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Preference-based English reverse auctions

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ABSTRACT

This paper studies English reverse auctions within a unified framework for preference-based English reverse auctions. In this context, and particularly for electronic auctions, representing and handling the buyer's preferences, so as to enable him/her to obtain the best possible outcome, is a major issue. Existing auction mechanisms, which are based on single or multi-attribute utility functions, are only able to represent transitive and complete preferences. It is well known, however, in the preference modeling literature that more general preference structures, allowing intransitivity and incomparability, are more appropriate to capture preferences. On the other hand, we must also consider properties on the evolution and, above all, on the outcome of any auction executed by an auction mechanism. These properties, as well as properties of non-dominance and fair competition defined for multiple criteria auctions, impose restrictions on the preference relation. This leaves room for interesting preference models to be implemented within English reverse auction mechanisms.

1. Introduction

The automation of auctions over the web has been raising new research perspectives, ranging across various domains such as auction theory [12,21], agent technology [19,22,35,36], and decision theory [5,15,25,32]. An auction is a competitive mechanism to allocate resources to a buyer based on predefined rules. These rules define the bidding process, how the winner is determined, and the final agreement. Software agents are increasingly being used to represent humans in electronic auctions [1,4,14,18–20,23,35]. These agents can systematically conduct a wide variety of auctions, on behalf of buyers, mediators, or sellers, and can make rapid decisions about bid selection, winner determination, or bid submission.

The four basic auction protocols are English, Dutch, first-price, and second-price or Vickrey (see, e.g., [21]). The reverse version of these protocols, used in e-procurement markets, is when a buyer plays the role of the auctioneer, whereas sellers play the role of bidders. Among these, the English reverse auction protocol is the most popular one for procurement processes. In this paper we focus on the English reverse auction protocol. The price-only English reverse auction protocol, which is prevailing, is an iterative process with a deadline, where sellers compete on the price in order to sell a single item to a unique buyer [11,21]. The buyer specifies the opening bid price and a bid decrement. At each round, each seller may overbid by proposing a bid which is cheaper than the current best bid by at least the bid decrement. The auction stops with the current best bid and the corresponding seller when no other seller can overbid. Multi-attribute auctions represent an extension to standard auction theory [7,27]. They allow negotiating on multiple attributes, involving not only the price, but also other attributes such as quality, delivery terms and conditions. The buyers reveal their preferences on the item to be purchased and sellers compete both on price and non-price attributes to win the contract. The multi-attribute English

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reverse auction protocol has been applied in various domains related to provision of goods [6] and allocation of services [8, 13,34]. In particular, it has been adopted officially by the European Community through directives on public procurement whose stated objectives are to decrease contracting costs, increase transparency and achieve better economic outcomes as a result of increased competition [10,30].

A crucial issue in the design of multi-attribute auctions is the way of modeling and exploiting the buyer's preferences so as to ensure the best possible outcome for the buyer. Multi-attribute auction mechanisms proposed in the literature are often based on a linear or quasi-linear utility function representing the buyer's preferences. Che [9] first introduced first-price and second-price auction mechanisms for two-dimensional reverse auctions in which suppliers bid on both price and quality. Then, David et al. [13,14] extended Che's work to an arbitrary number of attributes; they also designed an English reverse auction protocol for the general case of multi-attribute auctions. Still in the context of iterative reverse auctions, Parkes and Kalagnanam [28] designed an auction mechanism, where sellers are iteratively required to set a price for each attribute value. Engel and Wellman [16] extended this work considering dependencies among attributes. Other approaches are based on the explicit construction of criteria. Bichler [6] implemented and experimented multi-attribute auctions in a market place, where criterion values are aggregated using a weighted sum. As an alternative to the weighted sum, Bellosta et al. proposed a multiple criteria English reverse auction mechanism based on reference points [3,4], where the buyer's preferences and relative importance of criteria are not expressed in terms of weights, but more directly in terms of required values on the criteria.

All the above-mentioned auction mechanisms are based on a value or scoring function, which amounts to considering that the preference relation is transitive and complete. However, as well known in the literature on preference modeling, imposing these properties is questionable when aiming to represent preferences. Indeed, transitivity of indifference is often contradicted in practice. This occurs, in particular, when slight differences between two alternatives are not deemed significant and give rise to an indifference between these alternatives. In this case, a chain of such indifferences may correspond to a large difference between the first and last alternatives of this chain, leading to a preference for one over the other (see, e.g., [24]). Even for strict preference, one may observe intransitivities [26,33], particularly when preferences are multidimensional. Moreover, it is sometimes relevant to model preferences using incomparability, e.g. when the objects to be compared have strongly conflicting evaluations [29]. Consider for instance a buyer willing to purchase a car on the basis of two criteria, price and speed. This buyer may not wish to compare a fast but expensive car to a cheap but slow car, above all if he/she is interested in cars with medium price and speed. In the specific context of multiple criteria auctions, De Smet [15] also suggested considering incomparability situations when bids to be compared are quite different. For these reasons, we assume in this paper that the buyer's preferences are represented by a binary preference relation which is not necessarily transitive and complete.

We must take into account, however, that we are representing preferences in the context of auctions for which we should also consider some natural properties regarding the evolution and, above all, the final result of the auction. This last point refers to the issue of efficiency of the auction [38]. In this paper, we study the impact of intransitivity and/or incomparability on these properties and identify minimal conditions on the buyer's preference relation which ensure such properties.

Moreover, since many auction mechanisms are based on multidimensional preferences, we also focus on the case where the buyer's preference relation results from the aggregation of several criteria. In this context, additional properties in terms of *non-dominance of the winning bid* and *fair competition between non-dominated bids* should be satisfied. Such properties are commonly used in multiple criteria decision analysis [37] to characterize a decision procedure:

- *Non-dominance* requires that a decision procedure selects a non-dominated alternative, i.e. an alternative such that any other alternative which is better on one criterion, is worse on another criterion.
- *Fair competition* is satisfied by a decision procedure if for any non-dominated alternative there exists at least one set of values of the aggregation model parameters which enables the procedure to select the alternative.

In the context of preference-based English reverse auctions, we need to adapt these properties taking into account the fact that an auction is an iterative process where bids are progressively available. Moreover, a given potential bid is not necessarily proposed during an auction since this depends on each seller strategy and on the pressure of the competition between sellers. We investigate, here again, minimal conditions on the buyer's preference relation which ensure satisfaction of these properties. Considering these conditions, typical classes of preference relations are evaluated and discussed.

The main contribution of this paper is threefold. First, we propose a conceptual framework, called *PERA* (**P**reference-based **E**nglish **R**everse **A**uctions), for designing preference-based English reverse auctions within which buyer's preferences are represented by a binary preference relation. The basic purpose of this framework is to study all the auction mechanisms in a unified way. This framework can take into account price-only and existing multi-attribute auctions mechanisms, as well as mechanisms based on more general preference relations, relaxing transitivity and completeness. Second, we focus on mechanisms which involve preference relations resulting from the aggregation of multiple criteria. The framework allows us to analyze them according to two fundamental properties (non-dominance of the winning bid and fair competition between bids). Since most classical auction mechanisms do not satisfy both properties, we show how to design mechanisms which satisfy them. Finally, this framework integrates a generic algorithm that allows a buyer agent to manage English reverse auctions providing bid evaluation, bid selection and request formulation.

The remainder of this paper is structured as follows. Section 2 introduces framework *PERA* and the generic algorithm, which supports the execution of preference-based English reverse auctions, both in asynchronous and synchronous modes. Section 3 identifies and investigates properties related to the improving nature and efficiency of auctions executed by *PERA*. A specialization of *PERA*, called *MERA*, in the case where preferences result from the aggregation of the multiple criteria is introduced in Section 4. Properties of non-dominance of the winning bid and fair competition between non-dominated bids are introduced and studied respectively in Sections 5 and 6. Section 7 shows how to design auction mechanisms ensuring these properties and provides a detailed illustrative example. Conclusions are provided in a final section.

2. Framework *PERA*

Framework *PERA* integrates a generic algorithm that allows a buyer agent to manage Preference-based English reverse auctions. In this section, we present this algorithm and show how it can be customized for asynchronous and synchronous auctions. Finally, we define the notion of a *PERA* mechanism.

We first introduce the following notations.

- B, the *set of potential bids*, where any bid is characterized by a vector of attribute values.
- B^s, the set of potential bids of seller s.
- B^{-s}, the set of potential bids of all sellers except seller s.
- P^i, the set of bids received at round i.
- $\overline{P^i}$, the set of bids received until round i: $\overline{P^i} = \bigcup_{j=1}^{i} P^j$.
- P, the set of bids received during the auction.
- $\succsim$, the preference relation defined on B that models the buyer's preferences on the item to be purchased. Three basic relations can be defined from $\succsim$:
  - a strict preference relation $\succ$, corresponding to the asymmetric part of $\succsim$, where $a \succ b$ if and only if $a \succsim b$ and $\neg(b \succsim a)$,
  - an indifference relation $\sim$, corresponding to the symmetric part of $\succsim$, where $a \sim b$ if and only if $a \succsim b$ and $b \succsim a$,
  - an incomparability relation ?, where $a?b$ if and only if $\neg(a \succsim b)$ and $\neg(b \succsim a)$.
- Given two binary relations $\mathcal{R}$ and $\mathcal{S}$ defined on B, $\mathcal{R} \subset \mathcal{S}$ is equivalent to $a\mathcal{R}b \Rightarrow a\mathcal{S}b$.

2.1. *Algorithm* PERA

Algorithm *PERA*, described in Algorithm 1, generalizes the price-only English reverse auction algorithm in two ways:

- The price criterion, used to compare bids, is replaced by the buyer's preference relation $\succsim$.
- The beat-the-quote rule [38], requiring that a new bid has a lower price than the current best bid by a given decrement, is generalized taking into account a relation $\succ_r$ defined on B, called the request relation. Relation $\succ_r$ is asymmetric and stronger than relation $\succ$, that is such that $\succ_r \subset \succ$. Thus, the generalized rule requires that a new bid must be $\succ_r$-preferred to the current best bid.

The main assumptions of algorithm *PERA* are:

- Each auction deals with a single unit of an item, a fixed set of sellers, and has a fixed deadline.
- Each seller proposes at most one bid at each round.
- The seller who proposed the best bid at the current round is not called upon for the next round.
- At any round, if none of the called upon sellers has provided a bid before the round time limit then none of them owns a bid satisfying the request constraint.

