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Closer towards inflation in string theory

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Abstract

In brane inflation, the relative brane position in the bulk of a brane world is the inflaton. For branes moving in a compact manifold, the approximate translational (or shift) symmetry is necessary to suppress the inflaton mass, which then allows a slow-roll phase for enough inflation. Following recent works, we discuss how inflation may be achieved in superstring theory. Imposing the shift symmetry, we obtain the condition on the superpotential needed for inflation and suggest how this condition may be naturally satisfied.

Keywords: Inflation; Superstring theory; Cosmology

1. Introduction

It is very likely that our universe has gone through an inflationary epoch. In slow-roll inflation [1] the slow-roll condition imposes a very strong constraint on inflationary model building; in particular, it requires a very small inflaton mass. In simple brane world scenarios inspired by string theory, brane inflation is a natural way to achieve this [2]. For a BPS brane moving in the bulk of the brane world, the translational, or shift, symmetry implies a massless inflaton, that is, the brane position is a massless Goldstone boson. A weak brane–brane interaction breaks this symmetry slightly, so that the shift symmetry is still an approximate symmetry, or equivalently, the inflaton is a pseudo-Goldstone boson. This results in a relatively flat inflaton potential, allowing enough inflation to take place before the branes collide and give birth to the radiation-dominated big bang.

It is obviously important to see how brane inflation may be realized in a more realistic situation, where the compactification of the extra dimensions in superstring theory is dynamically stabilized. Recently, there was just such an attempt (KKLMMT) [3]. In the supergravity approximation, KKLMMT find that the Kähler potential contribution to the inflaton mass is of order $m_\phi^2 \simeq 2H^2/3$, while the slow-roll condition requires that $|m_\phi^2| \leqslant H^2/100$. The slow-roll condition may be reached if the superpotential contribution to m_ϕ^2 cancels that from the Kähler potential. In this Letter, we keep track of the shift symmetry, and identify the condition imposed by the shift symmetry on W and the inflaton potential V. We then suggest that the shift symmetry in W required for slow-roll inflation may appear quite naturally. In short, symmetry argument

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alone may be enough to tell us whether slow-roll inflation will take place or not, even though an explicit determination of W and V may be very difficult.

Here is a brief review and a summary of the scenario. Start with a 4-fold Calabi–Yau manifold in F-theory, or, equivalently, a type IIB orientifold compactified on a 3-fold Calabi–Yau manifold with fluxes to stabilize all but the volume modulus [4]. For large volume, supergravity provides a good description. Next, one introduces a non-perturbative superpotential W that stabilizes the volume modulus in a supersymmetric AdS vacuum. The introduction of $\bar{\text{D}}$3-branes in a warped type IIB background breaks supersymmetry and lifts the AdS vacuum to a metastable de Sitter (dS) vacuum [5] (the KKLT vacuum). To realize inflation, KKLMMT introduces a D3–$\bar{\text{D}}$3-brane pair, whose vacuum energy drives inflation [6]. The $\bar{\text{D}}$3-brane is fixed with the other $\bar{\text{D}}$3-branes and the inflaton ϕ (complex $\phi_i,\ i = 1, 2, 3$) is the (6-dimensional) position (relative to the $\bar{\text{D}}$3-branes) of the D3-brane. Consider the potential for the mobile D3-brane in the D3–$\bar{\text{D}}$3-brane inflationary scenario

$$V(\rho,\phi) = V_{\text{F}}\big(K(r), W(\rho,\phi)\big) + \frac{B}{r^2} + V_{\text{D}\bar{\text{D}}}(\phi), \tag{1}$$

$$K(r) = -3\ln(2r) = -3\ln(\rho + \bar{\rho} - \phi\bar{\phi}), \tag{2}$$

where r is the physical size of the compactified volume and ρ the corresponding bulk modulus. The form of the Kähler potential came from Ref. [7], where r is invariant under the constant shift symmetry:

$$\phi_i \to \phi_i + c_i, \qquad \rho \to \rho + \bar{c}_i\phi_i + \sum c_i\bar{c}_i/2. \tag{3}$$

$V_{\text{F}}(K(r), W(\rho,\phi))$ is the F-term potential, where the superpotential W is expected to stabilize the volume modulus in an AdS supersymmetric vacuum. KKLT then introduces $\bar{\text{D}}$3-branes (the Br^{-2} term) in a Klebanov–Strassler throat that breaks supersymmetry to lift the AdS vacuum to a de Sitter vacuum (a metastable vacuum with a very small cosmological constant and a lifetime larger than the age of the universe). Note that this term leaves the shift symmetry intact. The D3–$\bar{\text{D}}$3 potential $V_{\text{D}\bar{\text{D}}}(\phi)$ is presumably very weak due to warped geometry. This inflaton potential is designed to break the shift symmetry slightly, so inflation can end after slow-roll.

