Minimal flavour violation and multi-Higgs models

F.J. Botella\textsuperscript{a}, G.C. Branco\textsuperscript{b}, M.N. Rebelo\textsuperscript{b,\*}

\textsuperscript{a} Departament de Física Teòrica and IFIC, Universitat de València-CSIC, E-46100, Burjassot, Spain
\textsuperscript{b} Departamento de Física and Centro de Física Teórica de Partículas (CFTP), Instituto Superior Técnico, Av. Rovisco Pais, P-1049-001 Lisboa, Portugal

\textbf{A R T I C L E   I N F O}

Article history:
Received 7 December 2009
Received in revised form 25 February 2010
Accepted 3 March 2010
Available online 9 March 2010
Editor: G.F. Giudice

Keywords:
Minimal Flavour Violation
Multi-Higgs models
Flavour symmetries

\textbf{A B S T R A C T}

We propose an extension of the hypothesis of Minimal Flavour Violation (MFV) to general multi-Higgs models without the assumption of Natural Flavour Conservation (NFC) in the Higgs sector. We study in detail under what conditions the neutral Higgs couplings are only functions of $V_{\text{CKM}}$ and propose a MFV expansion for the neutral Higgs couplings to fermions.

© 2010 Elsevier B.V. Open access under CC BY license.

\section{1. Introduction}

The Standard Model (SM) of the electroweak and strong interactions has had an impressive success in accounting for most of the presently available experimental data. The discovery of non-vanishing neutrino masses provided a notable exception \cite{1}, pointing towards New Physics (NP), since in the SM neutrinos are strictly massless.

In spite of its great success, there is a general consensus that the SM including its simple extension incorporating neutrino masses, cannot be the "final theory". One of the reasons for this, has to do with the large number of free parameters, most of them arising from the flavour sector of the SM. This proliferation of free parameters reflects the fact that the flavour structure of Yukawa couplings is not constrained by gauge invariance. In the SM, Flavour Changing Neutral Currents (FCNC) are forbidden at tree level both in the gauge and the Higgs sectors. From the early stages of gauge theories, some principles of flavour conservation by neutral currents have been introduced both in the gauge sector through a generalisation of the GIM mechanism \cite{2}, as well as in the scalar sector through the principle of Natural Flavour Conservation (NFC) proposed by Glashow and Weinberg \cite{3}. It is interesting to note that one may have non-zero but naturally suppressed FCNC in the gauge sector in models where vector-like quarks \cite{4,5,6} are added to the SM. In this case, gauge mediated FCNC arise at tree level, suppressed by the small ratio $m^2/M^2$ where $m$ and $M$ denote standard quark masses and vector-like quark masses, respectively. Vector-like quarks arise in various extensions of the SM, including $E_6$ grand-unified theories and extra-dimension models. Other motivations for considering vector-like quarks include the possibility of finding a solution to the strong CP problem \cite{7,5} and accounting for \cite{6} the potentially large CP asymmetry recently observed in $B_s \rightarrow J/\Psi \phi$ decays \cite{8,9}. Recently, a different possibility was considered \cite{10} to avoid tree-level FCNC processes in the framework of two Higgs doublet models, allowing for new sources of CP violation.

All the flavour changing transitions in the SM are mediated by charged weak currents with the flavour mixing controlled by the Cabibbo–Kobayashi–Maskawa (CKM) matrix, $V_{\text{CKM}}$ \cite{11}. Any extension of the SM which attempts at solving the flavour puzzle has to confront the strict limits on FCNC processes as well as limits on CP violating transitions leading, for example, to electric dipole moments of quarks and leptons \cite{12}.

In the scalar sector, it has been considered the possibility of allowing for deviations of strict NFC by invoking the presence of suppression factors \cite{13,14} involving small off-diagonal elements of the quark mixing matrix $V_{\text{CKM}}$. The first models of this type were proposed by Branco, Grimus and Lavoura (BGL) \cite{15} who have shown that there are extensions of the SM with two Higgs doublets and an additional discrete symmetry, where there are FCNC at tree level, with couplings entirely determined in terms of the CKM matrix elements, with no other free parameters. In some
variants of these models [15] the Higgs particles can be relatively light, without entering in conflict with the stringent limits on FCNC processes.

