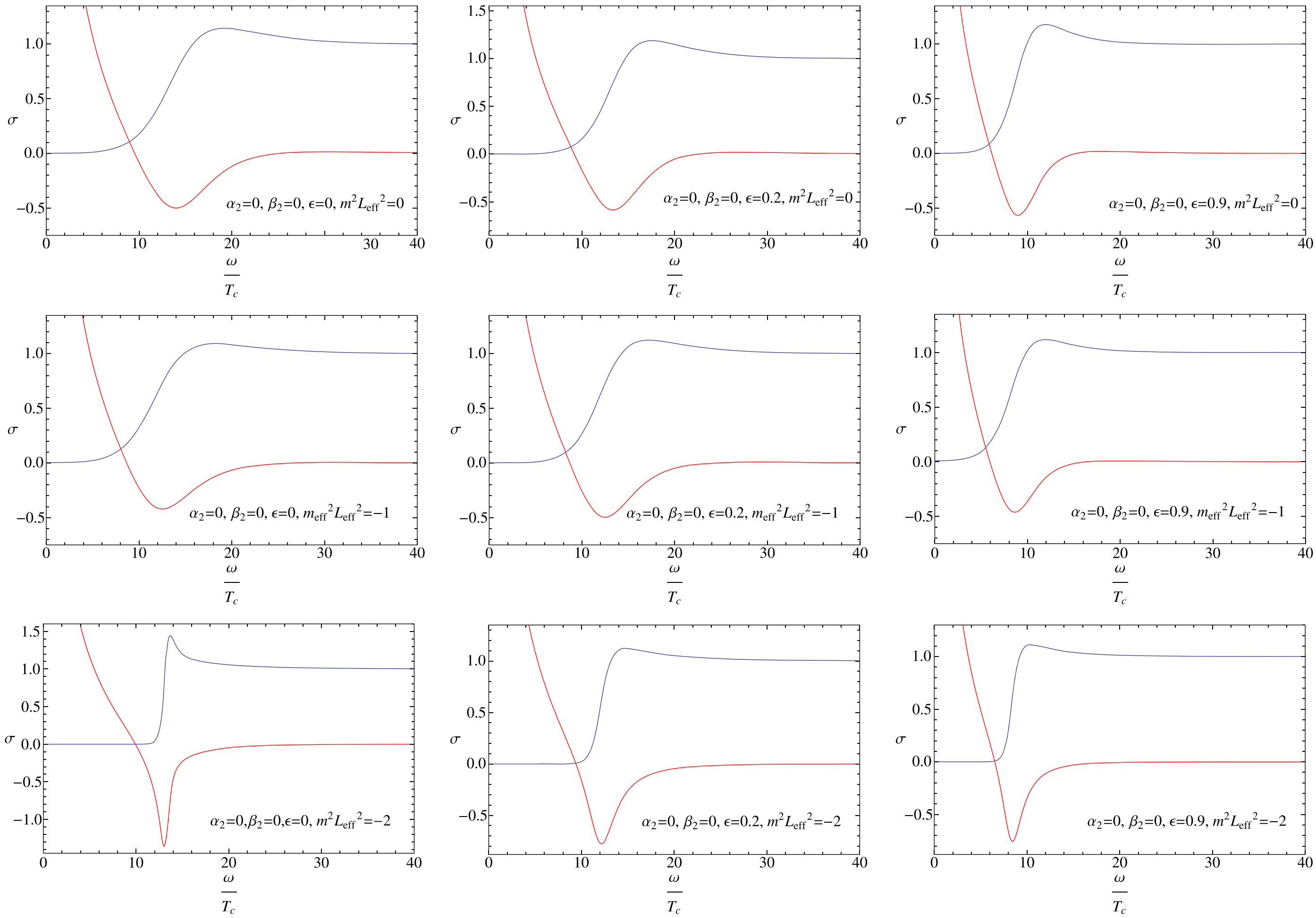

Fig. 3. (Color online.) The conductivity of the superconductors for $\epsilon = 0$, 0.2 and 0.9 with $\beta_2 = 0$, $\alpha_2 = 0$ and $m^2_{eff} L^2_{AdS} = 0, -1, -2$. The solid (blue) line represents the real part of the conductivity, $\mathrm{Re}(\sigma)$, and dashed (red) line is the imaginary part of the conductivity, $\mathrm{Im}(\sigma)$.