To realize slow-roll inflation in the early universe, $V(\rho,\phi)$ must be very flat in some of the ϕ directions around its minimum where the compactification size is stabilized, that is, some of the ϕ components must have very small masses $|m_\phi^2| \leqslant H^2/100$, where H is the Hubble constant during the inflationary epoch, while the remaining components can be more massive. Hopefully inflation takes place as the D3-brane moves slowly towards the $\bar{\text{D}}$3-branes. Inflation ends when they collide and annihilate. In this scenario, the inflaton potential comes from 2 sources: (1) the inter-brane potential $V_{\text{D}\bar{\text{D}}}(\phi)$, which is sufficiently weak due to the warped geometry, and (2) the D3-brane coupling inside V_{F}. KKLMMT showed that K inside V_{F} alone contributes $2H^2/3$ to m_ϕ^2 for all components of ϕ, while the non-perturbative $W(\rho)$ they used to stabilize r, as it does not depend on ϕ, does not contribute to m_ϕ^2. Since the contributions to m_ϕ^2 by the remaining terms in $V(\rho,\phi)$ around its minimum is negligible, $m_\phi^2 \simeq 2H^2/3$, so the slow-roll condition $\eta \simeq m_\phi^2/H^2 \leqslant 1/60$ is not satisfied. Without a better knowledge of W, this requires a 1 part in 100 fine-tuning. Actually, small η is required during the whole inflationary epoch as the D3-brane moves slowly towards the $\bar{\text{D}}$3-branes, not just at a single value of ϕ. This condition is much more stringent than the 1 in 100 fine-tuning. Such a condition must come from a symmetry argument. Here we keep track of the shift symmetry needed for inflation and identify the condition imposed by the shift symmetry on W.

To realize slow-roll inflation, we should consider a modified W in a way that some of the shift symmetry (3) is left intact in V_{F}. Recall that $W(\rho)$ used in KKLT is obtained from non-perturbative effects in the absence of ϕ. In the presence of a mobile D3-brane, some ϕ dependence in W is quite natural. Motivated by the shift symmetry, we consider

$$W(\rho,\phi) = -w_0 + Ae^{-a(\rho-\kappa_i\phi_i^2/2)}, \tag{4}$$

where w_0, A and a are real parameters. In the absence of ϕ, Eq. (4) reduces to the $W(\rho)$ used in KKLT. For generic parameters κ_i, it is clear that slow-roll conditions is not satisfied. However, demanding that the volume modulus is stabilized in a supersymmetric vacuum in V_{F}, we show that either $\kappa_i = 0$, which yields the original KKLT vacuum (which has no shift symmetry left), or

$$SUSY \to |\kappa_i| = 1 \to \eta \simeq 0 \tag{5}$$

that is, the slow-roll condition is satisfied. More precisely, the supersymmetric vacuum has a degeneracy where 3 components of ϕ are massless while the other 3 have $m^2 \simeq 2H^2/3$, so, during inflation, the D3-brane will fall rapidly to the minimum in the massive ϕ directions and then move slowly along the flat directions. We may set $\kappa_i = 1$ without loss of generality.

More generally, the potential $V_{\rm F}$ from a W of the form (for any real constants β_i)

$$W = F(\sigma)e^{i\beta_i\phi_i}, \quad \sigma = \rho - \phi^2/2 \tag{6}$$

has the remnant shift symmetry ($\phi_i \to \phi_i + \mathrm{Re}(c_i)$) that leaves the 3 real components of ϕ massless and so slow-roll inflation can take place. This is the condition on the D3–D̄3-brane inflation in this scenario. (More precisely, we actually need only one of the $|\kappa_i| = 1$.)