Algorithm 1 *PERA*($\succsim, \succ_r$)

Announce to the sellers requirements on the item to be purchased, the round time limit, and the closing time
$best^0 \leftarrow nil$
$i \leftarrow 1$
repeat
    Collect the set of valid bids P^i until the round time limit is reached
    if $P^i \neq \emptyset$ **then**
        Select the current best bid: $best^i \leftarrow select(\succsim, P^i)$
        Compute the new request constraint: $c^i \leftarrow request(\succ_r, best^i)$
        Announce the new request c^i to the sellers
        $i \leftarrow i + 1$
    end if
until ($P^i = \emptyset$) or ($t >$ closing time)
$b^* \leftarrow best^{i-1}$
return b^*

At the beginning of the auction, the buyer indicates to the set of sellers the requirements on the item to be purchased, the round time limit, and the closing time. The buyer collects the set of bids until the round time limit is reached. When the set of bids proposed during the current round is not empty, the buyer selects the current best bid as the reference bid and formulates feedback information for the next round. Feedback information consists of a *request constraint* that forces any new bid to beat the current best bid. The auction ends either when no bid has been proposed within the round time or when the closing time is reached. In the first case, the auction is said to end *naturally*. At the end of the auction, the current best bid becomes the winning bid.

Selection of the current best bid *(step 7 of Algorithm 1).* The current best bid is selected applying a function called *select* which takes as arguments a preference relation $\succsim$ and a non-empty set X of bids, and returns one bid from set X:

$$\mathit{select}(\succsim, X) \in X$$

This function is defined precisely according to the synchronization mode.

Definition of the request constraint *(step 8 of Algorithm 1).* The request constraint is defined applying a function called *request* which takes as arguments a request relation $\succ_r$ and a bid a, and returns a constraint imposing that bids should be $\succ_r$-preferred to bid a:

$$\mathit{request}(\succ_r, a)(b) \quad \Leftrightarrow \quad b \succ_r a$$

At each round $i \geqslant 1$, the request constraint $c^i = \mathit{request}(\succ_r, \mathit{best}^i)$ asks for bids that are $\succ_r$-preferred to the current best bid best^i. The bids that satisfy the request constraint c^i are said to be *valid*.

Owing to the definition of the request constraint, we get the following remarks.

Remark 1. In any *PERA* auction where the request relation $\succ_r$ is transitive, any seller unable to satisfy the current request constraint at round i, is unable to satisfy further request constraints, unlike where the request relation $\succ_r$ is intransitive.

Remark 2. In any *PERA* auction, we have $\mathit{best}^{i+1} \succ_r \mathit{best}^i$, for $i \geqslant 1$.

This remark provides an obvious necessary and sufficient condition in order to prevent cycling in the algorithm.

Proposition 1. *Algorithm PERA($\succsim, \succ_r$) does not cycle if and only if relation $\succ_r$ is acyclic.*

As a consequence, we impose that relation $\succ_r$ used in algorithm *PERA*($\succsim, \succ_r$) is acyclic.

In the next two subsections, we introduce two specialized versions of algorithm *PERA* called respectively asynchronous-*PERA* and synchronous-*PERA*.

2.2. *Asynchronous* PERA *auctions*

Asynchronous English reverse auctions often occur in the context of real-time bid submissions and are largely used in sourcing of heterogeneous goods and services [2,14,31,34].

In an asynchronous *PERA* auction, any seller may propose a valid bid at any moment before the closing time is reached. The round time limit is reached as soon as one bid is received. Thus, at each round, the buyer collects only one bid. Therefore, function *select* merely consists of returning this element as the current best bid.

The following remark outlines the behavior of any asynchronous *PERA* auction.

Remark 3. In any asynchronous *PERA* auction, we have $P^i = \{\mathit{best}^i\}$ and $\overline{P^i} = \{\mathit{best}^1, \ldots, \mathit{best}^i\}$, for $i \geqslant 1$.

2.3. *Synchronous* PERA *auctions*

Synchronous English reverse auctions often occur in the context of sealed bid auctions where each seller proposes his/her bid without knowing the bids of the other sellers. They are recommended in government procurement procedures [10,30] because they ensure fair competition between sellers.

At each round of a synchronous *PERA* auction, each seller either sends one valid bid before the round time limit or informs that he/she does not participate at this round. The buyer collects the set of bids proposed by the sellers and selects the current best bid as the reference bid. Therefore, function *select* is defined as follows.

Given a preference relation $\succsim$ and a non-empty set of bids X, function *select* selects arbitrarily any element in the set of maximal elements of X:

$$\mathit{select}(\succsim, X) \in M(\succsim, X) = \{b \in X \mid \forall b' \in X, \neg(b' \succ b)\}$$

We recall that acyclicity of relation $\succ$ is a necessary and sufficient condition to ensure that $M(\succsim, X)$ is not empty for any non-empty set X. Thus, we impose that relation $\succ$ is acyclic to ensure that function *select* always returns one bid in any non-empty set of bids.

2.4. PERA *mechanisms*

Various mechanisms can be defined as restrictions of algorithm *PERA* by specifying a class of preference relations and a class of request relations. A *PERA* mechanism, denoted by $PERA_{\mathcal{P},r}$, is defined by:

- $\mathcal{P}$, a *class of preference relations*,
- r, a *request mapping function* that associates to any preference relation $\succsim \in \mathcal{P}$ a class of request relations denoted by $r(\succsim)$.

Depending on the synchronization mode, we get the following definition of a well-defined mechanism.

Definition 1. A mechanism $PERA_{\mathcal{P},r}$ is well-defined if and only if:

1. For any $\succsim \in \mathcal{P}$, any $\succ_r \in r(\succsim)$ is stronger than $\succ$, i.e. such that $\succ_r \subset \succ$.
2. For any $\succsim \in \mathcal{P}$, any $\succ_r \in r(\succsim)$ is *acyclic*.
3. In the synchronous mode, for any $\succsim \in \mathcal{P}$, $\succ$ is *acyclic*.

Condition 1 corresponds to the generalization of the beat-the-quote rule. Conditions 2 and 3 guarantee that algorithm *PERA* does not cycle.

Considering that any transitive and complete relation defined on a finite or countable set can be represented by a utility (or value) function u (see, e.g., [17]), we introduce the following mechanism.

Example 1. Mechanism $PERA_{\mathcal{P}_U,r_U}$, denoted by $PERA_U$, is defined by:

- $\mathcal{P}_U$, the class of preference relations $\succsim$ such that
$$a \succsim b \quad \Leftrightarrow \quad u(a) \geqslant u(b)$$
where u denotes a utility (or value) function.
- $r_U(\succsim)$, the class of request relations $\succ_r$ such that
$$a \succ_r b \quad \Leftrightarrow \quad a \succ b \quad \text{and} \quad u(a) \geqslant u(b) + \varepsilon$$
where $\varepsilon \geqslant 0$ is an increment on u.

Observe that when $\varepsilon = 0$, relation $\succ_r$ coincides with $\succ$, i.e. $a \succ_r b$ *if and only if* $u(a) > u(b)$ and when $\varepsilon > 0$, we get $a \succ_r b$ *if and only if* $u(a) \geqslant u(b) + \varepsilon$. Choosing a request relation $\succ_r \in r(\succsim)$ amounts to setting an auction step ε. In this way the beat-the-quote rule is directly satisfied.

Moreover, mechanism $PERA_U$ is well-defined since it satisfies all the conditions of Definition 1.

In this section, we introduced framework *PERA* allowing the management of auctions while accepting general preference relations not necessarily transitive and complete. The main restriction is acyclicity of the request relation $\succ_r$, for the asynchronous mode, and acyclicity of the asymmetric part of the preference relation $\succ$, for the synchronous mode.

3. Properties of *PERA* auctions

In this section, we first study properties related to the evolution of *PERA* auctions. Finally, we investigate efficiency of *PERA* auctions.

3.1. *Evolution of* PERA *auctions*

Evolution of reverse English auctions refers to properties which could be considered as the auction progresses [38,15]. The price-only English auction achieves its improving nature by requiring that a new bid be cheaper than the current best bid. In *PERA* auctions, this property is generalized into three properties in order to take into account preference relations which are not necessarily complete and transitive:

- **[MBB]** *Maximality of the current best bid*. At each round $i \geqslant 1$, none of the bids received until round i is preferred to the current best bid: $best^i \in M(\succsim, \overline{P^i})$.

- **[IBB]** *Improved best bids.* At each round $i \geqslant 2$, the current best bid is strictly preferred to any previous best bid: $best^i \succ best^j$, for $j = 1, \ldots, i-1$.
- **[NCB]** *No cycling on bids.* At each round $i \geqslant 2$, any proposed bid has not been proposed previously: $P^i \cap \overline{P^{i-1}} = \emptyset$.

For both synchronization modes, the only dependence between the above properties is:

Proposition 2. [MBB] $\Rightarrow$ [NCB].

Proof. Assume, by contradiction, that [NCB] is not satisfied. Then, there exists $i \geqslant 2$ and $b \in \overline{P^{i-1}}$ such that $b \in P^i$, i.e. such that $b \succ_r best^{i-1}$ and thus such that $b \succ best^{i-1}$, since $\succ_r \subset \succ$. This contradicts [MBB] for $best^{i-1}$. □

The three above properties are trivially satisfied by any auction $PERA(\succsim, \succ_r)$, when both relations $\succ$ and $\succ_r$ are transitive, but might not be satisfied otherwise as shown in the illustrative example presented in Section 7 .

3.1.1. Evolution of asynchronous PERA *auctions*

We provide conditions on relations $\succsim$ and $\succ_r$ so that asynchronous-*PERA* ensures satisfaction of properties [MBB], [IBB], and [NCB]. First we give the following results.

Proposition 3. *In any asynchronous PERA auction we have*:

1. [IBB] $\Rightarrow$ [MBB].
2. [NCB] *is satisfied.*

Proof. See Appendix A.1. □

Proposition 4. *Algorithm asynchronous-PERA*$(\succsim, \succ_r)$ *ensures*:

- [MBB] *if and only if*

$$\forall n \geqslant 3,\ \forall b^1, \ldots, b^n \in B, \quad b^{j+1} \succ_r b^j, \quad j = 1, \ldots, n-1 \quad \Rightarrow \quad \neg\big(b^1 \succ b^n\big) \tag{1}$$

- [IBB] *if and only if*

$$\forall n \geqslant 3,\ \forall b^1, \ldots, b^n \in B, \quad b^{j+1} \succ_r b^j, \quad j = 1, \ldots, n-1 \quad \Rightarrow \quad b^n \succ b^1 \tag{2}$$

Proof. See Appendix A.2. □

As corollaries of Proposition 3, we provide now more interpretable sufficient conditions to ensure satisfaction of [MBB] and [IBB].