Table 2
The ratio ω_g/T_c for different values of the parameters ϵ, α_2 and β_2 with $m^2_{eff} L^2_{AdS} = 0, -1$ and -2.

$\alpha_2 = 0$, $\beta_2 = 0$						
	$\epsilon = 0$	$\epsilon = 0.1$	$\epsilon = 0.2$	$\epsilon = 0.5$	$\epsilon = 0.9$	$\epsilon = 0.99$
$m^2_{eff} L^2_{AdS} = 0$	14.6	13.9	13.2	11.3	9.0	8.6
$m^2_{eff} L^2_{AdS} = -1$	13.8	13.2	12.6	10.8	8.6	8.2
$m^2_{eff} L^2_{AdS} = -2$	12.9	12.4	11.9	10.3	8.4	8.1
$\alpha_2 = 0.2$, $\beta_2 = 0$						
	$\epsilon = 0$	$\epsilon = 0.1$	$\epsilon = 0.2$	$\epsilon = 0.5$	$\epsilon = 0.9$	$\epsilon = 0.99$
$m^2_{eff} L^2_{AdS} = 0$	13.0	11.8	11.5	9.7	7.8	7.2
$m^2_{eff} L^2_{AdS} = -1$	11.2	10.5	10.1	9.1	7.2	6.9
$m^2_{eff} L^2_{AdS} = -2$	10.3	9.7	9.3	8.6	6.8	6.5
$\alpha_2 = 0.47$, $\beta_2 = 0$						
	$\epsilon = 0$	$\epsilon = 0.1$	$\epsilon = 0.2$	$\epsilon = 0.5$	$\epsilon = 0.9$	$\epsilon = 0.99$
$m^2_{eff} L^2_{AdS} = 0$	6.9	6.4	5.8	5.0	4.3	4.1
$m^2_{eff} L^2_{AdS} = -1$	6.5	5.9	5.4	4.8	4.0	3.8
$m^2_{eff} L^2_{AdS} = -2$	5.6	5.3	5.0	4.6	3.7	3.5
$\alpha_2 = 0.1$, $\epsilon = 0.1$						
	$\beta_2 = 0$	$\beta_2 = 0.5$	$\beta_2 = 1$	$\beta_2 = 1.5$	$\beta_2 = 2$	$\beta_2 = 2.5$
$m^2_{eff} L^2_{AdS} = 0$	13.0	13.5	14.0	14.5	15.0	15.8
$m^2_{eff} L^2_{AdS} = -1$	12.2	12.8	13.2	13.6	14.0	14.5
$m^2_{eff} L^2_{AdS} = -2$	11.4	11.8	12.1	12.3	12.5	12.8

$\epsilon = 0$, 0.5 and 0.9 (or 0.99) with $\beta_2 = 0$, $\alpha_2 = 0$, 0.2, 0.47 and $m^2_{eff} L^2_{AdS} = 0, \ -1$ and -2. We find that, for the same value of $m^2_{eff} L^2_{AdS}$, the gap frequency ω_g decreases with the increase of the parameters ϵ or α_2. In each plot, the real part of the conductivity, $\mathrm{Re}[\sigma]$, approaches to a limit when the frequency grows. The limit for the case $\epsilon = 0$ and $\alpha_2 = 0$ is one, but generally it increases as parameters ϵ or α_2 increases. The imaginary part of conductivity $\mathrm{Im}[\sigma]$ becomes zero when $\omega \to \infty$, but it goes to infinity when the frequency approaches zero.

In Fig. 5 we plot the frequency dependent conductivity for $\beta_2 = 0$, 1 and 2 with $\alpha_2 = 0.1$, $\epsilon = 0.1$ and $m^2_{eff} L^2_{AdS} = 0, \ -1, -2$. We note that, for the same values of ϵ, α_2 and $m^2_{eff} L^2_{AdS}$, the gap frequency ω_g increases with the increase of the parameters β_2. That is to say, the ratio of the gap frequency in conductivity ω_g to the critical temperature T_c increases as the parameters β_2 increases with fixed α_2, ϵ and m^2_{eff}.

In Table 2 we also present how the ratio ω_g/T_c relate to the balance parameter and non-relativistic parameter with fixed values $m^2_{eff} L^2_{AdS} = 0, -1$ and -2, which shows that the ratio ω_g/T_c decreases with the increase of the balance parameter or the non-relativistic parameter α_2, but increases with the increase of the parameter β_2.