The question reduces to how natural/likely for W to depend (up to the phase factor) only on σ. To address this question, let us consider the non-perturbative interactions of the $SU(N)$ super-Yang–Mills theory due to the N D7-branes wrapping a 4-cycle in the Calabi–Yau manifold. In the absence of the D3-brane, $8\pi^2/g^2_{\rm YM} = \mathrm{Re}(\rho)$. $SU(N)$ strong interaction generically generates $W(\rho) = Ae^{-a\rho}$ in the absence of ϕ. The presence of a D3-brane introduces open string modes stretching between the D3-brane and the N D7-branes, which generically yields a bi-fundamental mode that behaves as a quark charged under $SU(N)$. The mass of the quark is $m = |\phi_{(\rm D7)}|$, where $\phi_{(\rm D7)}$ is ϕ measured with respect to the position of the wrapped D7-branes. (See Fig. 1.) So W should depend on ϕ as well. There should be a subspace of ϕ which has the same quark mass m and so the same value for the $W(\rho,\phi) = W(\Lambda, m)$, where Λ is the scale for $SU(N)$. Naively, this subspace might be the S^5 surface around the D7-branes with a radius given by $m = |\phi_{(\rm D7)}|$, that is, the components of ϕ tangent to this surface are massless Goldstone modes. However, as pointed out above, supersymmetry and the holomorphicity of W allows at most 3 flat directions. For slow-roll inflation, a single flat direction is enough. In this sense, the presence of a remnant shift symmetry in W is quite natural. Once one shows that the shift symmetry is present in the supersymmetric vacuum, the slow-roll condition is generically satisfied. The precise form of W may not be necessary.

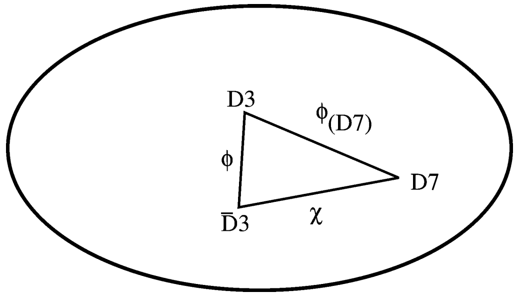

Fig. 1. The relative positions of the wrapped D7-branes, the D3-brane and the D̄3-branes inside a Calabi–Yau manifold. ϕ is the position of the D3-brane relative to the D̄3-brane.

Recently, Ref. [8] discusses the shift symmetry for the D7-brane in the D3–D7 inflationary scenario [9]. We shall comment on this interesting work.

2. The shift symmetry and *W*

Given the Kähler potential and the superpotential W, the potential $V_{\rm F}$ is given by

$$V_{\rm F} = e^K\big(K^{i\bar{j}} D_i W \overline{D_j W} - 3W\overline{W}\big), \tag{7}$$

where

$$K_i = \frac{\partial K}{\partial \varphi_i}, \qquad D_i W = \frac{\partial W}{\partial \varphi_i} + K_i W. \tag{8}$$

Physics is determined by the particular combination G,

$$G = K + \ln(W) + \overline{\ln(W)} \tag{9}$$

since it is invariant under the Kähler transformation

$$K \to K + \xi(\varphi_i) + \overline{\xi(\varphi_i)}, \qquad W \to We^{-\xi(\varphi_i)}. \tag{10}$$

The potential is given uniquely in term of G

$$V_{\rm F} = e^G\left[G^{i\bar{j}}\frac{\partial G}{\partial \varphi_i}\overline{\frac{\partial G}{\partial \varphi_j}} - 3\right], \tag{11}$$

where $G_{i\bar{j}} = K_{i\bar{j}}$.

For a D3-brane with position ϕ in a compact manifold, the Kähler potential is [7]

$$K = -3\ln\big(\rho + \bar{\rho} - k(\phi, \bar{\phi})\big), \tag{12}$$

where ρ is the bulk modulus and ϕ is the position of a mobile D3-brane (in 6 dimensions, we have ϕ_i ($i = 1, 2, 3$)). The physical radius of the Calabi–Yau manifold is given by

$$2r = \rho + \bar{\rho} - k(\phi, \bar{\phi}). \tag{13}$$

Note that r is invariant under the transformation:

$$\begin{aligned} &\rho \to \rho + f(\phi), \\ &\bar{\rho} \to \bar{\rho} + \bar{f}(\phi), \\ &k(\phi, \bar{\phi}) \to k(\phi, \bar{\phi}) + f + \bar{f}. \end{aligned} \tag{14}$$