Corollary 1. *If* $\succ$ *is* acyclic *then algorithm asynchronous-PERA*$(\succsim, \succ_r)$ *ensures* [MBB].

Proof. Since $\succ_r \subset \succ$, $b^{j+1} \succ_r b^j$ implies $b^{j+1} \succ b^j$, $j = 1, \ldots, n-1$. Acyclicity of $\succ$ implies then $\neg(b^1 \succ b^n)$ and establishes (1) in Proposition 4. □

Corollary 2. *If* $\succ$ *is* transitive *or if* $\succ_r$ *is* transitive, *then algorithm asynchronous-PERA*$(\succsim, \succ_r)$ *ensures* [MBB] *and* [IBB].

Proof. If $\succ$ is transitive or if $\succ_r$ is transitive, (2) in Proposition 4 is satisfied due to $\succ_r \subset \succ$. Thus [IBB] is satisfied. From Proposition 3, satisfaction of [IBB] implies satisfaction of [MBB]. □

3.1.2. Evolution of synchronous PERA *auctions*

We first provide necessary and sufficient conditions on relations $\succsim$ and $\succ_r$ so that synchronous-*PERA* ensures satisfaction of properties [MBB], [IBB], and [NCB] for any auction it executes.

Proposition 5. *Algorithm synchronous-PERA*$(\succsim, \succ_r)$ *ensures*:

- [MBB] *if and only if*

$$\forall n \geqslant 2,\ \forall a, b^1, \ldots, b^n \in B, \quad \neg\big(a \succ b^1\big) \quad \textit{and} \quad b^{j+1} \succ_r b^j, \quad j = 1, \ldots, n-1 \quad \Rightarrow \quad \neg\big(a \succ b^n\big) \tag{3}$$

Table 1
Sufficient conditions on $\succ$ and $\succ_r$ ensuring [MBB], [IBB], and [NCB].

	[MBB]	[IBB]	[NCB]
Asynchronous-*PERA* (minimal condition: $\succ_r$ acyclic)			
$\succ_r$ acyclic	–	–	✓
$\succ$ acyclic	✓	–	✓
$\succ_r$ transitive	✓	✓	✓
$\succ$ transitive	✓	✓	✓
Synchronous-*PERA* (minimal condition: $\succ$ acyclic)			
$\succ$ acyclic	–	–	–
$\succ_r$ transitive	–	✓	✓
$\succ$ transitive	✓	✓	✓

- [IBB] *if and only if*

$$\forall n \geqslant 3,\ \forall b^1, \ldots, b^n \in B, \quad b^{j+1} \succ_r b^j, \quad j = 1, \ldots, n-1 \quad \Rightarrow \quad b^n \succ b^1 \tag{4}$$

- [NCB] *if and only if*

$$\forall n \geqslant 2,\ \forall a, b^1, \ldots, b^n \in B, \quad \neg\big(a \succ b^1\big) \quad and \quad b^{j+1} \succ_r b^j, \quad j = 1, \ldots, n-1 \quad \Rightarrow \quad \neg\big(a \succ_r b^n\big) \tag{5}$$

Proof. See Appendix A.3. □

As corollaries of the previous propositions, we provide now more interpretable sufficient conditions on relations $\succsim$ and $\succ_r$ to ensure satisfaction of [MBB], [IBB], and [NCB].

Corollary 3. *If* $\succ$ *is* transitive *then algorithm synchronous-PERA*($\succsim, \succ_r$) *ensures* [MBB], [IBB], *and* [NCB].

Proof. Observe that, since $\succ_r \subset \succ$, $b^{j+1} \succ_r b^j$ implies $b^{j+1} \succ b^j$, $j = 1, \ldots, n-1$. Then transitivity of $\succ$ implies $b^n \succ b^1$. This establishes (4) in Proposition 5, ensuring [IBB].

Using the same observation, for all $a \in B$, such that $\neg(a \succ b^1)$, $b^n \succ b^1$ and transitivity of $\succ$ imply $\neg(a \succ b^n)$. This establishes (3) in Proposition 5, ensuring [MBB]. Finally, from Proposition 2, [NCB] is also satisfied. □

Corollary 4. *If* $\succ_r$ *is* transitive *then algorithm synchronous-PERA*($\succsim, \succ_r$) *ensures* [IBB] *and* [NCB].

Proof. Observe that, by transitivity of $\succ_r$, $b^{j+1} \succ_r b^j$, $j = 1, \ldots, n-1$, implies $b^n \succ_r b^1$.

Then inclusion $\succ_r \subset \succ$ implies $b^n \succ b^1$ and this establishes (4) in Proposition 5, ensuring [IBB].

From inclusion $\succ_r \subset \succ$, we get $\neg(a \succ b^1)$ implies $\neg(a \succ_r b^1)$. Moreover, using the initial observation, we get $b^n \succ_r b^1$. Then, transitivity of $\succ_r$ implies $\neg(a \succ_r b^n)$. This establishes (5) in Proposition 5, ensuring [NCB]. □

The above results, summarized in Table 1, allow us to draw some general conclusions. First, when $\succ$ is transitive, the three properties are satisfied for both synchronization modes. Therefore, classical auction mechanisms, which are based on transitive and complete preference relations, do satisfy these properties. Nevertheless, this shows also that we can consider auction mechanisms, handling non-transitive indifference or admitting incomparability, while preserving the three properties. Second, conditions to satisfy these properties are always stronger in the synchronous mode than in the asynchronous mode.

3.2. Efficiency

Efficiency of a price-only auction requires that one of the bidders with the cheapest bid wins the auction. Equivalently, none of the bidders, except maybe the winning bidder, can propose a bid cheaper than the winning bid. These two definitions, although equivalent, generalize into two different formulations in the context of preference-based auctions: *efficiency* requires first that if one of the bidders can provide a bid strictly preferred to any other bid, then he/she wins the auction, and second that none of the bidders, except may be the winning bidder, can propose a bid which is strictly preferred to the winning bid. However, the use of a bid decrement ε in a price-only auction, does not ensure *efficiency* but only a weaker form of efficiency within ε. In a preference-based auction, which uses a relation $\succ_r$ to formulate requests, *efficiency* is extended in order to take into account relation $\succ_r$ instead of $\succ$. This leads to the following definition.

Definition 2. A preference-based auction is said to be *quasi efficient* if and only if:

- The winning bid b^*, proposed by the winning seller s^* is $\succ_r$-maximal in the set B^{-s^*}: $\forall b \in B^{-s^*}, \neg(b \succ_r b^*)$.
- If one of the sellers, $\tilde{s}$, owns at least one bid $\succ_r$-preferred to any potential bid in $B^{-\tilde{s}}$, then seller $\tilde{s}$ wins the auction.

Obviously, any form of *efficiency* cannot be ensured if the auction is stopped prematurely due to the closing time. Therefore, such properties are meaningful only under the assumption that the auction ends naturally. We will not state the assumption for the sake of brevity.

Proposition 6. *Algorithm PERA ensures* quasi efficiency.

Proof.

- The winning bid is such that no seller, except possibly the winning seller, can provide a bid that is $\succ_r$-preferred to the winning bid.
- If one of the seller, $\tilde{s}$, owns one bid, $b^{\tilde{s}}$, $\succ_r$-preferred to any other bid, then he/she wins the auction. Indeed, if needed, seller $\tilde{s}$ is always able to propose $b^{\tilde{s}}$ to win the auction. □

One should notice that *quasi efficiency* is ensured without imposing any additional restriction on $\succsim$ or $\succ_r$. Actually, this is an intrinsic property of algorithm *PERA*.

4. Multiple criteria English Reverse Auctions

From now on, we focus on auctions using preference relations resulting from the aggregation of multiple criteria. This corresponds to situations where preferences regarding the item to be purchased are multidimensional and conflicting. In the following, this specific class of *PERA* mechanisms is referred to as *MERA* (Multiple criteria English Reverse Auctions). Some of these *MERA* mechanisms are based on a utility function which aggregates these criteria. In this case, the preference relations are transitive and complete. However, we also consider *MERA* mechanisms which involve preference relations using veto thresholds, which are not necessarily transitive and complete.

In this context, any bid $b \in B$ is characterized by p criterion values $(c_1(b), \ldots, c_p(b))$, where c_j is a criterion function that associates to any bid a value in domain $C_j \subset \mathbb{R}$ such that for any $b, b' \in B$, $c_j(b) \geqslant c_j(b')$ implies that b is at least as good as b' for the buyer, regarding the viewpoint represented by criterion c_j, $j = 1, \ldots, p$. In the following, we denote for simplicity $b_j = c_j(b)$.

Let us now recall some common concepts and notations.

- $C = C_1 \times \cdots \times C_p \subset \mathbb{R}^p$, the criterion space.
- $\underline{\Delta}$, the *dominance relation* defined on B such that for any $b, b' \in B$, $b \underline{\Delta} b'$ if and only if $b_j \geqslant b'_j$, $j = 1, \ldots, p$. We denote by Δ the asymmetric part of $\underline{\Delta}$, where $b \Delta b'$ if and only if $b \underline{\Delta} b'$ and there exists $j \in \{1, \ldots, p\}$ such that $b_j > b'_j$. A bid $b \in X \subset B$ is *non-dominated* in X if and only if there is no $b' \in X$ such that $b' \Delta b$. Moreover, we assume that any preference relation $\succsim$ defined on B does not violate dominance, i.e. satisfies $\underline{\Delta} \subset \succsim$.
- Considering a function $u : \mathbb{R}^p \to \mathbb{R}$, u is:
  - monotonically increasing if and only if for any $z, z' \in \mathbb{R}^p$, $z_j \geqslant z'_j$, $j = 1, \ldots, p$, implies that $u(z) \geqslant u(z')$,
  - strongly monotonically increasing if and only if for any $z, z' \in \mathbb{R}^p$, $z_j \geqslant z'_j$, $j = 1, \ldots, p$ and $z \neq z'$, implies that $u(z) > u(z')$.

  By reversing the previous inequality, we obtain definitions for monotonically decreasing and strongly monotonically decreasing functions.