From Figs. 3, 4 and 5 and Table 2, we find that the ratio of the gap frequency in conductivity ω_g to the critical temperature T_c in this black hole reduces to Cai's result $\omega_g/T_c \approx 13$ [38] found in the

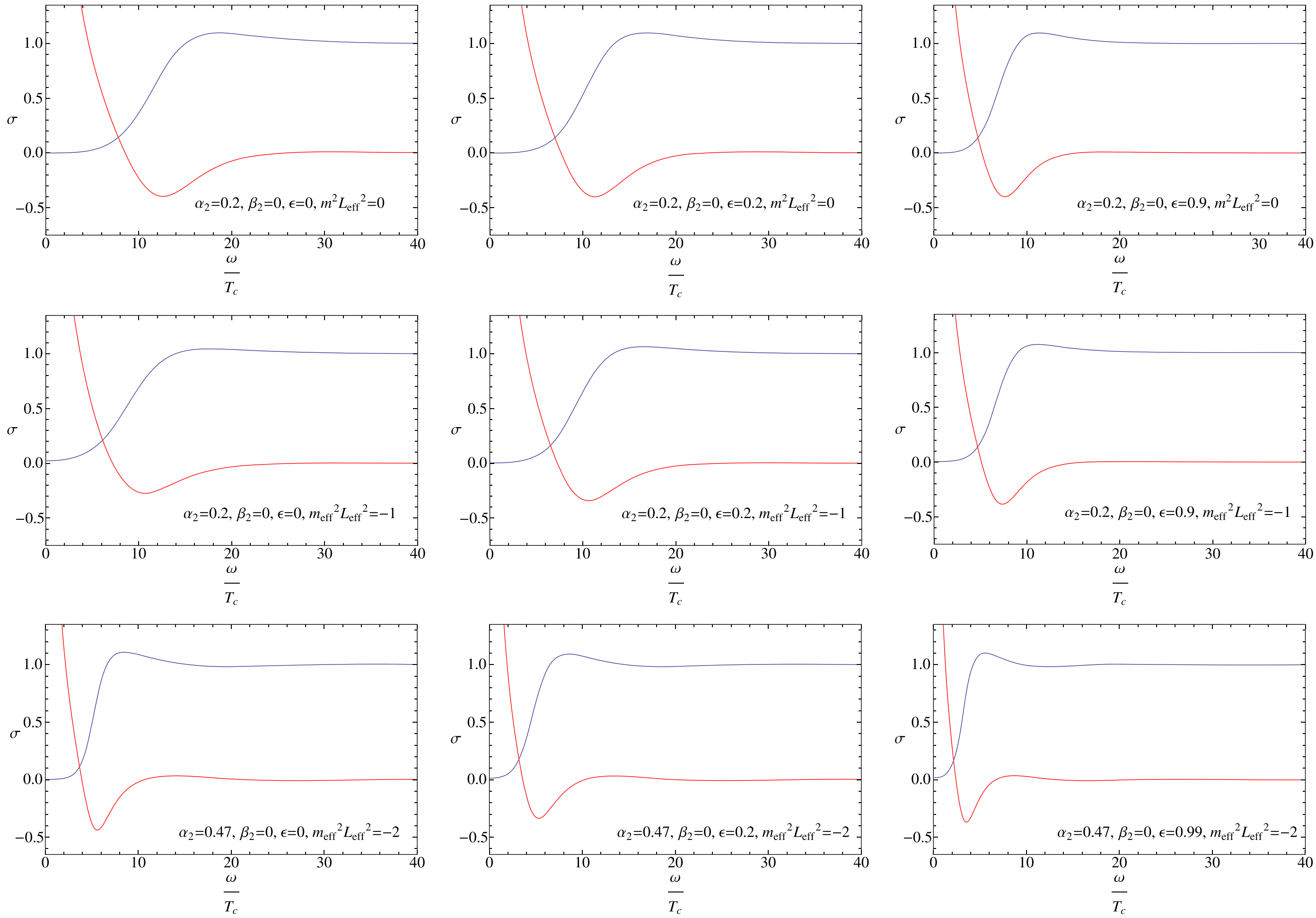

Fig. 4. (Color online.) The conductivity of the superconductors for $\epsilon = 0$, 0.2, 0.9 and 0.99 with $\beta_2 = 0$, $\alpha_2 = 0.2$, 0.47 and $m^2_{eff}L^2_{AdS} = 0$, −1, −2. The solid (blue) line represents the real part of the conductivity, Re(σ), and dashed (red) line is the imaginary part of the conductivity, Im(σ).