Let us focus on the simplest choice of $k(\phi, \bar{\phi})$,

$$k(\phi, \bar{\phi}) = \phi\bar{\phi} = \sum \phi_i \bar{\phi}_i. \tag{15}$$

So the above symmetry reduces to the shift symmetry (3). The supergravity effective action is valid only for large r. This is the regime we are interested in. For a constant W, it is easy to check that $V_{\mathrm{F}} = 0$. This is an example of no-scale SUGRA and we have the full shift symmetry. However, we are interested in supersymmetric vacua. Imposing the supersymmetry condition,

$$\begin{aligned} &D_i W = \frac{\partial W}{\partial \phi_i} + W \frac{\partial K}{\partial \phi_i} = 0, \\ &D_\rho W = 0, \end{aligned}$$

we obtain

$$\begin{aligned} &\frac{\partial}{\partial \phi_i} \ln(W) = \frac{-3\bar{\phi}_i}{\rho + \bar{\rho} - \phi\bar{\phi}}, \\ &\frac{\partial}{\partial \rho} \ln(W) = \frac{3}{\rho + \bar{\rho} - \phi\bar{\phi}}. \end{aligned} \tag{16}$$

Let us first consider a toy model with maximum shift symmetry and where the bulk modulus is not stabilized. In this case we can treat Eq. (16) as differential equations. Imposing the holomorphic condition on the supersymmetric W_{s}, we obtain

$$W_{\mathrm{s}} = w_0 \big(\rho - \kappa_i \phi_i^2/2\big)^{3/2}, \tag{17}$$

where the constant w_0 is the normalization and $|\kappa_i| = 1$. The line of the continues solutions of Eq. (16) is given by $\kappa_i = \bar{\phi}_i/\phi_i$, $\rho = \bar{\rho}$. This leaves a subset of the shift symmetry intact: the shift symmetry (3) where the c_i satisfies $\kappa_i = \bar{c}_i/c_i$. Without loss of generality, we can choose $\kappa_i = 1$ by the field redefinition $\phi_{i\,\mathrm{new}} = \sqrt{\kappa_i}\phi_i$. At the supersymmetric AdS vacuum, $V_{\mathrm{F}} = -\frac{3}{8} w_0^2$ and the remnant shift symmetry is given by real c_i $(c_i = \bar{c}_i)$:

$$\begin{aligned} &\phi_i \to \phi_i + c_i, \\ &\bar{\phi}_i \to \bar{\phi}_i + c_i, \\ &\rho \to \rho + \phi c + c^2/2. \end{aligned} \tag{18}$$

Eq. (17) suggests that instead of (ρ, ϕ_i), it is natural to use (σ, ϕ_i), where

$$\sigma = \rho - \frac{\phi^2}{2} = \rho - \sum \frac{\phi_i^2}{2}. \tag{19}$$

That is, σ is invariant under the above $\mathrm{Re}(c_i)$ shift. The transition from (ρ, ϕ) to (σ, ϕ) is always non-singular. In term of (σ, ϕ), $W_{\mathrm{s}}(\sigma) = \omega_0 \sigma^{3/2}$ and

$$2r = \sigma + \bar{\sigma} + \frac{(\phi - \bar{\phi})^2}{2}. \tag{20}$$

We see that the remnant $c = \bar{c}$ shift symmetry will be left intact for any $W(\sigma)$.

Next, we like to find the most general W that stabilizes the compactification volume in a supersymmetric vacuum with some remnant shift symmetry. The physics is more transparent in the (σ, ϕ) coordinate. Let $\ln W(\sigma, \phi) = f(\sigma, \phi)$, the solution at the supersymmetric minimum is given by

$$\begin{aligned} &f_i|_0 \equiv f_{\phi_i}|_0 = \frac{3(\phi_i - \bar{\phi}_i)|_0}{2r_0}, \\ &f_\sigma|_0 = \frac{3}{2r_0}, \end{aligned} \tag{21}$$

where the subscript 0 indicates the supersymmetric minimum point. Under the $\mathrm{Re}(c_i)$ shift transformation, these equations remain invariant only if

