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ORIGINAL ARTICLE

On *q*-analogues of the Mangontarum transform for certain *q*-Bessel functions and some application

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Abstract Several *q*-analogues of certain integral transforms have been recently investigated by many authors in the recent past. In this paper, we introduce certain analogues of the so-called *q*-Mangontarum transform and implement the proposed variants to given classes of *q*-Bessel functions. The results of this paper are new and complement the previously known results of Mangontarum (2014). Some results related to *q*-Laplace transforms are also obtained.

1. Introduction

Quantum calculus or *q*-calculus is receiving an increase of interest, mainly due to its applications in mathematics and physical sciences. It is a version of calculus where derivatives are differences and antiderivatives are sums and, no further limit nor smoothness is required. Since Jackson (1905) defined the *q*-differential operator, which is considered the outset of quantum calculus, it, compared to differential and integral calculus, is very recent and hence some rules and definitions have to be presented.

In this article, we spread our results into six sections. In Section 2, we recall some known definitions and notations from the *q*-theory. In Section 3, we give definitions of some *q*-analogues of the *q*-Mangontarum transform. In Section 4, we recall some series representation of a class of *q*-Bessel functions. In Section 5, we apply the *q*-Mangontarum transform of first type to a given class of *q*-Bessel functions. In Section 6, we compose some further hypergeometric series of the *q*-Mangontarum transform and employ the given series to the same class of *q*-Bessel functions. Finally, we are discussing some corollaries.

2. Definitions and preliminaries

We recall some definitions and notations from the *q*-calculus. Wherever it appears in this paper, a is a fixed complex number, $a \in \mathbb{C}$.

The *q*-shifted factorials are defined as

$$(a;q)_0 = 1; (a;q)_n = \prod_{k=0}^{n-1}(1-aq^k), n = 1,2,\ldots; (a;q)_\infty = \lim_{n\to\infty}(a;q)_n. \quad (1)$$

Notations that usually appear in this article are as follows

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$$[x]_q = \frac{1-q^x}{1-q}, x \in \mathbb{C}; \left([n]_q\right)! = \frac{(q;q)_n}{(1-q)^n}, n \in \mathbb{N}; (a;q)_x = \frac{(a;q)_\infty}{(aq^x;q)_\infty}, x \in \mathbb{R} \Big\}. \quad (2)$$

A q-analogue of the exponential function of type two was introduced in Albayrak et al. (2013) as

$$e_q(x) = \sum_0^\infty \frac{x^n}{(q;q)_n} = \frac{1}{(x;q)_\infty}, |x| < 1, \quad (3)$$

and that analogue of type one was also raised in Albayrak et al. (2013) as

$$\hat{e}_q(x) = \sum_0^\infty \frac{(-1)^n q^{\frac{n(n-1)}{2}} x^n}{(q;q)_n} = (x;q)_\infty, x \in \mathbb{C}. \quad (4)$$

Jackson integrals from 0 to x and from 0 to ∞ were defined by Jackson (1905)

$$\int_0^x f(t) d_q t = x(1-q) \sum_{k=0}^\infty q^k f(xq^k) \quad (5)$$

$$\int_0^{\infty/A} f(t) d_q t = (1-q) \sum_{k\in\mathbb{Z}} \frac{q^k}{A} f\left(\frac{q^k}{A}\right). \quad (6)$$

The straightforward conclusion of the q-shifted factorials is that

$$(q^{x+n};q)_\infty = \frac{(q^x;q)_\infty}{(q^x;q)_n}, n \in \mathbb{N}. \quad (7)$$