Hořava–Lifshitz gravity with the condition of the detailed balance for the relativistic matter when $\epsilon = 0$, $\beta_2 = 0$ and $\alpha_2 = 0$, while it tends to the Horowitz–Roberts relation $\omega_g/T_c \approx 8$ obtained in the Einstein gravity as $\epsilon \to 1$ with $\alpha_2 = 0$ and $\beta_2 = 0$. Especially, the ratio can arrive at the value of the BCS theory $\omega_g/T_c \approx 3.5$ if we take right value for ϵ, α_2, β_2 and m^2_{eff}, say $\epsilon = 0.99$, $\alpha_2 = 0.47$, $\beta_2 = 0$ and $m^2_{eff}L^2_{AdS} = -2$.

5. Conclusions

The behavior of the holographic superconductors in the Hořava–Lifshitz gravity has been investigated in this manuscript by introducing the non-relativistic scalar and electromagnetic fields in a planar black-hole background. We first present a detailed analysis of the condensation of the operator $\mathcal{O}_+$ by the numerical method for the Hořava–Lifshitz black hole without the condition of the detailed balance. It is found that, as the parameter of the detailed balance ϵ increases with fixed the non-relativistic parameter α_2 and effective mass of the scalar field m^2_{eff}, the condensation gap becomes smaller, corresponding to the larger critical temperature, which means that the scalar hair can be formed easier for the larger ϵ. Similarly, the scalar hair can be formed easier as the non-relativistic parameter α_2 becomes larger with fixed detailed balance ϵ and effective mass of the scalar field. And it is also shown that, for the same ϵ or α_2, the condensation gap becomes larger if m^2_{eff} becomes less negative, which means that it is harder for the scalar hair to form as the effective mass of the scalar field becomes larger. It is interesting to note that the parameter β_2 does not affect the condensation. We then studied the electrical conductivity for the non-relativistic matter in the Hořava–Lifshitz black-hole background and find that the ratio of the gap frequency in conductivity to the critical temperature, ω_g/T_c, decreases with the increase of the balance parameter ϵ or the non-relativistic parameter α_2, but increases with the increase of the parameter β_2. The ratio reduces to Cai's result $\omega_g/T_c \approx 13$ [38] found in a Hořava–Lifshitz gravity with the condition of the detailed balance for the relativistic matter when $\epsilon = 0$, $\alpha_2 = 0$ and $\beta_2 = 0$, while it tends to the Horowitz–Roberts relation $\omega_g/T_c \approx 8$ [8] obtained in the Einstein gravity if we take $\epsilon \to 1$, $\alpha_2 = 0$ and $\beta_2 = 0$. Especially, the ratio can arrive at the value of the BCS theory $\omega_g/T_c \approx 3.5$ if we take right values of ϵ, α_2, β_2 and m.

Acknowledgements

This work is supported by the National Natural Science Foundation of China under Grant No. 11475061; the SRFDP under Grant No. 20114306110003; the Open Project Program of State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, China (No. Y5KF161CJ1); S. Chen's work was partially supported by the National Natural Science Foundation of China under Grant No. 11275065; and Q. Pan's work was partially supported by the National Natural Science Foundation of China under Grant No. 11275066.