$$f_{\sigma i} = f_{ij} = 0. \tag{22}$$

So a solution which has a degenerate supersymmetric minimum has the form

$$f(\sigma, \phi) = h(\sigma) + i\beta_i \phi_i, \tag{23}$$

where $h(\sigma)$ is any function of σ, except being a pure constant, as can be seen from Eq. (21). Furthermore, Eq. (21) indicates that β_i are real numbers. Finally, the form of the superpotential is (for real β_i)

$$W(\sigma, \phi_i) = F(\sigma) e^{i\beta_i \phi_i}. \tag{24}$$

The above superpotential is not $\mathrm{Re}(c_i)$ shift invariant, because of the phase term. However, the combination $\ln W + \overline{\ln W}$ is. This is exactly the contribution of the superpotential to G, as can be seen from Eq. (9). That is, the minimum (the AdS supersymmetric vacuum) of $V_{\rm F}$ is $\mathrm{Re}(c_i)$ shift invariant.

As a comment, let us look at the kinetic part of the action in terms of (σ,ϕ). Calculating the Kähler metrics $K_{i\bar{j}}$ from the Kähler potential (12), we find

$$\begin{aligned}\mathcal{L}_{\rm k} &= \frac{1}{2}\sqrt{g}\,K_{i\bar{j}}\partial_\mu\varphi_i\partial^\mu\bar{\varphi}_j \\ &= \frac{3\sqrt{g}}{8r^2}\big(\partial_\mu\rho\partial^\mu\bar{\rho} + (2r\delta_{ij} + \bar{\phi}_i\phi_j)\partial_\mu\phi_i\partial^\mu\bar{\phi}_j \\ &\qquad - \phi_i\partial_\mu\rho\partial^\mu\bar{\phi}_i - \bar{\phi}_i\partial_\mu\phi_i\partial^\mu\bar{\rho}\big) \\ &= \frac{3\sqrt{g}}{8r^2}\big(\big[2r\delta_{ij} - (\phi_i-\bar{\phi}_i)(\phi_j-\bar{\phi}_j)\big]\partial_\mu\phi_i\partial^\mu\bar{\phi}_j \\ &\qquad + (\phi_i-\bar{\phi}_i)\big(\partial_\mu\phi_i\partial^\mu\bar{\sigma} - \partial_\mu\sigma\partial^\mu\bar{\phi}_i\big) \\ &\qquad + \partial_\mu\sigma\partial^\mu\bar{\sigma}\big).\end{aligned} \tag{25}$$

By construction σ is $\mathrm{Re}(c_i)$ shift invariant. It is clear that the kinetic term $\mathcal{L}_{\rm k}$ is shift invariant, when expressed in either (ρ,ϕ) or (σ,ϕ) basis. In the (σ,ϕ) basis, each term in $\mathcal{L}_{\rm k}$ is explicitly shift invariant.

3. Kähler modulus stabilization and dS vacuum

Following KKLT, we shall introduce non-perturbative quantum corrections to the superpotential that can lead to supersymmetric AdS vacua in which the Kähler modulus is fixed. Such a correction can come from a number of sources: Euclidean D3-branes wrapping some 4-cycles [10], or strongly coupled $SU(N)$ super-Yang–Mills theory associated with the N D7-branes wrapping a 4-cycle in a Calabi–Yau manifold in a type IIB orientifold (or F-theory). The gauge coupling is related to the dilaton and the size of the 4-cycle that the D7-branes wrap on. Since the dilaton and the complex moduli are already stabilized [4], the gauge coupling depends only on the size of the 4-cycle, which is proportional to the size of the Calabi–Yau manifold. So $8\pi^2/g^2_{\rm YM} = \mathrm{Re}(\rho)$. $SU(N)$ strong interaction generically generates $W(\Lambda) = W(\rho)$, where Λ is the QCD scale of the $SU(N)$ super-Yang–Mills theory. In the absence of ϕ and in the large volume regime, $W(\rho) = A\exp(-a\rho)$. For large N, $a \simeq 1/N$.