4.1. Multiple criteria English Reverse Auction mechanisms based on an aggregation function

Mechanism $MERA_{\mathcal{P}_{U}, r_U}$, denoted by $MERA_U$, is a $PERA_U$ mechanism based on a real aggregation function u defined on $\mathbb{R}^p$.

We illustrate now some particular $MERA_U$ mechanisms based on aggregation functions often used or proposed in practice: the weighted sum [10,6], reference point-based functions [4,3] and the lexicographic order [10].

Example 2. Mechanism $MERA_{\mathcal{P}_{\Sigma}, r_{\Sigma}}$, denoted by $MERA_{\Sigma}$ is a $MERA_U$ mechanism based on a weighted sum function u_ω, which is a strongly monotonically increasing function defined as follows:

$$u_\omega(a) = \sum_{j=1}^{p} \omega_j a_j \quad \text{where } \omega_j > 0 \text{ is the weight associated to criterion } j,\ j = 1, \ldots, p$$

Table 2
Impacts of veto thresholds.

$\succsim$	$\succsim_v$
$a \succ b$	$a\,(\succ_v \cup ?_v)\,b$
$a\,?\,b$	$a\,?_v\,b$
$a \sim b$	$a\,(\succ_v \cup \succ_v^{-1} \cup\, ?_v \cup \sim_v)$

Example 3. Mechanism $MERA_{\mathcal{P}_R,r_R}$, denoted by $MERA_R$, is a $MERA_U$ mechanism using a reference point-based function u_α, which is a monotonically decreasing function defined as follows:

$$u_\alpha(b) = \max_{j=1,\dots,p} \bigl\{\lambda_j(\alpha_j - b_j)\bigr\}$$

where $\alpha \in C$ specifies the aspiration point of the buyer on the item to be purchased, and λ_j, $j = 1,\dots,p$, is a scaling factor aiming at normalizing differences expressed on heterogeneous criterion scales.

Example 4. Mechanism $MERA_{\mathcal{P}_L,r_L}$, denoted by $MERA_L$, is a mechanism based on the lexicographic order which can be viewed as a strongly monotonically increasing function. This mechanism is defined by:

- $\mathcal{P}_L$, the class of preference relations $\succsim$ such that:

$$a \succsim b \quad \Leftrightarrow \quad \begin{cases} a = b \quad \text{or} \\ \exists k \in \{1,\dots,p\}, \quad a_{\pi(k)} > b_{\pi(k)} \quad \text{and} \\ a_{\pi(j)} = b_{\pi(j)}, \quad \text{for } j \leqslant k-1 \end{cases}$$

  where π denotes a permutation of set $\{1,\dots,p\}$.
- $r_L(\succsim)$, the class of preference relations $\succ_r$ such that:

$$a \succ_r b \quad \Leftrightarrow \quad \begin{cases} a \succ b \quad \text{and} \\ \exists k \in \{1,\dots,p\}, \quad a_{\pi(k)} \geqslant b_{\pi(k)} + \theta_{\pi(k)} \quad \text{and} \\ a_{\pi(j)} \geqslant b_{\pi(j)}, \quad \text{for } j \leqslant k-1 \end{cases}$$

  where $\theta \in \mathbb{R}^{p+}$ is an increment vector.
  Observe that when the increment vector $\theta = 0$, $\succ_r$ coincides with $\succ$.

4.2. Mechanisms based on veto thresholds

In some situations, the buyer may consider that even if bid a is better than bid b on most criteria, a is so worse than b on one criterion, say criterion j, that assertion '*a is preferred to b*' cannot be accepted. In this case criterion j opposes a veto to the assertion '*a is preferred to b*'. This corresponds to the idea of discordance often used in multiple criteria decision analysis (see, e.g., [29]). To implement this concept, one needs to define a veto threshold v_j associated to each criterion j such that $a_j < b_j - v_j$ implies '*a is not preferred to b*'. This leads to the definition of mechanisms based on veto thresholds.

Definition 3. From any mechanism $MERA_{\mathcal{P},r}$, a mechanism $MERA_{\mathcal{P}_V,r_V}$ is defined by:

- $\mathcal{P}_v$, the class of preference relations $\succsim_v$ such that

$$a \succsim_v b \quad \Leftrightarrow \quad a \succsim b \quad \text{and} \quad a_j \geqslant b_j - v_j, \quad j = 1,\dots,p$$

  where $v = (v_1,\dots,v_p)$, $v_j > 0$, $j = 1,\dots,p$, is a vector of veto thresholds.
- $r_v(\succsim_v)$, the class of request relations $\succ_{rv}$ such that

$$a \succ_{rv} b \quad \Leftrightarrow \quad a \succ_r b \quad \text{and} \quad a_j \geqslant b_j - v_j, \quad j = 1,\dots,p$$

The asymmetric part of $\succsim_v$ is defined by:

$$a \succ_v b \quad \Leftrightarrow \quad a_j \geqslant b_j - v_j, \quad j = 1,\dots,p \quad \text{and} \quad \begin{cases} a \succ b \quad \text{or} \\ \bigl(a \sim b \text{ and } \exists k \in \{1,\dots,p\},\ b_k < a_k - v_k\bigr) \end{cases}$$

The possible impacts of applying veto thresholds to a given relation $\succsim$ in order to obtain a relation $\succsim_v$ are summarized in Table 2. Applying veto thresholds to relation $\succsim$ leads to a relation $\succsim_v$ such that $\succ_v$ is not necessarily acyclic. This is due to the fact that from $a \sim b$ we can obtain $a \succ_v b$ and thus create cycles in $\succ_v$. This impacts on the well-definedness of mechanism $MERA_{\mathcal{P}_V,r_V}$ as follows.

Proposition 7.

- *Any mechanism* $MERA_{\mathcal{P}_V, r_V}$ *is well-defined in asynchronous mode.*
- *Any mechanism* $MERA_{\mathcal{P}_V, r_V}$ *is well-defined in synchronous mode if and only if any* $\succ_v \in \mathcal{P}_V$ *is acyclic.*

Proof. From the above definitions of $\succ_v$ and $\succ_{rv}$, we have $\succ_{rv} \subset \succ_v$. We have also $\succ_{rv} \subset \succ_r$, which involves that $\succ_{rv}$ is acyclic, since $\succ_r$ is imposed to be acyclic. Referring to conditions of well-definedness, presented in Definition 1, we get the results for each synchronization mode. □

The following corollary applies this result to the mechanisms previously defined.

Corollary 5. *Let* $MERA_{\Sigma V}$, $MERA_{RV}$ *and* $MERA_{LV}$ *denote mechanisms resulting from applying veto thresholds respectively on mechanism* $MERA_{\Sigma}$, $MERA_R$ *and* $MERA_L$.

1. *Mechanisms* $MERA_{\Sigma V}$ *and* $MERA_{RV}$ *are not well-defined in synchronous mode.*
2. *Mechanism* $\mathcal{P}_L$ *is well-defined in synchronous mode.*

Proof. 1. Consider an item described using three criteria, the weighted sum function u_ω defined using $\omega_1 = \omega_2 = \omega_3 = 1/3$, and the following bids $a(10, 20, 30)$, $b(20, 30, 10)$, $c(30, 10, 20)$. We have $a \sim b$, $b \sim c$, and $a \sim c$. However, when introducing veto thresholds $v_1 = v_2 = v_3 = 15$, we get a cycle $a \succ_v b$, $b \succ_v c$, and $c \succ_v a$. The same result is obtained considering the reference point-based function u_α with $\alpha = (40, 40, 40)$ and $\lambda_1 = \lambda_2 = \lambda_3 = 1$.

2. The symmetric part $\sim$ of any $\succsim \in \mathcal{P}_L$ is restricted to identity on the criterion values. Therefore, we get $\sim_v = \sim$ and $\succ_v \subset \succ$. In this case, starting from a transitive and complete relation $\succsim \in \mathcal{P}_L$, we obtain a partial relation $\succsim_v$, whose asymmetric part $\succ_v$ is acyclic. □

5. Non-dominance of the winning bid

In multiple criteria decision analysis, a natural requirement is that a decision procedure should select a non-dominated alternative. In the context of auctions, we need to adapt this requirement by imposing non-dominance of the winning bid with respect to all bids that could be proposed by the non-winning sellers. However, we do not require that the winning bid be non-dominated with respect to bids that could be proposed by the winning seller. This leads to the following definition.

Definition 4. A multiple criteria auction satisfies *non-dominance* if and only if the winning bid b^* proposed by the winning seller s^* is non-dominated in B^{-s^*}: $\forall b \in B^{-s^*}, \ \neg(b \Delta b^*)$.

As for efficiency, *non-dominance* cannot be ensured if the auction is stopped prematurely due to the closing time. Therefore, all propositions and corollaries presented in this section assume that the auction ends naturally. Here again, we will not state explicitly this assumption for the sake of brevity.

Proposition 8. *Algorithm* $MERA(\succsim, \succ_r)$ *ensures* non-dominance *if and only if* $\Delta \subset \succ_r$.

Proof. See Appendix A.4. □

The following corollaries are a direct application of the previous result to mechanisms based on an aggregation function u.

Corollary 6. *When function u is strongly monotonic, mechanism* $MERA_U$ *ensures* non-dominance *if* $\succ_r$ *coincides with* $\succ$, *i.e. if* $\varepsilon = 0$.

Proof. Since u is strongly monotonic, we have $a \Delta b$ implies $u(a) > u(b)$, i.e. $\Delta \subset \succ = \succ_r$. □

From this corollary, we get that mechanism $MERA_{\Sigma}$ ensures *non-dominance* if $\varepsilon = 0$. In the same way, mechanism $MERA_L$ ensures *non-dominance* if $\theta = 0$.

Corollary 7. *If function u is monotonic, but not strongly monotonic, mechanism* $MERA_U$ *does not ensure* non-dominance.