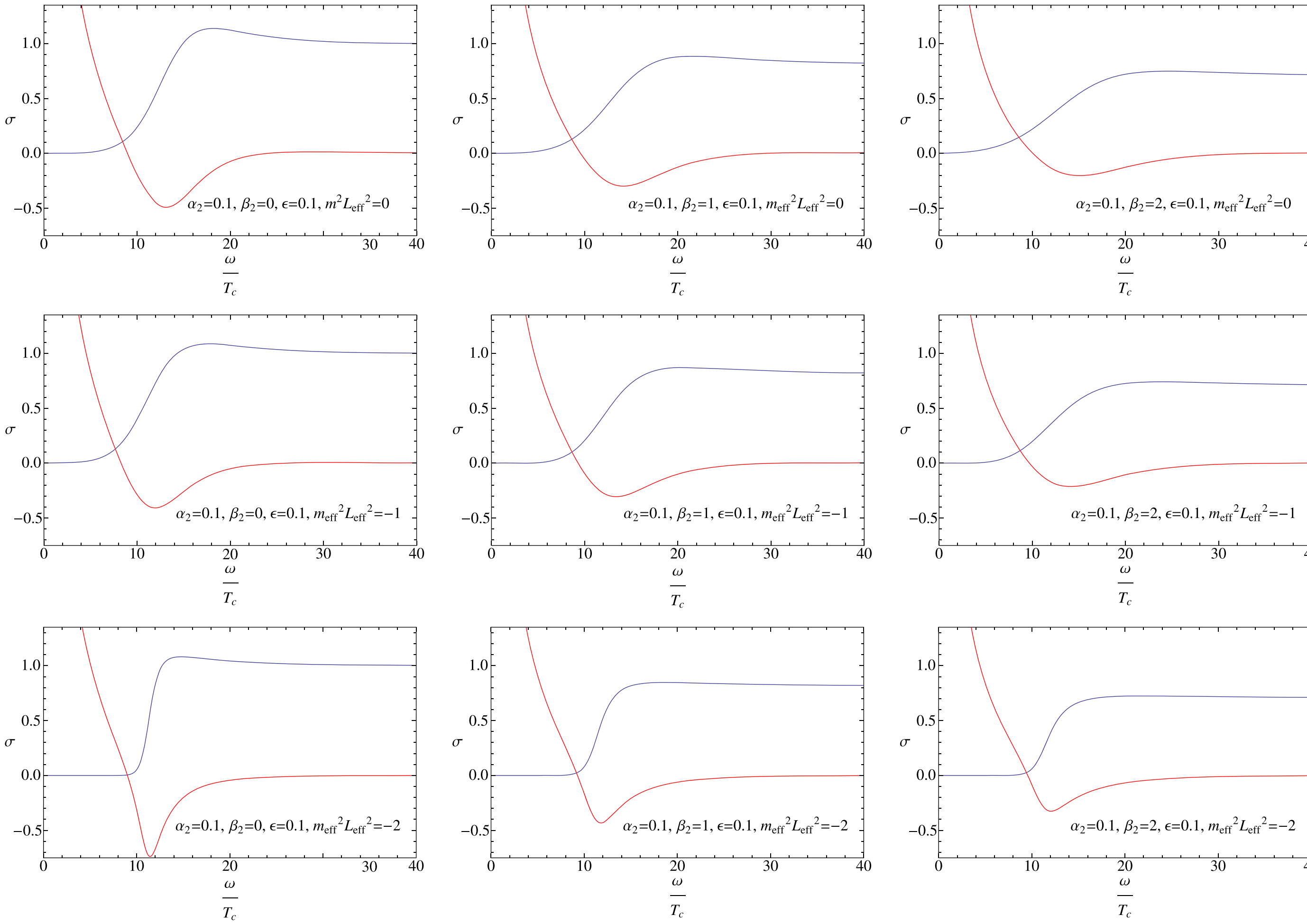

Fig. 5. (Color online.) The conductivity of the superconductors for $\beta_2 = 0$, 1 and 2 with $\alpha_2 = 0.1$, $\epsilon = 0.1$ and $m^2_{eff} L^2_{AdS} = 0, -1, -2$. The solid (blue) line represents the real part of the conductivity, Re(σ), and dashed (red) line is the imaginary part of the conductivity, Im(σ).