The presence of a D3-brane introduces ϕ, its position. As shown in Fig. 1, let χ be the separation between the D7-branes and the $\bar{\rm D}$3-branes. Let $\phi_{\rm (D7)}$ be the position of the D3-brane measured relative to the D7-branes, so $|\phi_{\rm (D7)}|$ is the separation distance between the D3-brane and the D7-branes. Then $\phi = \phi_{\bar{\rm D}3}$ is the inflaton. We see that $\rho = \rho_{\bar{\rm D}3}$ and $\rho_{\rm (D7)}$ are different but related:

$$\begin{aligned}\phi_i &= \phi_{{\rm (D7)}i} + \chi_i, \\ \rho &= \rho_{\rm (D7)} + \bar{\chi}\phi_{\rm (D7)} + \frac{\chi\bar{\chi}}{2}.\end{aligned} \tag{26}$$

Using the shift symmetry (3), we see that the physical size of the Calabi–Yau manifold, $2r = \rho + \bar{\rho} - \phi\bar{\phi} = \rho_{\rm (D7)} + \bar{\rho}_{\rm (D7)} - \phi_{\rm (D7)}\bar{\phi}_{\rm (D7)}$, is invariant under the change in coordinates, as it should be.

The presence of a D3-brane introduces open string modes stretching between the D3-brane and the N D7-branes, which generically yields a bi-fundamental mode that behaves as a quark charged under $SU(N)$. The mass of the quark is

$$m = |\phi_{\rm (D7)}|. \tag{27}$$

Since the strong interaction dynamics of the super-Yang–Mills theory depends on the quark mass m, W must depend on ϕ as well [11]. There should be a subspace of ϕ which has the same quark mass m and so the same value for the $W(m,\Lambda) = W(\rho,\phi)$. Treating (naively) the 4-cycle as a point in the Calabi–Yau manifold, one may expect this subspace to be the S^5 surface with radius given by $m = |\phi_{\rm (D7)}|$. That is, the 5 modes tangent to this surface are massless Goldstone modes. However, as we have shown, supersymmetry and the holomorphicity of W allows only 3 flat directions. Still, the above argument suggests that the flat directions should be orthogonal to the $\phi_{\rm (D7)}$ direction. Although the interaction between D3–$\bar{\rm D}$3-branes are weak (and given by $V_{{\rm D}\bar{\rm D}}$), the interaction between $\bar{\rm D}$3-branes and D7-branes is important for the $SU(N)$ strong interaction. There are also bi-fundamental modes stretching between the $\bar{\rm D}$3-branes and the D7-branes that behave like additional quarks of $SU(N)$, with mass m'. Presumably, the stabilization of the dilaton and complex structure moduli fixes m', up to an overall volume scaling which is a function of ρ. This means that $W(m,m',\Lambda) = W(\rho,\phi)$. In general, W can be quite complicated. However, the above argument suggests that its supersymmetric vacuum should have a number of flat directions. For small ϕ, this suggests that the flat direction is orthogonal to the χ direction. A simple extension suggests (with real positive parameters w_0, A and a)

$$W = -w_0 + Ae^{-a\rho} \to \left(-w_0 + Ae^{-a\sigma}\right)e^{i\sqrt{\kappa_i}\,\beta_i\phi_i} \tag{28}$$

that is, the supersymmetric non-perturbative correction to the superpotential W is a function of

$$\begin{aligned}
&\sigma = \rho - \kappa_i\phi_i^2/2,\\
&\kappa_i = -\bar{\chi}_i/\chi_i,\\
&\beta_i = \bar{\beta}_i.
\end{aligned} \tag{29}$$

Other possible solutions lift at least one of the flat directions: for example, we may end up with $\kappa_1 = 0$, $|\kappa_2| = |\kappa_3| = 1$ or $\kappa_1 = \kappa_2 = 0$, $|\kappa_3| = 1$. For slow-roll inflation, all we need is one flat direction. On the other hand, the original solution of KKLT, with $\kappa_1 = \kappa_2 = \kappa_3 = 0$, has no flat direction.

Let us consider the above solution with 3 flat directions. In the absence of ϕ (say after inflation and the annihilation of the D3–$\bar{\text{D}}$3-brane pair), the form of the potential reduces to the well-known one. In the coordinate where χ_i are purely imaginary, $\sigma = \rho - \phi^2/2$. In the large volume limit and small ϕ, the difference between ρ and σ is only a small quantitative correction. Depending on how the 4-cycle is embedded in the Calabi–Yau manifold, it is possible that part of the rotational/translational symmetry is broken. Once the dynamics are better understood, one may be able to put more precise constraints on the 4-cycles that D7-branes can wrap. It will be most interesting to see if the inclusion of a mobile D3-brane yields this form for W. Here, let us simply assume that there are such supersymmetric non-perturbative correction that leaves the shift symmetry intact. Now we have

$$G = \ln\left(\frac{\left|(-\omega_0 + Ae^{-a\sigma})e^{i\beta\phi}\right|^2}{\left(\sigma + \bar{\sigma} + \frac{(\phi-\bar{\phi})^2}{2}\right)^3}\right). \tag{30}$$