The success of the SM and its CKM mechanism of mixing and CP violation shows that if there are New Physics contributions to flavour changing interactions at the TeV scale its couplings should occur at a much higher scale or else should be strongly non-generic. This is natural and in a certain sense to be expected if one takes into account that flavour changing transitions in the SM have a special flavour structure, not predicted within its framework. For example, in the SM there is no explanation for the pattern of flavour mixings and in particular why \((V_{\text{CKM}})_{12} \sim (m_d/m_s)^{1/2}\) while \((V_{\text{CKM}})_{13} \sim (m_d/m_b)\).

One of the suggestions for the flavour structure of New Physics is the proposal of Minimal Flavour Violation (MFV) [16,17] where one of the ingredients is the assumption that all new flavour changing transitions are controlled by the CKM matrix. The gauge sector of the Standard Model (SM) with three generations of quarks and leptons has a large \(G_F = U(3)^3\) flavour symmetry which is only broken by Yukawa couplings. One may formally recover [17] this flavour symmetry by promoting Yukawa couplings to auxiliary fields \(Y\), transforming under \(G_F\) in such a way that Yukawa interactions become \(G_F\) invariant. Then an effective theory arising from New Physics is of MFV type if all higher order operators, constructed from SM fields and \(Y\) fields are formally invariant under \(G_F\). This hypothesis, together with the realization that in the SM Yukawa couplings for all fermions, except the top, are small, leads to specific predictions [18].

If one regards the SM as an effective theory, valid up to some energy scale \(\Lambda\), then in order to have a solution of the hierarchy problem, one expects the scale \(\Lambda\) of New Physics to be of the order of a few TeV. The above considerations have motivated the idea of Minimal Flavour Violation (MFV) both in the quark [16,17] and lepton sectors [19,20]. The MFV hypothesis requires that all flavour and CP violating interactions be related to the structure of Yukawa couplings and controlled by \(V_{\text{CKM}}\).

The MFV idea has been applied to two Higgs doublet extensions of the SM where there is Natural Flavour Conservation (NFC) in the Higgs sector at tree level, as it is the case in the minimal supersymmetric extension of the Standard Model (MSSM).

In this Letter we examine how to implement the MFV ingredient of having the CKM matrix as the only non-trivial flavour mixing matrix in the scalar sector with two and three Higgs doublets, without the assumption of Natural Flavour Conservation in the Higgs sector. We also investigate how the above scheme could result from a family symmetry imposed on the full Lagrangian. As a result, the implementation of the above condition leads to zero textures in the Yukawa couplings. Notice however that these zeros are stable under renormalisation, since they result from a symmetry. Although the scenario we are considering has some of the ingredients of what is usually called “Minimal Flavour Violation” it involves a somewhat weaker condition. It has, however, the interesting feature of resulting from an exact symmetry of the Lagrangian, with no further assumptions. This Letter is organised as follows: in Section 2 we recall the important requirement of rephasing invariance, and in Section 3 we analyse in detail how the requirement of MFV can be fulfilled in the context of an extension of the SM where two Higgs doublets are introduced. In Section 4 we propose a general MFV expansion of the neutral Higgs couplings to quarks and we stress the important role of discrete symmetries in fixing the parameters of this expansion. The case of three Higgs doublets in the context of MFV is analysed in Section 5 and our conclusions are presented in Section 6.