References

[1] J. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231.
[2] S.S. Gubser, I.R. Klebanov, A.M. Polyakov, Phys. Lett. B 428 (1998) 105, arXiv:hep-th/9802109.
[3] E. Witten, Adv. Theor. Math. Phys. 2 (1998) 253.
[4] S.S. Gubser, Class. Quantum Gravity 22 (2005) 5121.
[5] S.S. Gubser, Phys. Rev. D 78 (2008) 065034.
[6] S.A. Hartnoll, C.P. Herzog, G.T. Horowitz, Phys. Rev. Lett. 101 (2008) 031601.
[7] S.A. Hartnoll, C.P. Herzog, G.T. Horowitz, J. High Energy Phys. 0812 (2008) 015.
[8] G.T. Horowitz, M.M. Roberts, Phys. Rev. D 78 (2008) 126008.
[9] E. Nakano, Wen-Yu Wen, Phys. Rev. D 78 (2008) 046004.
[10] I. Amado, M. Kaminski, K. Landsteiner, J. High Energy Phys. 0905 (2009) 021.
[11] G. Koutsoumbas, E. Papantonopoulos, G. Siopsis, J. High Energy Phys. 0907 (2009) 026.
[12] K. Maeda, M. Natsuume, T. Okamura, Phys. Rev. D 79 (2009) 126004.
[13] Julian Sonner, Phys. Rev. D 80 (2009) 084031.
[14] S.A. Hartnoll, Class. Quantum Gravity 26 (2009) 224002, arXiv:0903.3246.
[15] C.P. Herzog, J. Phys. A 42 (2009) 343001.
[16] Qiyuan Pan, Bin Wang, Eleftherios Papantonopoulos, J. Oliveira, A. Pavan, Phys. Rev. D 81 (2010) 106007, arXiv:0912.2475.
[17] M. Ammon, J. Erdmenger, M. Kaminski, P. Kerner, Phys. Lett. B 680 (2009) 516.
[18] S.S. Gubser, C.P. Herzog, S.S. Pufu, T. Tesileanu, Phys. Rev. Lett. 103 (2009) 141601.
[19] Songbai Chen, Liancheng Wang, Chikun Ding, Jiliang Jing, Nucl. Phys. B 836 (2010) 222, arXiv:0912.2397.
[20] P. Horava, Phys. Rev. D 79 (2009) 084008, arXiv:0901.3775.
[21] P. Horava, J. High Energy Phys. 0903 (2009) 020, arXiv:0812.4287.
[22] A. Kehagias, K. Sfetsos, Phys. Lett. B 678 (2009) 123, arXiv:0905.0477.
[23] R.G. Cai, Y. Liu, Y.W. Sun, J. High Energy Phys. 0906 (2009) 010, arXiv:0904.4104.
[24] R.G. Cai, N. Ohta, Phys. Rev. D 81 (2010) 084061, arXiv:0910.2307.
[25] Y.S. Piao, Phys. Lett. B 681 (2009) 1, arXiv:0904.4117.
[26] Remo Garattini, arXiv:0912.0136.
[27] Anzhong Wang, Yumei Wu, J. Cosmol. Astropart. Phys. 0907 (2009) 012, arXiv:0905.4117.
[28] E.O. Colgain, H. Yavartanoo, J. High Energy Phys. 0908 (2009) 021, arXiv:0904.4357.
[29] Y.S. Myung, Y.W. Kim, arXiv:0905.0179.
[30] Y.S. Myung, Phys. Lett. B 684 (2010) 158, arXiv:0908.4132.
[31] T. Nishioka, Class. Quantum Gravity 26 (2009) 242001, arXiv:0905.0473.
[32] R.G. Cai, L.M. Cao, N. Ohta, Phys. Lett. B 679 (2009) 504, arXiv:0905.0751.
[33] Songbai Chen, Jiliang Jing, Phys. Lett. B 687 (2010) 124, arXiv:0905.1409.
[34] Songbai Chen, Jiliang Jing, Phys. Rev. D 80 (2009) 024036, arXiv:0905.2055.
[35] Chikun Ding, Songbai Chen, Jiliang Jing, Phys. Rev. D 81 (2010) 024028.
[36] E. Kiritsis, G. Kofinas, Nucl. Phys. B 821 (2009) 467.
[37] I. Kimpton, A. Padilla, J. High Energy Phys. 1304 (2013) 133, arXiv:1301.6950.
[38] Rong-Gen Cai, Hai-Qing Zhang, Phys. Rev. D 81 (2010) 066003.
[39] K. Lin, E. Abdalla, A.Z. Wang, arXiv:1406.4721.
[40] P. Horava, Phys. Rev. D 79 (2009) 084008.
[41] H. Lu, J. Mei, C.N. Pope, Phys. Rev. Lett. 103 (2009) 091301, arXiv:0904.1595.
[42] Rong-Gen Cai, Li-Ming Cao, Nobuyoshi Ohta, Phys. Rev. D 80 (2009) 024003, arXiv:0904.3670.
[43] I.R. Klebanov, E. Witten, Nucl. Phys. B 556 (1999) 89, arXiv:hep-th/9905104.
[44] R. Gregory, S. Kanno, J. Soda, J. High Energy Phys. 0910 (2009) 010.