Proof. Function u being monotonic, but not strongly monotonic, there exist $z, z' \in \mathbb{R}^p$ such that $z > z'$ and $u(z) = u(z')$. Then, there may exist $a, b \in B$ such that $a_j = z_j$ and $b_j = z'_j$ for $1, \ldots, p$, and thus such that $a \Delta b$ and $u(a) = u(b)$. Therefore we have $a \Delta b$ and $\neg(a \succ_r b)$, for any $\succ_r$ associated to u. This shows that $\neg(\Delta \subset \succ_r)$. □

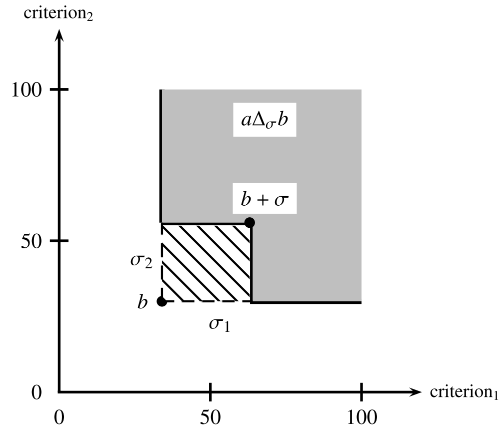

Fig. 1. Relation Δ_σ. The hatched area represents the set of bids a such that ($a\Delta b$ *and* $\neg(a\Delta_\sigma b)$) and the grey area represents the set of bids a such that $a\Delta_\sigma b$.

From this corollary, we get that mechanism $MERA_R$ cannot ensure *non-dominance*.

However, launching an auction with a step $\varepsilon = 0$ may make the auction too slow to end naturally. As a consequence, *non-dominance* may not be satisfied. This is why we may consider a weaker form of *non-dominance* when using strictly positive auction steps. To this end, we need to allow the winning bid to be dominated, provided that this dominance is not too strong. This requires to define a concept of strong dominance. While several definitions are possible, we suggest the following definition of *quasi non-dominance* which uses a significance threshold on each criterion. The corresponding idea of strong dominance is then that, besides classical strict dominance, we should observe a significant improvement on at least one criterion so as to accept strong dominance.

Definition 5. Let σ be a vector of p positive significance thresholds, an associated strong dominance relation Δ_σ (see Fig. 1 in the bicriteria case) is defined by:

$$a\Delta_\sigma b \quad \Leftrightarrow \quad \begin{cases} a\Delta b \quad \text{and} \\ \exists k \in \{1,\dots,p\}, \quad a_k > b_k + \sigma_k \end{cases} \tag{6}$$

This concept of *strong dominance* relation allows us to relax the condition of Definition 4.

Definition 6. A multiple criteria auction satisfies *quasi non-dominance* for a given significance threshold vector σ, if and only if the winning bid b^* proposed by the winning seller s^* is Δ_σ*-non-dominated* in B^{-s^*}: $\forall b \in B^{-s^*}$, $\neg(b\Delta_\sigma b^*)$.

Proposition 9. *Algorithm MERA*($\succsim, \succ_r$) *ensures* quasi non-dominance *for a given significance threshold vector* σ, *if and only if* $\Delta_\sigma \subset \succ_r$.

Proof. The proof is similar to the one given for Proposition 8 (Appendix A.4) by replacing Δ by Δ_σ. $\square$

Since results on mechanisms based on an aggregation function u depend on the precise definition of u, the following corollaries deal with mechanisms based on the weighted sum function, the lexicographic order, and the reference point-based function, respectively.

Corollary 8.

1. *Mechanism* $MERA_\Sigma$ *ensures* quasi non-dominance *if we set* $\sigma_j = \varepsilon/\omega_j$, $j = 1,\dots,p$.
2. *Mechanism* $MERA_L$ *ensures* quasi non-dominance *if we set* $\sigma_j = \theta_j$, $j = 1,\dots,p$.

Proof. 1. Consider $a, b \in B$ such that $a\Delta_\sigma b$, with $\sigma_j = \varepsilon/\omega_j$, $j = 1,\dots,p$. We have $a_j \geqslant b_j$, $j = 1,\dots,p$, and there exists $k \in \{1,\dots,p\}$, $a_k > b_k + \varepsilon/\omega_k$, which implies $\sum_{j=1}^p \omega_j a_j > \sum_{j=1}^p \omega_j b_j + \varepsilon$, i.e. $u_\omega(a) > u_\omega(b) + \varepsilon$, and thus $a \succ_r b$. This proves that $\Delta_\sigma \subset \succ_r$.

2. Consider $a, b \in B$ such that $a\Delta_\sigma b$, with $\sigma_j = \theta_j$, $j = 1,\dots,p$. We have $a_j \geqslant b_j$, $j = 1,\dots,p$, and there exists $k \in \{1,\dots,p\}$, such that $a_k > b_k + \sigma_k$. Considering $k' = \pi^{-1}(k)$, we get $a_{\pi(k')} > b_{\pi(k')} + \theta_{\pi(k')}$ and $a_{\pi(j)} \geqslant b_{\pi(j)}$, for $j \leqslant k'-1$, which implies $a \succ_r b$. This proves that $\Delta_\sigma \subset \succ_r$. $\square$

Corollary 9. *Mechanism $MERA_R$ does not ensure* quasi non-dominance.

Proof. We show, for any strong dominance relation Δ_σ, the possible existence of $a, b \in B$ such that $a \Delta_\sigma b$ and $u_\alpha(a) = u_\alpha(b)$. Consider for this, bid $b(b_1, b_2) \in B$, $\lambda_1 = \lambda_2 = 1$ and the reference point $\alpha(b_1, b_2)$, there may exists $t > \sigma_1$ such that $a(b_1 + t, b_2) \in B$. We have then $a \Delta_\sigma b$ and $u_\alpha(a) = u_\alpha(b) = 0$.

Since $u_\alpha(a) = u_\alpha(b)$, we have $\neg(a \succ_r b)$ for any $\succ_r$ associated to $\succsim$. Therefore, $\Delta_\sigma \subset \succ_r$ is not satisfied. □

The use of veto thresholds affects *quasi non-dominance* in the following way.

Proposition 10. *If algorithm MERA($\succsim, \succ_r$) ensures* quasi non-dominance *for a given significance threshold vector σ, algorithm MERA($\succsim_V, \succ_{rV}$) also ensures* quasi non-dominance *for σ:*

- *in the asynchronous mode,*
- *in the synchronous mode if $\succ_V$ is acyclic.*

Proof. We show that $\Delta_\sigma \subset \succ_r$ implies $\Delta_\sigma \subset \succ_{rV}$. Indeed, $a \Delta_\sigma b$ implies $a_j \geqslant b_j$ and thus $a_j \geqslant b_j - v_j$, $j = 1, \ldots, p$, which, together with $a \succ_r b$, imply $a \succ_{rV} b$. Moreover, in the synchronous mode, we need to impose acyclicity of $\succ_V$ in order to get a well-defined mechanism (see condition 3 of Definition 1). □

The following corollary applies this result on mechanisms $MERA_{LV}$ and $MERA_{\Sigma V}$.

Corollary 10. *Mechanism $MERA_{LV}$ ensures* quasi non-dominance *in both synchronization modes and $MERA_{\Sigma V}$ in the asynchronous mode only.*

6. Fair competition between non-dominated bids

In multiple criteria decision analysis, a decision procedure relies on an aggregation model to capture the decision maker's preferences and aims to select a preferred alternative among the set of non-dominated alternatives. A decision procedure satisfies *fair competition between non-dominated alternatives*, if for any non-dominated alternative, there exists at least one set of values of the aggregation model parameters which enables the procedure to select this alternative or any of its equivalents, i.e. any alternative with the same criterion values. This is clearly a crucial issue since, if *fair competition* is not satisfied, potentially interesting alternatives are rejected *a priori* because of technical limitations of the aggregation model.

In the context of auctions, *fair competition* should be evaluated between non-dominated bids from the set $B^S = \bigcup_{s \in S} B^s$ of bids that could be proposed by the competing sellers. We cannot ensure, however, that a given bid will be effectively proposed during the auction and, thus, will win. Therefore, we need to adapt this property by requiring only that any non-dominated bid is valid, i.e. liable to be proposed, at any round; moreover if any of its equivalents, including itself, is proposed then it wins the auction. This leads to the following definition.

Definition 7. Any mechanism $MERA_{\mathcal{P},r}$ satisfies *fair competition* if and only if for any bid a, non-dominated in B^S, there exists at least one preference relation $\succsim \in \mathcal{P}$ and one request relation $\succ_r \in r(\succsim)$ ensuring that any bid equivalent to bid a is valid at any round, and wins if proposed.

As observed before, *fair competition* does not ensure that any bid a, non-dominated in B^S, wins any auction *MERA*($\succsim, \succ_r$), where $\succsim$ and $\succ_r$ represent the relations associated to a. However, this property ensures that:

- Any auction *MERA*($\succsim, \succ_r$), which ends naturally, is won by one of the sellers which owns bid a or one of its equivalents.
- When at least two different sellers own bid a or one of its equivalents, any auction *MERA*($\succsim, \succ_r$), which ends naturally, is won by bid a or one of its equivalents.

Otherwise, in both cases, the current winning bid could be beaten by bid a or one of its equivalents.

We provide now a necessary and sufficient condition for *fair competition*.

Proposition 11. *Both for asynchronous and synchronous modes, any mechanism $MERA_{\mathcal{P},r}$ satisfies* fair competition *if and only if the following condition holds*:

$$\forall B,\ \forall a \in B,\ \exists \succsim \in \mathcal{P},\ \exists \succ_r \in r(\succsim),\ \forall b \in B, \quad \neg(b \underline{\Delta} a) \quad \Rightarrow \quad a \succ_r b \tag{7}$$

Proof. See Appendix A.5. □

The following corollaries apply this result to the mechanisms presented in Section 4.

Corollary 11. *Mechanism $MERA_{\Sigma}$, using the weighted sum function, and mechanism $MERA_L$, using the lexicographic order, do not satisfy* fair competition*.*

Proof. Let us consider the set of potential bids $B = \{a,\ b,\ c\}$ with criterion values $a(40, 40)$, $b(100, 0)$ and $c(0, 100)$ and let us show that *fair competition* cannot be obtained for bid a since condition (7) is violated.