It is beneficial here that we recall the q-analogues of the gamma function,

$$\left.\begin{array}{c} \Gamma_q(\alpha) = \int_0^{1/(1-q)} x^{\alpha-1} \hat{e}_q(q(1-q)x) d_q x, (\alpha > 0) \\ \text{and} \\ \tilde{\Gamma}_q(\alpha) = K(A;\alpha) \int_0^{\infty/A(1-q)} x^{\alpha-1} e_q(-(1-q)x) d_q x \end{array}\right\}, \quad (8)$$

where, $\alpha_1 > 0$, $K(A;\alpha)$ is the function

$$K(A;\alpha) = A^{\alpha-1} \frac{(-q/\alpha;q)_\infty (-\alpha;q)_\infty}{(-q^t/\alpha;q)_\infty (-\alpha q^{1-t};q)_\infty}. \quad (9)$$

For our benefit, we also state here some other properties of $\Gamma_q(\alpha)$ and $\tilde{\Gamma}_q(\alpha)$ functions:

$$\Gamma_q(\alpha) = \frac{(q;q)_\infty}{(1-q)^{\alpha-1}} \sum_{k=0}^\infty \frac{q^{k\alpha}}{(q;q)_k} = \frac{(q;q)_\infty}{(q^\alpha - q)_\infty} (1-q)^{1-x}, \quad (10)$$

$x \neq 0, -1, -2, \ldots$, and

$$\tilde{\Gamma}_q(\alpha) = \frac{K(A;\alpha)_\infty}{(1-q)^{\alpha-1}\left(-\frac{1}{A};q\right)_\infty} \sum_{k\in\mathbb{Z}} \left(\frac{q^k}{A}\right) \left(-\frac{1}{A};q\right)_k. \quad (11)$$

3. Mangontarum q-integral transform

Integral transforms have different q-analogues in the theory of q-calculus. Authors such as Abdi (1961), Purohit and Kalla (2007), Uçar and Albayrak (2011), Exton (1978) defined two types of q-analogues of the Laplace transform (of type one and of type two, rep.) as

$$L_q(f(t))(u) = \frac{1}{1-q} \int_0^{\frac{1}{v}} f(t) \hat{e}_q(qut) d_q t \quad (12)$$

and

$${}_qL(f(t))(u) = \frac{1}{1-q} \int_0^\infty f(t) e_q(-ut) d_q t. \quad (13)$$

In that manner, Albayrak et al. (2013), Fitouhi and Bettaibi (2006), Fitouhi and Bettaibi (2007), Hatem and Nadia (2009), Koornwinder and Swarttouw (1992), Fitouhi and Bouzeffour (in press), and some others, define various types of q-analogues of various integral transforms. In the sequence of these integrals, the q-Mangontarum integral transform was recently introduced in Mangontarum (2014) as the q -analogue of the Elzaki transform (Elzaki, 2011).

The q-Mangontarum transform of type one was defined over the set A as (Mangontarum, 2014, Def. 1)

$$T_q(f(t))(u) = u \int_0^\infty f(t) \hat{e}_q\left(-q\frac{t}{u}\right) d_q t \quad (14)$$

where

$$A = \left\{ f(t) : \exists M, k_1, k_2 > 0, |f(t)| < M \hat{e}_q\left(\frac{|t|}{k_j}\right), t \in (-1)^j \times [0,\infty) \right\}, \quad (15)$$

$f(t) \in A$, $k_1 \leqslant u \leqslant k_2$ and $0 \leqslant t$.

On the other hand, the q-Mangontarum transform of type two was defined as (Mangontarum, 2014, Def. 15),

$$\tilde{T}_q(f(t))(u) = u \int_0^\infty f(t) e_q\left(-\frac{t}{u}\right) d_q t \quad (16)$$

over the set

$$\bar{A} = \left\{ f(t) : \exists M, k_1, k_2 > 0, |f(t)| < M e_q\left(\frac{|t|}{k_j}\right), t \in (-1)^j \times [0,\infty) \right\} \quad (17)$$

where $f(t) \in \bar{A}$, $k_1 \leqslant u \leqslant k_2$ and $0 \leqslant t$.