The supersymmetric minima is given by $G_\sigma = G_\phi = 0$, which for real σ has the solution

$$\begin{aligned}
&\mathrm{Im}(\phi_{i0}) = r_0\beta_i/3,\\
&\omega_0 = \left(1 + \frac{2a}{3}r_0\right)Ae^{-a(r_0 + \frac{\beta^2}{9}r_0^2)},\\
&V_{\mathrm{AdS}} = -\frac{1}{6r_0}a^2A^2e^{-2ar_0}e^{-\frac{2}{3}\beta^2 r_0(1+\frac{a}{3}r_0)}.
\end{aligned} \tag{31}$$

Next, to lift the AdS vacuum to a dS vacuum, we must break supersymmetry. Again following KKLT, we introduce a $\bar{\text{D}}$3-brane (or more than one), with its position fixed by the fluxes. (One must adjust the 5-form flux so tadpole cancellation remains valid.) This means no additional translational moduli are introduced. The net effect is simply the addition of an energy density of the form $V_{\bar{\text{D}}3} = B/r^2$. Since r is invariant under the remnant shift symmetry, no modification of this term is needed here. To simplify the result we may consider the case where $\beta_i = 0$, corresponding to $(\phi_i - \bar{\phi}_i)_0 = 0$. For $\mathrm{Im}(\phi_i) = 0$, the potential takes the form:

$$\begin{aligned}
V &= V_{\mathrm{F}}(K, W) + V_{\bar{\text{D}}3}\\
&= \frac{aAe^{-ar}}{2r^2}\left(-\omega_0 + Ae^{-ar} + \frac{1}{3}aAre^{-ar}\right) + \frac{B}{r^2},
\end{aligned} \tag{32}$$

which is clearly independent of $\mathrm{Re}(\phi_i)$. Of course, one must fine-tune to obtain a very small cosmological constant. One may choose another mechanism to lift the AdS vacuum to the dS vacuum. The shift symmetry is maintained as long as the lifting is a function of r only.

4. D3–$\bar{\text{D}}$3 inflation

We can now introduce an extra D3–$\bar{\text{D}}$3-brane pair, where the $\bar{\text{D}}$3-brane is stuck with other $\bar{\text{D}}$3-branes while the D3-brane moves towards them. The above analysis for the mobile D3-brane can be used for this additional D3-brane. The resulting vacuum energy drives inflation. The remnant shift symmetry is broken only by the very weak D3–$\bar{\text{D}}$3-brane interaction, namely, $V_{\text{D}\bar{\text{D}}}(\phi)$ in Eq. (1).

In the case where $\beta_i = 0$, the D3-brane rapidly falls to $\mathrm{Im}(\phi) = 0$ at the early stage of the inflationary epoch, and then moves slowly towards $\phi = 0$. For very small $|\phi|$, a tachyonic mode appears and inflation ends quickly, as in hybrid inflation.

For the case where $\beta_i \neq 0$, the situation may be different, since the $\bar{\text{D}}$3-brane is not sitting along the flat direction the D3-brane is moving. For large $\beta_i r_0$, the D3 may not collide with the $\bar{\text{D}}$3, at least classically. Let us consider the case when the D3-brane along the flat direction is close to the $\bar{\text{D}}$3-brane, as described by the following simple potential:

$$V = \frac{1}{2}m_\phi^2\left(\phi - \frac{r_0\beta}{3}\right)^2 - \frac{C}{\phi^4} + \text{const}, \tag{33}$$

where, to simplify the analysis, we consider only one component of ϕ. The last term comes from the $V_{D\bar{D}}(\phi)$. Here $m_\phi \sim H$. In the absence of $V_{D\bar{D}}(\phi)$, the D3-brane will move along $\phi = \text{Im}(\phi) = r_0\beta/3$. However, $V_{D\bar{D}}(\phi)$ will tend to drive the D3-brane towards the $\bar{D}$3-brane. We like $V_{D\bar{D}}(\phi)$ to overcome $m_\phi^2(\phi - \frac{r_0\beta}{3})^2/2$ so that inflation can end quickly.