- For all weight vector $(\omega_1,\omega_2) \in [0,1]^2$ such that $\omega_1 + \omega_2 = 1$, we have $u_\omega(a) = 40$, $u_\omega(b) = 100\omega_1$, and $u_\omega(c) = 100 - 100\omega_1$. Then we can check easily that b is optimal when $0.5 \leqslant \omega_1 \leqslant 1$ and c is optimal when $0 \leqslant \omega_1 \leqslant 0.5$. Hence, we have

$$\forall \succsim \in \mathcal{P}_{\Sigma},\ \exists b \in B, \quad \neg(b \underline{\Delta} a) \quad \text{and} \quad \neg(a \succ b)$$

  and thus $\neg(a \succ_r b), \forall \succ_r \in r(\succsim)$, since $\succ_r \subset \succ$.
- For any lexicographic order, bid $a(40, 40)$ cannot be optimal and thus condition (7) is violated. □

Corollary 12. *Mechanism $MERA_R$, using the reference point-based function, satisfies* fair competition*.*

Proof. Mechanism $MERA_R$ satisfies condition (7) for any set of potential bids B and any bid $a \in B$ taking $\alpha = (a_1, \ldots, a_p)$ as aspiration point and $\varepsilon = 0$ as step of the request. We have indeed, $u_\alpha(a) = 0$ and $u_\alpha(b) > 0$ for any $b \in B$ such that $\neg(b \underline{\Delta} a)$. □

The use of veto thresholds affects *fair competition* in the following way.

Proposition 12. *Considering $\mathcal{P}$ a class of preference relations, mechanism $MERA_{\mathcal{P}_V, r_V}$, if well-defined, satisfies* fair competition *if and only if mechanism $MERA_{\mathcal{P},r}$ satisfies* fair competition*.*

Proof. Assuming that $MERA_{\mathcal{P}_V, r_V}$ is well-defined and that $MERA_{\mathcal{P},r}$ satisfies *fair competition*, condition (7) associates to any $a \in B$ a relation $\succsim \in \mathcal{P}$ and a relation $\succ_r \in r(\succsim)$. Then condition (7) remains satisfied by relations $\succsim_v \in \mathcal{P}_V$ and $\succ_{rv} \in r_v(\succsim_v)$ respectively defined from $\succsim$ and $\succ_r$, using any veto thresholds such that bid a remains comparable to any bid that does not dominate it. Such thresholds, if chosen large enough, always exist.

Assuming that $MERA_{\mathcal{P}_V, r_V}$ satisfies *fair competition*, condition (7) associates a relation $\succsim_v \in \mathcal{P}_V$ and a relation $\succ_{rv} \in r_v(\succsim_v)$ to any $a \in B$. Then taking relations $\succsim$ and $\succ_r$ underlying respectively $\succsim_v$ and $\succ_{rv}$ allows us to satisfy condition (7), since we have $\succ_{rv} \subset \succ_r$. □

As a consequence of this proposition, we get:

Corollary 13. *Mechanism asynchronous-$MERA_{RV}$, using the reference point-based function and veto thresholds, satisfies* fair competition*.*

Remark 4.

1. When using veto thresholds with the reference point-based function, we must pay attention to setting these thresholds in such a way that the aspiration point is preferred to any bid that does not dominate it. This is not restrictive since the concept of aspiration point, by its very definition, imposes that the only bids that are preferred to it are the ones which dominate it.
2. As shown in Section 4, mechanism $MERA_{RV}$ cannot be used in the synchronous mode, since veto thresholds may introduce cycles into relations defined from the reference point-based function.

7. Hybrid mechanisms

As shown in the previous section, standard aggregation models satisfy either *quasi non-dominance* or *fair competition*. In order to satisfy both properties, it is natural to try to combine models of each type. To define such a combination, it should be pointed out that aggregation models satisfying *fair competition* but not *quasi non-dominance* are able to return all non-dominated bids but also dominated ones. More precisely, provided that such a model corresponds to a monotonic function, which is usually the case, the only situation where dominated bids are returned is when there also exists at least one non-dominated bid achieving the same optimal value on the function. Conversely, aggregation models satisfying *quasi non-dominance* but not *fair competition* only return non-dominated bids but may miss some of them. Therefore, the only possible combination consists in using first a model satisfying *fair competition* and, in case of multiple candidate bids, filtering them using a model satisfying *quasi non-dominance*.

In this section, we first define such a hybrid mechanism. Then, we present a detailed illustrative example of an auction process using this mechanism.

Table 3
Value functions.

Car type								
Value	hatchback	convertible	roadster	SUV	coupé	sedan	large SUV	large sedan
Score	20	40	40	60	60	80	80	100
Car color								
Value	white	yellow	blue	grey	red	purple	black	green
Score	40	40	60	60	80	80	100	100

7.1. Mechanism $MERA_{RLV}$

Mechanism $MERA_{RLV}$ is a hybrid mechanism with veto thresholds which uses the reference point-based function as the main aggregation model and resolves ties with the lexicographic model. It is defined with:

- $\mathcal{P}_{RLV}$, the class of preference relations $\succsim_v$ such that

$$a \succsim_v b \quad \Leftrightarrow \quad \begin{cases} a_j \geqslant b_j - v_j,\ j = 1, \ldots, p \text{ and } (\\ u_\alpha(a) < u_\alpha(b) \text{ or} \\ (u_\alpha(a) = u_\alpha(b) \text{ and } (a = b \text{ or } (\exists k \in \{1, \ldots, p\},\ a_{\pi(k)} > b_{\pi(k)} \text{ and} \\ a_{\pi(j)} = b_{\pi(j)}, \text{for } j \leqslant k-1)))) \end{cases}$$

- $r_{RLV}(\succsim)$, the class of request relations $\succ_{rv}$ such that

$$a \succ_{rv} b \quad \Leftrightarrow \quad \begin{cases} a \succ_v b \text{ and } (\\ u_\alpha(a) \leqslant u_\alpha(b) - \varepsilon \text{ or} \\ (u_\alpha(b) - \varepsilon < u_\alpha(a) \leqslant u_\alpha(b) \text{ and} \\ \exists k \in \{1, \ldots, p\}, a_{\pi(k)} \geqslant b_{\pi(k)} + \theta_{\pi(k)} \text{ and } a_{\pi(j)} \geqslant b_{\pi(j)}, \text{for } j \leqslant k-1)) \end{cases}$$

  where $\varepsilon \geqslant 0$ is a decrement on u_α and $\theta \in \mathbb{R}^{p+}$ an increment vector on the criterion values.

Observe that when we have $\varepsilon = 0$ and $\theta_j = 0$, $j \in \{1, \ldots, p\}$, relation $\succ_{rv}$ coincides with $\succ_v$.

Moreover, the use of the lexicographic order in the preference relation, even at the second place, prevents cycles in the asymmetric part, $\succ_v$, of any preference relation $\succsim_v \in \mathcal{P}_{RLV}$. Therefore mechanism $MERA_{RLV}$ is well-defined in both synchronization modes. Finally, mechanism $MERA_{RLV}$ ensures *quasi non-dominance* from Corollary 8 and Proposition 10, and *fair competition* from Corollary 12 and Proposition 12.

7.2. The auction context

The auction context is a synchronous auction, where one buyer agent negotiates with three sellers s^1, s^2, and s^3 over a car, described by three attributes: price, type, and color. The set of potential bids for each seller is given in Table 5, Appendix B. Before starting the auction, the buyer defines his/her preference relation on the item to be purchased as follows:

- Each attribute is encoded so as to reflect preferences on the corresponding viewpoint. This gives rise to the three criterion functions: price, type, and color. For the sake of simplicity, we use a linear transformation for the criterion price:

$$b_1 = \big(50000 - \text{price(b)}\big)/(50000 - 10000) \times 100.$$

  Moreover, for the attribute type, the buyer expresses the following preferences:
  large sedan $\succ$ large SUV $\succ$ sedan $\succ$ SUV $\sim$ coupé $\succ$ convertible $\sim$ roadster $\succ$ hatchback which are encoded in Table 3.
  For the attribute color, the buyer expresses the following preferences:
  green $\sim$ black $\succ$ purple $\sim$ red $\succ$ blue $\sim$ grey $\succ$ yellow $\sim$ white which are encoded in Table 3.
  This way, we define criterion functions to be maximized and taking values in $[0, 100]$.
- $\alpha = (65, 60, 60)$, the aspiration point;
- $\lambda = (1, 1, 1)$, the vector of scaling factors;
- (price, type, color), the lexicographic order of the criteria;
- $v = (40, 45, 45)$, the vector of veto thresholds;

Table 4
Auction process.

i	Current constraint c^i	s^1	s^2	s^3
1		$(*)$ (34000, large sedan, purple) $(\underline{40}, 100, 80)$ $u_\alpha(b^{s1,1}) = 25$	(41000, coupé, purple) $(\underline{23}, 60, 80)$ $u_\alpha(b^{s2,1}) = 42$	(30000, hatchback, blue) $(50, \underline{20}, 60)$ $u_\alpha(b^{s3,1}) = 40$
2	$(b_2 \geqslant 55$ and $b_3 \geqslant 35)$ and $(u_\alpha(b) \leqslant 20$ or $(20 < u_\alpha(b) \leqslant 25$ and $(b_1 \geqslant 45$ or $(b_1 \geqslant 40$ and $b_2 \geqslant 100$ and $b_3 \geqslant 85))))$	current winner	$(*)$ (32000, coupé, green) $(\underline{45}, 60, \boldsymbol{100})$ $u_\alpha(b^{s2,2}) = 20$	(13500, coupé, yellow) $(91, 60, \underline{\boldsymbol{40}})$ $u_\alpha(b^{s3,2}) = 20$
3	$(b_1 \geqslant 5$ and $b_2 \geqslant 15$ and $b_3 \geqslant 55)$ and $(u_\alpha(b) \leqslant 15$ or $(15 < u_\alpha(b) \leqslant 20$ and $(b_1 \geqslant 50$ or $(b_1 \geqslant 45$ and $b_2 \geqslant 65))))$	$(*)$ (21000, roadster, grey) $(73, \underline{40}, 60)$ $u_\alpha(b^{s1,3}) = 20$	current winner	(21000, convertible, blue) $(73, \underline{40}, 60)$ $u_\alpha(b^{s3,3}) = 20$
4	$(b_1 \geqslant 33$ and $b_3 \geqslant 15)$ and $(u_\alpha(b) \leqslant 15$ or $(15 < u_\alpha(b) \leqslant 20$ and $(b_1 \geqslant 78$ or $(b_1 \geqslant 73$ and $b_2 \geqslant 45)$ or $(b_1 \geqslant 73$ and $b_2 \geqslant 40$ and $b_3 \geqslant 65))))$	current winner	$(*)$ (28000, sedan, grey) $(\underline{55}, 80, 60)$ $u_\alpha(b^{s2,4}) = 10$	(13500, coupé, yellow) $(91, 60, \underline{40})$ $u_\alpha(b^{s3,4}) = 20$
5	$(b_1 \geqslant 10$ and $b_2 \geqslant 35$ and $b_3 \geqslant 15)$ and $(u_\alpha(b) \leqslant 5$ or $(5 < u_\alpha(b) \leqslant 10$ and $(b_1 \geqslant 60$ or $(b_1 \geqslant 55$ and $b_2 \geqslant 85)$ or $(b_1 \geqslant 55$ and $b_2 \geqslant 80$ and $b_3 \geqslant 65))))$	no bid	current winner	$(*)$ (22000, large SUV, blue) $(70, 80, \underline{60})$ $u_\alpha(b^{s3,5}) = 0$
6	$(b_1 \geqslant 30)$ and $b_2 \geqslant 35$ and $b_3 \geqslant 15$ and $(u_\alpha(b) \leqslant -5$ or $-5 < u_\alpha(b) \leqslant 0$ and $(b_1 \geqslant 75$ or $(b_1 \geqslant 70$ and $b_2 \geqslant 85)$ or $(b_1 \geqslant 70$ and $b_2 \geqslant 80$ and $b_3 \geqslant 65)))$	no bid	no bid	current winner

Observe that, we have $b_j \leqslant 100$ for any potential bid b and $\alpha_j + v_j > 100$, $j = 1, \ldots, p$. Thus, as required in Remark 4, the aspiration point α is preferred to any bid that does not dominate it. Therefore, it is liable to win at any round, if it is non-dominated in $B^{s^1} \cup B^{s^2} \cup B^{s^3}$.