In this article, we introduce two analogues of the q- Mangontarum transform in the following manner:

Definition 1. (i) Over the set A in (15), we define the q-Mangontarum transform of type one as

$$T_q(f(t))(u) = \frac{1}{(1-q)u} \int_0^u f(t) \hat{e}_q\left(q\frac{t}{u}\right) d_q t, \quad (18)$$

where $f(t) \in A$, $k_1 \leqslant u \leqslant k_2$ and $0 \leqslant t$.

(ii) We define the q-Mangontarum transform of type two as

$$\tilde{T}_q(f(t))(u) = \frac{1}{(1-q)} \int_0^\infty f(t) e_q\left(-\frac{t}{u}\right) d_q t, \quad (19)$$

where $f(t) \in \bar{A}, k_1 \leqslant u \leqslant k_2$ and $0 \leqslant t$.

4. q-Bessel functions

Bessel functions were first used by Bessel to describe three body motion appearing in series expansion on planetary perturbation. As the best known q-analogues of the Bessel function, type one and type two are respectively due to Jackson (1905), Exton (1978) and Ismail (1982) given as

$$J_\mu^{(1)}(z;q) = \left(\frac{z}{2}\right)^\mu \sum_{n=0}^\infty \frac{\left(\frac{-z^2}{4}\right)^n}{(q,q)_{\mu+n}(q;q)_n}, \ |z| < 2, \quad (20)$$

$$J_\mu^{(2)}(z;q)=\left(\frac{z}{2}\right)^\mu\sum_{n=0}^\infty\frac{q^{n(n+\mu)}\left(\frac{-z^2}{4}\right)^n}{(q,q)_{\mu+n}(q;q)_n},\ z\in\mathbb{C}. \tag{21}$$

By the idea of q-hypergeometric functions, (20) and (21) can respectively be defined as

$$J_\mu^{(1)}(z;q)=\frac{(q^{\mu+1};q)_\infty}{(q;q)_\infty}\left(\frac{z}{2}\right)^\mu{}_2\phi_1\left[\begin{matrix}0\\q^{\mu+1}\end{matrix};q,\frac{-z^2}{4}\right], \tag{22}$$

$$J_\mu^{(2)}(z;q)=\frac{(q^{\mu+1};q)_\infty}{(q;q)_\infty}\left(\frac{z}{2}\right)^\mu{}_0\phi_1\left[\begin{matrix}-\\q^{\mu+1}\end{matrix};q,\frac{q^{\mu+1}z^2}{4}\right]. \tag{23}$$

Hahn–Exton q-Bessel function (of type three) was introduced by Hahn (1953) and Exton (1978) as

$$J_\mu^{(3)}(z;q)=z^\mu\sum_{n=0}^\infty\frac{(-1)^n q^{\frac{n(n-1)}{2}}(qz^2)^n}{(q,q)_{\mu+n}(q;q)_n},z\in\mathbb{C}. \tag{24}$$

This kind of q-analogue has been presented in terms of q-hypergeometric functions as

$$J_\mu^{(3)}(z;q)=\frac{(q^{\mu+1};q)_\infty}{(q;q)_\infty}z^\mu{}_1\phi_1\left[\begin{matrix}0\\q^{\mu+1}\end{matrix};q,qz^2\right]. \tag{25}$$

5. q-Mangontarum transform T_q of q-Bessel functions

In this section of this paper, we focus our attention to the type one of q-Mangontarum transform.

On taking account of (5), the transform T_q can be written in terms of a series expansion as $T_q(f(t))(u)=\frac{1}{u}\sum_{k=0}^\infty q^k f(uq^k)\hat{e}_q(q^{k+1})$ and by (4) and (2) has a generic form as

$$T_q(f(t))(u)=(q;q)_\infty\sum_{k=0}^\infty\frac{q^k}{(q;q)_k}f(q^k u). \tag{26}$$

Now we aim to estimate some values of the q-Mangontarum transform of type one of a class of q-Bessel functions.