The condition that the D3-brane collides with $\bar{D}$3-brane along its trajectory is that $V' > 0$, that is V' has no local minimum away from $\phi = 0$. This in turn implies that

$$r_0\beta < \frac{18}{5}\left(\frac{20C}{m_\phi^2}\right)^{1/6}. \tag{34}$$

To estimate this bound, we recall the KKLMMT scenario: $C = \frac{4\pi^2}{\mathcal{N}^2}\varphi_0^8$, where $\mathcal{N}$ is the number of the D3-branes (or 5-form charge) in the original KKLT vacuum, and φ_0 indicates the position of the $\bar{D}$3-brane in the Klebanov–Strassler throat. We require $\phi = \varphi_0$ for inflation to end. We find

$$r_0\beta < 4 \times 10^{-6} m_\phi^{-1/3}. \tag{35}$$

Assuming that inflation take places at the GUT scale and restoring the powers of M_{Pl} in the calculations, we obtain $r_0(\beta M_{\text{Pl}}) < 4 \times 10^4$, which is not very restrictive.

5. D3–D7

Ref. [8] considers the shift symmetry for the D7-branes. Using the Kähler potential for a D7-brane (described by S), they consider the D-term potential [12] and the shift symmetry in a supersymmetric vacuum. The superpotential which has a supersymmetric minimum and is compatible with the shift symmetry $S \to S + c$ for real c has the form

$$W = e^{-\frac{1}{2}S^2} e^{i\gamma S}, \tag{36}$$

where S represents the position of the D7-brane on the Calabi–Yau manifold and γ is a real number, similar to β in D3 case.

For the system of non-interacting D3- and D7-brane, the form of the Kähler potential and the superpotential are expected to be

$$K = -3\ln(\rho + \bar{\rho} - \phi\bar{\phi}) + \bar{S}S,$$
$$W = F(\sigma)e^{-\frac{1}{2}S^2} e^{i(\beta\phi + \gamma S)}. \tag{37}$$

Now the system has four flat directions, three along the (ϕ_i) subspace and one along $\text{Re}(S)$.

Following the above argument, we do expect that the ϕ in Eq. (37) corresponds to $\phi_{(D7)}$ measured with respect to the initial D7-brane position. The inflaton for such a D3–D7-system is the relative positions of the D3-brane and the D7-branes. However, as discussed earlier, we do expect a non-trivial non-perturbative potential between the D3-brane and the D7-branes due to the quark mass in the super-Yang–Mills theory.

6. Discussions

Instead of Eq. (28), one may also use the racetrack idea to stabilize the Kähler modulus [13]. In this scenario, there are more than one strongly interacting gauge groups. The number of flat directions may be reduced by the presence of multi-centers of super-Yang–Mills theories. It will be interesting to find the constraint on the super-Yang–Mills theories due to the shift symmetry.

There will be quark fields coming from the open string modes stretching between the $\bar{D}$3-branes and the D7-branes. They will also have impact on the non-perturbative dynamics of the super-Yang–Mills theory. The dynamical effect of this quark is similar, if not identical, to that for the quark field due to the D3-brane. Since the open string modes between the D3-brane and the $\bar{D}$3-branes are not involved in the super-Yang–Mills dynamics, one may expect that there is an almost flat direction that connects them, that is, the dynamics may place the D3-brane along an almost flat direction that ends on the $\bar{D}$3-branes.

Suppose a fine-tuning is needed to place the $\bar{D}$3-branes along the almost flat direction of the D3-brane. Generically, let us say this is a 1 in 100 type of fine-tuning. It is known that there are many metastable KKLT type vacua, simply by changing the flux quanta. If the early universe is composed of

100 (or more) causally disconnected regions (a rather generic situation), each region may correspond to a different vacuum. If at least one of them goes through a slow-roll inflationary phase, this region grows by an exponential factor, completely dominates over the regions that do not inflate. It is exponentially unlikely that one will end up in anyone of the regions that do not inflate. That is, cosmologically, there is no fine-tuning.

To conclude, even though the determination of W may be quite difficult, physics becomes more transparent when we view from the shift symmetry perspective, which alone may be enough to tell us whether slow-roll inflation will take place or not. If slow-roll brane inflation takes place, the inflationary properties may not be very sensitive to the details of W.

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