The buyer defines his/her request relation by setting:

- $\varepsilon = 5$, the increment on u_α;
- $\theta = (5, 5, 5)$, the increment vector on the criterion values for the lexicographic aggregation function.

At the beginning of the auction, the buyer agent indicates to the sellers the criterion functions, the aspiration point and the criterion order. Then, each seller identifies his/her proposals that match the item to be purchased.

7.3. The auction process

The auction takes place in 6 rounds reported in Table 4. At the first round, each seller provides a bid that satisfies the initial requirement. Since we have $b^{s^1,1} \succ_v b^{s^2,1}$ and $b^{s^1,1} \succ_v b^{s^3,1}$, $M(\succsim_v, P^1) = \{b^{s^1,1}\}$ and bid $b^{s^1,1}$ is selected as the current best bid. Its corresponding seller s^1 becomes the current winner and is not called upon at the next round. The current constraint c^1 (described in Table 4), asking for bids that are $\succ_{rv}$-preferred to $b^{s^1,1}$, is sent to sellers s^2 and s^3.

At round 2, $b^{s^2,2}$ and $b^{s^3,2}$ are incomparable. Indeed, we have $b^{s^3,2} \succ b^{s^2,2}$ (the same value on u_α, but a better value of $b^{s^3,2}$ on price), but the third criterion opposes a veto to $b^{s^3,2} \succ_v b^{s^2,2}$ since $b_3^{s^3,2} = 40 < b_3^{s^2,2} - v_3 = 100 - 45$. Thus, $M(\succsim, P^2) = \{b^{s^2,2}, b^{s^3,2}\}$. We assume that bid $b^{s^2,2}$ is arbitrary selected as the current best bid.

At round 3, since bid $b^{s^1,3}$ and $b^{s^3,3}$ are equivalent, we have $M(\succsim, P^3) = \{b^{s^1,3}, b^{s^3,3}\}$. We assume that bid $b^{s^1,3}$ is arbitrary selected as the current best bid.

At round 4, seller s^3 is able to propose the same bid as at round 2. Since we have $b^{s^2,4} \succ_v b^{s^3,4}$, $M(\succsim, P^3) = \{b^{s^2,4}\}$ and bid $b^{s^2,4}$ is selected as the current best bid.

At round 5, seller s^1 cannot propose a valid bid. Thus, we have $M(\succsim, P^3) = \{b^{s^3,5}\}$ and bid $b^{s^3,5}$ is selected as the current best bid. At round 6, sellers s^1 and s^2 cannot propose a valid bid. Thus, bid $b^{s^3,5} = (70, 80, 60)$ becomes the winning bid.

The winning bid corresponds to a blue large SUV with a price of \$22000. Thus, the buyer obtains an agreement on an item with two attributes, type and price, which are over his/her aspirations.

It is interesting to observe the evolution of the auction and the properties that are satisfied.

- **[MBB]** is not satisfied. Indeed, we have $best^3 = b^{s^1,3}$ and $b^{s^3,2} \succ_v best^3$. Thus, $best^3$ is not a maximal element among the set of bids previously received.
- **[IBB]** is not satisfied. Indeed, we have $best^1 = b^{s^1,1}$ and $best^1 \; ? \; best^3$. Thus, $best^3$ is not preferred to $best^1$.
- **[NCB]** is not satisfied, since seller s^3 has proposed the same bid twice, first at round 2 and second at round 4.

Even if this evolution does not respect any of the 'natural' properties [MBB], [IBB], and [NCB], the final result of the auction is satisfactory. Indeed, as expected, the auction is *quasi efficient*. First, none of the non-winning sellers can propose a bid which is $\succ_r$-preferred to the winning bid. Second, observing from Table 5, Appendix B, that seller s^3 owns the potential bid $(65, 80, 80)$, which is $\succ_r$-preferred to any potential bid of sellers s^1 and s^2, he/she should win any auction. Indeed, seller s^3 does win the auction described in Table 4. Observe finally that quasi non-dominance is satisfied since the winning bid $b^{s^3,5}$ is *non-dominated* in B^{-s^3}.

8. Conclusions

This paper proposed a study of English reverse auction mechanisms in a unified framework by considering that the buyer's preferences are represented by a binary preference relation. Existing mechanisms are based on a single or multidimensional utility function, which corresponds to a transitive and complete preference relation. One of our main goals in this study was to allow for more general preference relations, relaxing transitivity and completeness, in order to be able to model more elaborate and more realistic preferences. On the other hand, the necessity of integrating preferences within an auction mechanism which should respect some properties imposes restrictions on the preference relations. After precisely defining such properties and providing minimal conditions on the preference relations so as to satisfy these properties, it appears that the main restriction is acyclicity of the request relation or of the asymmetric part of the preference relation. Moreover, we showed that quasi efficiency is satisfied with no additional restriction on the preference relations. This leaves room for interesting preference models that accept intransitivity and incomparability. In particular, when considering auction mechanisms based on multiple criteria, we showed that aggregation models including veto thresholds can be implemented while preserving properties related to non-dominance and fair competition. Regarding these two basic properties, we pointed out that classical multiple criteria auction mechanisms usually satisfy one of them, but not both of them. We showed, however, how to satisfy both properties by using hybrid aggregation models.

Even if our framework is devoted to the reverse version of the English protocol, it can be adapted to other protocols. This is particularly easy for one-round protocols. Indeed, we only need a simplified version of our framework, without defining a *request* function. In *first-preferred* auctions, which would generalize first-price auctions, the winning seller must provide an item corresponding to the winning bid. In *second-preferred* auctions, which would generalize second-price or Vickrey auctions, the winning seller only has to provide an item corresponding to the second most preferred bid. In this case, we need to redefine concepts and properties using only the preference relation. On the contrary, adapting our framework to the reverse Dutch protocol would require focusing on the *request* function whereas the *select* function would not play any part. In this case, indeed, we need to define a scheme of less and less demanding constraints until the first seller who proposes a bid wins the auction. For all these protocols, it would be interesting to investigate conditions for efficiency, bid non-dominance, and fair competition.

Our framework can be deemed as the first milestone to providing an agent-based auction system where an autonomous buyer agent conducts auctions on behalf of a buyer and communicates with several autonomous seller agents that bid for selling an item on behalf of sellers. Thus, providing a framework which helps designing such seller agents would be a useful complementary research work. The three main tasks on the research agenda would be:

- modeling of the seller's preferences on items he/she sells using binary preference relations,
- automated definition of the bids taking into account requests from the buyer agent, seller's preferences and strategy,
- definition of desirable properties for the sellers and study of the conditions to achieve these properties.

Finally, integrating more elaborate preference models in more complex types of auctions, such as multi-item, double, or combinatorial auctions, would be a challenging research project.

Appendix A. Proofs

A.1. Proof of Proposition 3

[IBB] $\Rightarrow$ [MBB]. We have $\overline{P^i} = \{best^1, \ldots, best^i\}$ for $i \geqslant 1$. Thus, assuming [IBB], we have $best^i \succ best^j$, for $i \geqslant 2$ and $j < i$. So we get $M(\succsim, \overline{P^i}) = \{best^i\}$ and [MBB] is satisfied.

[NCB] is satisfied since relation $\succ_r$ is imposed to be acyclic.

A.2. Proof of Proposition 4

1. The condition is sufficient. We have $best^{j+1} \succ_r best^j$, for $i \geqslant 2$ and $j = 1, \ldots, i-1$, due to the request constraints. This implies, according to (1), that $\neg(best^j \succ best^i)$, for $i \geqslant 2$ and $j = 1, \ldots, i-1$. Thus, we have $best^i \in M(\succsim, \overline{P^i})$, for $i \geqslant 2$. Moreover, for $i = 1$, we have trivially $best^1 \in M(\succsim, \overline{P^1})$.
The condition is necessary. Let us assume by contradiction that (1) is false:
$$\exists n \geqslant 3, \exists b^1, \ldots, b^n \in B, \quad b^{j+1} \succ_r b^j, \quad j = 1, \ldots, n-1, \quad \text{and} \quad b^1 \succ b^n$$
We show now the existence of an auction which does not satisfy [MBB]. Consider for this, n sellers s^j with $b^j \in B^{s^j}$, $j = 1, \ldots, n$. At round j, seller s^j proposes b^j, $j = 1, \ldots, n$. This sequence of bids is valid, since it satisfies the request constraint at each round. Observe, however, that $\overline{P^n} = \{b^1, \ldots, b^n\}$ and $best^n = b^n \notin M(\succsim, \overline{P^n})$, since $b^1 \succ b^n$.
2. The condition is sufficient. We have $best^{j+1} \succ_r best^j$ for $i \geqslant 2$ and $j = 1, \ldots, i-1$, due to the request constraints. This implies, according to (2), that $best^i \succ best^j$ for $i \geqslant 2$ and $j = 1, \ldots, i-1$. Thus, [IBB] is satisfied.
The condition is necessary. Let us assume by contradiction that (2) is false:
$$\exists n \geqslant 3,\ \exists b^1, \ldots, b^n \in B, \quad b^{j+1} \succ_r b^j, \quad j = 1, \ldots, n-1, \quad \text{and} \quad \neg\big(b^n \succ b^1\big)$$
We show now the existence of a *PERA* auction which does not satisfy [IBB].
Consider for this, $n-1$ sellers s^j with $b^1, b^n \in B^{s^1}$ and $b^j \in B^{s^j}$, $j = 2, \ldots, n-1$. At round j, seller s^j proposes b^j, $j = 1, \ldots, n-1$. Then, at round n, s^1 proposes b^n. Observe that we have $b^1 = best^1$, $b^n = best^n$, and $\neg(b^n \succ b^1)$.