Theorem 2. *Let $f(t)=t^{\Delta-1}\prod_{j=1}^n J_{2\mu_j}^{(1)}(2\sqrt{a_jt};q)$ be given. Then, we have*

$$\begin{aligned}T_q(f(t))(u)=(\delta_\Delta^q)\prod_{j=1}^n\sum_{m_j=0}^\infty(1-q)^{2\mu_j+m_j-1}\\ \times\frac{\left(a_j^{\mu_j+m_j}\right)(q^{2\mu_j+m_j+1};q)_\infty}{(q;q)_{m_j}}(u)^{\mu_j+m_j}\\ \times\Gamma_q(2\mu_j+m_j-1),\end{aligned}$$

where $\delta_\Delta^q=\frac{(u(1-q))^\Delta}{u(q;q)_\infty}$.

Proof. By using (26) and (20) we get

$$\begin{aligned}T_q(f(t))(u)=u^{\Delta-1}(q;q)_\infty\prod_{j=1}^\infty(a_ju)^{\mu_j}\sum_{m_j=0}^\infty\frac{(a_ju)^{m_j}}{(q;q)_{2\mu_j+m_j}(q;q)_{m_j}}\\ \times\sum_{k=0}^\infty\frac{q^{k(\Delta+\mu_j+m_j)}}{(q;q)_k}.\end{aligned} \tag{27}$$

By aid of (10) and the parity of (2), (27) reveals

$$\begin{aligned}T_q(f(t))(u)=(\delta_\Delta^q)\prod_{j=1}^\infty(a_ju)^{\mu_j}\sum_{m_j=0}^\infty\frac{(a_ju)^{m_j}(q^{2\mu_j+m_j+1};q)_\infty}{(q;q)_{m_j}}\\ \times(1-q)^{\mu_j+m_j}\Gamma_q(\Delta+\mu_j+m_j),\end{aligned}$$

where $\delta_\Delta^q=\frac{(u(1-q))^\Delta}{u(q;q)_\infty}$.

A motivation of the previous equation completes the proof of the theorem.

Let us apply now the T_q transform to the family $\left\{J_{2\mu_i}^{(2)}(2\sqrt{a_it};q)\right\}_{i=1}^n$ of Bessel functions.

Theorem 3. *Let $f(t)=t^{\Delta-1}\prod_{j=1}^n J_{2\mu_j}^{(2)}(2\sqrt{a_nt};q)$ be given. Then, we have*

$$\begin{aligned}T_q(f(t))(u)=\left(\widehat{\delta_\Delta}\right)\prod_{j=1}^n\sum_{m_j=0}^\infty\frac{q^{m_j\left(m_j+2\mu_j\right)}\left(a_j\right)^{\mu_j+m_j}(q^{2\mu_j+m_j+1};q)_\infty}{(q;q)_{m_j}}\\ (1-q)^{\mu_j+m_j}\times\Gamma_q(\Delta+\mu_j+m_j)u,\end{aligned}$$

where $\widehat{\delta_\Delta^q}=\frac{(u(1-q))^{\Delta-1}}{(q;q)_\infty}$.

Proof. On account of (26), we obtain that

$$T_q(f(t))(u)=(q;q)_\infty\sum_{k=0}^\infty\frac{q^k}{(q;q)_k}\left(uq^k\right)^{\Delta-1}\prod_{j=1}^\infty J_{2\mu_j}^{(2)}\left(2\sqrt{a_juq^k};q\right). \tag{28}$$

Invoking (21) in (28) yields

$$T_q(f(t))(u)=(q;q)_\infty u^{\Delta-1}\sum_{k=0}^\infty\frac{q^{k\Delta}}{(q;q)_k}\prod_{j=1}^n(a_ju)^{\mu_j}q^{k\mu_j}\sum_{m_j=0}^\infty\frac{q^{(m_j+2\mu_j)m_j}\left(a_juq^k\right)^{m_j}}{(q;q)_{2\mu_j+m_j}(q;q)_{m_j}}. \tag{29}$$

By employing (20), (29) gives

$$T_q(f(t))(u)=u^{\Delta-1}\prod_{j=1}^n\sum_{m_j=0}^\infty\frac{q^{m_j\left(m_j+2\mu_j\right)}\left(a_ju\right)^{\mu_j+m_j}(q^{2\mu_j+m_j+1};q)_\infty}{(q;q)_{m_j}}\sum_{k=0}^\infty\frac{q^{k\left(\Delta+\mu_j+m_j\right)}}{(q;q)_k}. \tag{30}$$

On aid of (10), (30) fairly implies

$$\begin{aligned}T_q(f(t))(u)=\left(\widehat{\delta_\Delta^q}\right)\prod_{j=1}^n\sum_{m_j=0}^\infty\frac{q^{m_j\left(m_j+2\mu_j\right)}\left(a_j\right)^{\mu_j+m_j}(q^{2\mu_j+m_j+1};q)_\infty}{(q;q)_{m_j}}\\ \times(1-q)^{\mu_j+m_j}\Gamma_q(\Delta+\mu_j+m_j)u,\end{aligned}$$

where $\widehat{\delta_\Delta^q}$ has the usual meaning above.

This completes the proof of the theorem.

Finally in this section, we apply the q-Mangontarum transform to a class of q-Bessel functions of type three.

Theorem 4. *Let $f(t)=t^{\Delta-1}\prod_{j=1}^n q_{2\mu_j}^{\mu_j(3)}\left(\sqrt{q^{-1}a_nt};q\right)$ be given. Then, we have*

$$\begin{aligned}T_q(f(t))(u)=\left(\widehat{\delta_\Delta^q}\right)\prod_{j=1}^n\sum_{m_j=0}(-1)^{m_j}\frac{q^{m_j\binom{m_j-1}{2}}a_j^{\mu_j+m_j}}{(q;q)_{m_j}}(q^{2\mu_j+m_j+1};q)_\infty\\ \times(1-q)^{\mu_j+m_j}\Gamma_q(\Delta+\mu_j+m_j)(u)^{\mu_j+m_j},\end{aligned}$$

where $\widehat{\delta_\Delta^q} = \frac{(u(1-q))^{\Delta-1}}{(q;q)_\infty}$.

Proof. By (28) and (24) and direct computations we write

$$T_q(f(t))(u) = (q;q)_\infty u^{\Delta-1}\sum_{k=0}^{\infty}\frac{q^{k\Delta}}{(q;q)_k}\prod_{j=1}^{n}(a_ju)^{\mu_j}q^{k\mu_j}\sum_{m_j=0}^{\infty}\frac{(-1)^{m_j}q^{m_j\binom{m_j-1}{2}}(a_juq^k)^{m_j}}{(q;q)_{2\mu_j+m_j}(q;q)_{m_j}}$$
$$= (q;q)_\infty\frac{u^{\Delta-1}}{v^\Delta}\prod_{j=1}^{n}\sum_{m_j=0}^{\infty}\frac{(-1)^{m_j}q^{m_j\binom{m_j-1}{2}}\left(a_j\frac{u}{v}\right)^{\mu_j+m_j}}{(q;q)_{2\mu_j+m_j}(q;q)_{m_j}}\sum_{k=0}^{\infty}\frac{q^{k(\Delta\mu_j+m_j)}}{(q;q)_k}. \quad (31)$$

By further use of (2) and (10), (31) finally yields

$$T_q(f(t))(u) = \left(\widehat{\delta_\Delta^q}\right)\prod_{j=1}^{n}\sum_{m_j=0}(-1)^{m_j}\frac{q^{m_j\binom{m_j-1}{2}}a_j^{\mu_j+m_j}}{(q;q)_{m_j}}(q^{2\mu_j+m_j+1};q)_\infty$$
$$\times(1-q)^{\mu_j+m_j}\times\Gamma_q(\Delta+\mu_j+m_j)(u)^{\mu_j+m_j},$$

where $\widehat{\delta_\Delta^q}$ has the usual meaning.