A.3. Proof of Proposition 5

1. The condition is sufficient. For any $a \in \overline{P^i}$, there exists $k \leqslant i$, such that $a \in P^k$. We have $\neg(a \succ best^k)$ due to the selection of $best^k$ from P^k. Moreover we have $best^{j+1} \succ_r best^j$ for $j = k, \ldots, i-1$ due to the request constraints. Using (3), we get $\neg(a \succ best^i)$ and thus $best^i \in M(\succsim, \overline{P^i})$. Moreover, for $i = 1$, we have trivially $best^1 \in M(\succsim, \overline{P^1})$.
The condition is necessary. Let us assume by contradiction that (3) is false:
$$\exists n \geqslant 2,\ \exists a, b^1, \ldots, b^n \in B, \quad \neg\big(a \succ b^1\big) \quad \text{and} \quad b^{j+1} \succ_r b^j, \quad j = 1, \ldots, n-1, \quad \text{and} \quad a \succ b^n$$
We show now the existence of a synchronous *PERA* auction which does not satisfy [MBB]. Consider n sellers s^j with $B^{s^j} = \{a, b^1, \ldots, b^n\}$, $j = 1, \ldots, n$. The auction starts with s^1 proposing b^1 and the other sellers proposing a. Assume that bid b^1 is selected. At round i, $i = 2, \ldots, n$, all sellers except seller s^{i-1} propose b^i. Assume that bid b^i is accepted as $best^i$ for seller s^i. Thus, we have $\overline{P^n} = \{a, b^1, \ldots, b^n\}$ with $b^j = best^j$, $j = 1, \ldots, n$. Since $a \succ best^n$, we get $best^n \notin M(\succsim, \overline{P^n})$.
2. The condition is sufficient. We have $best^{j+1} \succ_r best^j$ for $i \geqslant 2$ and $j = 1, \ldots, i-1$, due to the request constraint. This implies, according to (4), that $best^i \succ best^j$ for $i \geqslant 2$ and $j = 1, \ldots, i-1$.
The condition is necessary. Let us assume by contradiction that (4) is false:
$$\exists n \geqslant 3,\ \exists b^1, \ldots, b^n \in B, \quad b^{j+1} \succ_r b^j, \quad j = 1, \ldots, n-1, \quad \text{and} \quad \neg\big(b^n \succ b^1\big)$$
Consider n sellers s^j with $B^{s^j} = \{b^1, \ldots, b^n\}$, $j = 1, \ldots, n$. The auction starts with all the sellers proposing b^1. Assume that bid b^1 is selected from seller s^1. At round i, $i = 2, \ldots, n$, all sellers except seller s^{i-1} propose b^i. Assume that bid b^i is accepted as $best^i$ for seller s^i. For this auction, $\neg(b^n \succ b^1)$ corresponds to $\neg(best^n \succ best^1)$. Thus [IBB] is not satisfied.
3. The condition is sufficient. For any $a \in \overline{P^{i-1}}$, there exists $k \leqslant i-1$, such that $a \in P^k$. We have $\neg(a \succ best^k)$ due to the selection of $best^k$ from P^k. Moreover we have $best^{j+1} \succ_r best^j$ for $j = k, \ldots, i-1$ due to the request constraints. Using (5), we get $\neg(a \succ_r best^i)$ for any $a \in \overline{P^{i-1}}$ and thus $P^i \cap \overline{P^{i-1}} = \emptyset$.
The condition is necessary. Let us assume by contradiction that (5) is false:
$$\exists n \geqslant 2,\ \exists a, b^1, \ldots, b^n \in B, \quad \neg\big(a \succ b^1\big) \quad \text{and} \quad b^{j+1} \succ_r b^j, \quad j = 1, \ldots, n-1, \quad \text{and} \quad a \succ_r b^n$$
We show now the existence of a synchronous *PERA* auction which does not satisfy [MBB]. Consider n sellers s^j with $B^{s^j} = \{a, b^1, \ldots, b^n\}$, $j = 1, \ldots, n$. The auction starts with s^1 proposing b^1 and the other sellers proposing a. Assume that bid b^1 is selected. At round i all sellers except seller s^{i-1} propose bid b^i. Assume that bid b^i is accepted as $best^i$ for seller s^i, $i = 2, \ldots, n$. Since $a \succ_r b^n$, all sellers except seller s^n propose bid a at round $n+1$. Observe that a has been proposed at rounds 1 and $n+1$, and thus [NCB] is not satisfied.

A.4. Proof of Proposition 8

Proof. The condition is sufficient. Indeed, consider an auction running according to Algorithm $MERA(\succsim, \succ_r)$. The assumption of natural end implies that for all $b \in B^{-s^*}$, we have $\neg(b \succ_r b^*)$ and thus $\neg(b \Delta b^*)$.

The condition is necessary. Let us assume by contradiction that we have $\neg(\Delta \subset \succ_r)$. Thus, there exists two bids $a, b \in B$ such that $a\Delta b$ and $\neg(a \succ_r b)$. Consider now two sellers s and s' with $B^s = B^{s'} = \{a, b\}$ and the following asynchronous and synchronous auctions. At the first round, sellers s proposes bid b (asynchronous auction) or sellers s and s' propose bid b which is selected from s (synchronous auction). At the second round, seller s' cannot propose any bid. Thus, seller s wins the auction with bid b which is dominated in B^{-s}. $\square$

A.5. Proof of Proposition 11

Proof. The condition is sufficient. Consider any bid $a \in B^s$ non-dominated in B^s and relations $\succsim$ and $\succ_r$ associated to a by (7). Let $Equ(a)$ denote the set of bids containing a and its equivalents. Considering any bid $b \in B^s \setminus Equ(a)$, we have $\neg(b\underline{\Delta}a')$, for any $a' \in Equ(a)$. This implies that $a' \succ_r b$, ensuring the validity of any bid a' equivalent to a, at any round and for any mode. Moreover, asymmetry of $\succ_r$ and the inclusion $\succ_r \subset \succ$ imply respectively $\neg(b \succ_r a')$ and $a' \succ b$, showing that any bid a' equivalent to a wins if proposed in any mode.

The condition is necessary. Let us consider a mechanism $MERA_{\mathcal{P},r}$ that does not satisfy (7), i.e. such that:

$$\exists B,\ \exists a \in B,\ \forall \succsim \in \mathcal{P},\ \forall \succ_r \in r(\succsim),\ \exists b \in B, \quad \neg(b\underline{\Delta}a) \quad \text{and} \quad \neg(a \succ_r b) \tag{A.1}$$

Let us consider bid a defined by (A.1), any $\succsim \in \mathcal{P}$, any $\succ_r \in r(\succsim)$ and two sellers s and s' such that $B^s = \{a, b\}$ and $B^{s'} = \{b\}$, where bid b satisfies (A.1). Let us assume that at the first round of an **asynchronous** auction seller s' proposes bid b and that at the first round of a **synchronous** auction, sellers s and s' propose bid b which is selected from s'.

In both cases, bid a is not valid at the second round, since we have $\neg(a \succ_r b)$ from (A.1). Seller s' wins the auction with bid b. Bid a, which is non-dominated by any bid in B^s, is not capable of winning when bid b is proposed at the first round. $\square$

Appendix B. Seller's potential bids

Table 5
Sellers' potential bids described on attribute, criterion, and aggregation values. Each bid has a price between the reserve price and the catalog price. u_α is computed with $\alpha = (60, 60, 65)$.

Seller s^1		
bid	criterion values	u_α
([13500, 15000], hatchback, yellow)	([88, 91], 20, 40)	40
([25000, 27000], hatchback, blue)	([58, 63], 20, 60)	40
([34000, 35500], hatchback, red)	([36, 40], 20, 80)	40
([38500, 40000], hatchback, black)	([25, 29], 20, 100)	40
([18000, 19500], convertible, yellow)	([76, 80], 40, 40)	20
([20000, 21500], roadster, grey)	([71, 75], 40, 60)	20
([30000, 31500], coupé, grey)	([46, 50] , 60, 60)	[15, 19]
([13000, 14500], convertible, yellow)	([89, 93], 40, 40)	20
([32500, 34000], large sedan, purple)	([40, 44], 100, 80)	[21, 25]
Seller s^2		
bid	criterion values	u_α
([13000, 15000], convertible, white)	([88, 93], 40, 40)	20
([20000, 22000], convertible, blue)	([70, 75], 40, 60)	20
([30000, 32000], convertible, green)	([45, 50], 40, 100)	20
([29000, 31000], roadster, black)	([53, 48], 40, 100)	20
([16000, 18000], SUV, yellow)	([80, 85], 60, 40)	20
([29000, 31000], SUV, grey)	([48, 53], 60, 60)	[12, 17]
([40000, 42000], coupé, purple)	([20, 25], 60, 80)	[40, 45]
([31000, 33000], coupé, green)	([43, 48], 60, 100)	[17, 22]
([25000, 27000], large sedan, yellow)	([58, 63], 100, 40)	20
([26000, 28000], sedan, grey)	([55, 60], 80, 60)	[5,10]
([24000, 26000], large SUV, red)	([60, 65], 80, 80)	[0, 5]
Seller s^3		
bid = (price, type, color)	criterion values	u_α
([11500, 13500], coupé, yellow)	([91, 96], 60, 40)	20
([30000, 32000], SUV, blue)	([45, 50], 60, 60)	[15, 20]
([38000, 40000], coupé, red)	([35, 40], 60, 80)	[25, 30]
([20000, 22000], large SUV, blue)	([70, 75], 80, 60)	0
([22000, 24000], sedan, purple)	**([65, 70], 80, 80)**	**[−5, 0]**
([19500, 21000], sedan, grey)	([73, 76], 80, 60)	0
([28000, 30000], hatchback, yellow)	([50, 55], 20, 40)	40
([34000, 36000], large sedan, purple)	([35, 40], 100, 80)	[25, 30]

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