This completes the proof of the theorem.

6. q-Mangontarum transform $\widetilde{T}_q$ of q-Bessel functions

In this section of this article, we focus our attention on the q-Mangontarum transform of type two. The series representation of the second type q-Mangontarum can be derived from (5) as

$$\widetilde{T}_q(f(t))(u) = \sum_{k\in\mathbb{Z}}\frac{q^kf(q^k)}{(-uq^k;q)_\infty}.$$

By (2), this expression can be written as

$$\widetilde{T}_q(f(t))(u) = \frac{1}{(-u;q)_\infty}\sum_{k\in\mathbb{Z}}(-u;q)_kq^kf(q^k). \quad (32)$$

We establish the following theorem.

Theorem 5. *Let* $f(t) = t^{\Delta-1}\prod_{j=1}^{n}J_{2\mu_j}^{(2)}(2\sqrt{a_jt};q)$. *Then, we have*

$$\widetilde{T}_q(f(t))(u) = (\alpha_\Delta^q)\prod_{j=1}^{n}\sum_{m_j=0}(-1)^{m_j}\frac{a_j^{\mu_j+m_j}q^{m_j(\mu_j+m_j)}(1-q)^{(\mu_j+m_j)}}{k(u;\Delta+\mu_j+m_j)}$$
$$\times\widetilde{\Gamma}_q(\Delta+\mu_j+m_j)(u)^{\mu_j+m_j},$$

where $(\alpha_\Delta^q) = \frac{(1-q)^{\Delta-1}u^\Delta}{(q;q)_\infty}$.

Proof. On account of (32) and (21), we by aid of (2) write

$$\widetilde{T}_q(f(t))(u) = \frac{1}{\left(-\frac{1}{u};q\right)_\infty}\prod_{j=1}^{n}\sum_{m_j=0}^{\infty}(-1)^{m_j}\frac{(a_j)^{(\mu_j+m_j)}q^{m_j(\mu_j+m_j)}}{(q;q)_{m_j}(q;q)_\infty}(q^{2\mu_j+m_j+1};q)_\infty$$
$$\times\sum_{k\in\mathbb{Z}}q^{k(\Delta+\mu_j+m_j)}\left(-\frac{1}{u};q\right)_k. \quad (33)$$

By using (11) and setting $A = u$ and $\alpha = \Delta+\mu_j+m_j$, we write (33) as

$$\widetilde{T}_q(f(t))(u) = \frac{(1-q)^{\Delta-1}u^\Delta}{(q;q)_\infty}\prod_{j=1}^{n}\sum_{m_j=0}^{\infty}(-1)^{m_j}$$
$$\times\frac{(a_j)^{(\mu_j+m_j)}q^{m_j(2m_j+2\mu_j)}(1-q)^{(\mu_j+m_j)}\Gamma(\Delta+\mu_j+m_j)}{K(u;\Delta+\mu_j+m_j)}(u)^{\mu_j+m_j}.$$

Hence the theorem is proved.

Theorem 6. *Let* $f(t) = t^{\Delta-1}\prod_{j=1}^{n}q^{m_j}J_{2\mu_j}^{(3)}(2\sqrt{q^{-1}a_jt};q)$. *Then, we have*

$$\widetilde{T}_q(f(t))(u) = (\alpha_\Delta^q)\prod_{j=1}^{n}\sum_{m_j=0}^{\infty}(-1)^{m_j}(a_j)^{(\mu_j+m_j)}q^{\frac{m_j(\mu_j+m_j)}{2}}$$
$$\times\left(q^{m_j(\mu_j+m_j)};q\right)_\infty$$
$$\times\frac{(1-q)^{(\mu_j+m_j)}\Gamma(\Delta+\mu_j+m_j)}{K(u;\Delta+\mu_j+m_j)}(u)^{\mu_j+m_j},$$

where $(\alpha_\Delta^q) = \frac{(1-q)^{\Delta-1}u^\Delta}{(q;q)_\infty}$.

Proof. By using (32) and (24) we by aid of (2) write

$$\widetilde{T}_q(f(t))(u) = \frac{1}{\left(-\frac{1}{u};q\right)_\infty(q;q)_\infty}\prod_{j=1}^{n}\sum_{m_j=0}^{\mu_j}(-1)^{m_j}$$
$$\times\frac{(a_j)^{(\mu_j+m_j)}(q^{2\mu_j+m_j+1};q)}{(q;q)_{m_j}}$$
$$\times q^{\frac{m_j(m_j-1)}{2}}\sum_{k\in\mathbb{Z}}q^{k(\Delta+\mu_j+m_j)}\left(-\frac{1}{u};q\right)_k.$$

Using of (11), for $A = u$ and $\alpha = \Delta+\mu_j+m_j$, gives

$$\widetilde{T}_q(f(t))(u) = (\alpha_\Delta^q)\prod_{j=1}^{n}\sum_{m_j=0}^{\infty}(-1)^{m_j}\frac{(a_j)^{(\mu_j+m_j)}q^{\frac{m_j(m_j-1)}{2}}(q^{2\mu_j+m_j+1};q)}{K(u;\Delta+\mu_j+m_j)}$$
$$\times(1-q)^{(\mu_j+m_j)}\Gamma_q(\Delta+\mu_j+m_j)\times\left(\frac{1}{u}\right)^{\mu_j+m_j},$$

where α_Δ^q has the usual meaning.

This completes the proof of the theorem.

Corollary 7. *Let* $J_1^{(1)}$ $J_1^{(2)}$ *be a Bessel function of type one and type two, respectively. Then,*

(i) $T_q\left(t^{\Delta-1}J_1^{(1)}(2\sqrt{at};q)\right)(u) = (1-q)^\Delta u^{\Delta-1}\sum_0^\infty\frac{(1-q)^m}{(q^m;q)_2(q^m;q)_m}(au)^m$.

(ii) $T_q\left(t^{\Delta-1}J_1^{(2)}(2\sqrt{at};q)\right)(u) = (au)^{\frac{1}{2}}\frac{u^{\Delta-1}}{(1-q)^{\frac{1}{2}}}\sum_{m=0}^{\infty}\frac{q^{m(m+1)}(q^m;q)_\infty}{(q^m;q)_2(q^{\Delta+m+1};q)_\infty}(u)^m$.

(iii) *Let* $f(t) = t^{\Delta-1}\prod_{j=1}^{n}q^{m_j}J_{2\mu_j}^{(3)}(2\sqrt{q^{-1}a_jt};q)$ and $\alpha_\Delta^q = \frac{(1-q)^{\Delta-1}u^\Delta}{(q;q)_\infty}$. *Then,*

(i′) $\widetilde{T}_q(f(t))(u) = (\alpha_\Delta^q)\prod_{j=1}^{n}\sum_{m_j=0}^{\infty}(-1)^{m_j}(a_j)^{(\mu_j+m_j)}q^{\frac{m_j(\mu_j+m_j)}{2}}\left(q^{m_j(\mu_j+m_j)};q\right)_\infty\times\frac{(1-q)^{(\mu_j+m_j)}\Gamma(\Delta+\mu_j+m_j)}{K(u;\Delta+\mu_j+m_j)}(u)^{\mu_j+m_j}$,

(ii″) $\widetilde{T}_q(f(t))(u) = (\alpha_\Delta^q)\prod_{j=1}^{n}\sum_{m_j=0}^{\infty}(-1)^{m_j}(a_j)^{(\mu_j+m_j)}q^{\frac{m_j(\mu_j+m_j)}{2}}\left(q^{m_j(\mu_j+m_j)};q\right)_\infty\times\frac{(1-q)^{(\mu_j+m_j)}\Gamma(\Delta+\mu_j+m_j)}{K(u;\Delta+\mu_j+m_j)}(u)^{\mu_j+m_j}$.

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