2. The requirement of rephasing invariance

As we have seen, the definition of MFV includes the requirement that all flavour transitions are controlled by the CKM matrix. Let us consider a FCNC transition connecting, for definiteness, a quark \(d_i\) to a different quark of the same charge \(d_j\). The transition could be mediated by a scalar or a vector boson:

\[
\mathcal{L}_{\text{scalar}} = d_i^* d_j J_5^S \Gamma d_k \bar{S},
\]

\[
\mathcal{L}_{\text{vector}} = d_i^* d_j J_5^V \Gamma d_k V^\mu.
\]

Note that the couplings \(J_5^S\), \(J_5^V\) may arise at tree level or in higher orders. Let us assume that the quark mass matrices have been diagonalised, so that \(d_j\) denote quark mass eigenstates. Under rephasing of the quark fields:

\[
d_j \rightarrow d_j' = \exp(-i\beta_j) d_j
\]

the couplings \(J_5^S\) and \(J_5^V\) have to transform in such a way that the interactions of Eqs. (1) and (2) remain rephasing invariant. This implies that under rephasing

\[
\Gamma_{jk} \rightarrow \Gamma'_{jk} = \exp\left[i(\beta_k - \beta_j)\right] \Gamma_{jk}.
\]

The fact that in MFV theories, the flavour dependence of \(\Gamma_{jk}\) is completely controlled by the CKM matrix, severely restricts the functional dependence of \(\Gamma_{jk}\) on \(V_{\text{CKM}}\). The simplest forms allowed by rephasing invariance are:

\[
\Gamma_{jk} = \sum_a c_a V_{aj} V_{ak}^*.
\]

where \(c_a\) are rephasing invariant coefficients. In the sequel, we shall see that the simplest two Higgs doublet (2HD) models which conform to the MFV requirement do have FCNC couplings with such functional dependence on \(V_{\text{CKM}}\).

3. The case of two Higgs doublets

In this section, we analyse in detail how the requirement of MFV can be fulfilled in the context of an extension of the SM, where two Higgs doublets are introduced. In order to fix our notation, we explicitly write the Yukawa interactions:

\[
L_Y = -\frac{\bar{d}_L^i \Gamma_1 \Phi_1 d_R^i}{\sqrt{2} \Lambda_1} - \frac{\bar{d}_L^i \Gamma_2 \Phi_2 d_R^i}{\sqrt{2} \Lambda_1} - \frac{\bar{d}_L^i \Phi_3 d_R^i}{\sqrt{2} \Lambda_2} - \frac{\bar{d}_L^i \Phi_4 d_R^i}{\sqrt{2} \Lambda_2} + \text{h.c.}
\]

where \(\Gamma_1\) and \(\Delta_1\) denote the Yukawa couplings of the lefthanded quark doublets \(Q_1^0\) to the righthanded quarks \(d_R^0\), \(u_R^0\) and the Higgs doublets \(\Phi_i\). The quark mass matrices generated after spontaneous gauge symmetry breaking are given by:

\[
M_d = \frac{1}{\sqrt{2}} (v_1 \Gamma_1 + v_2 \epsilon^{\alpha \gamma} \Gamma_2),
\]

\[
M_u = \frac{1}{\sqrt{2}} (v_1 \Delta_1 + v_2 \epsilon^{\alpha \gamma} \Delta_2),
\]

where \(v_1 = \langle 0 | \Phi_1^0 | 0 \rangle\) and \(\alpha\) denotes the relative phase of the vacuum expectation values (vevs) of the neutral components of \(\Phi_i\). The matrices \(M_d, M_u\) are diagonalised by the usual bi-unitary transformations:

\[
U_{dL}^* M_d U_{dR} = D_d \equiv \text{diag}(m_d, m_s, m_b),
\]

\[
U_{uL}^* M_u U_{uR} = D_u \equiv \text{diag}(m_u, m_c, m_t).
\]
In terms of the quark mass eigenstates $u, d$, the Yukawa couplings are:
\[ L_Y = \sqrt{2} H^+ \bar{u} (V N_d Y_R + N_u^T \gamma_2) d + \text{h.c.} - \frac{H^0}{\nu} (\bar{u} D_d u + \bar{d} D_d d) \]
\[ - \frac{R}{\nu} \left[ \bar{u} (N_u Y_R + N_d^T \gamma_2) y u + \bar{d} (N_d Y_R + N_u^T \gamma_2) d \right] \]
\[ + i \frac{\eta}{\nu} \left[ \bar{u} (N_u y_R - N_d^T \gamma_2) u - \bar{d} (N_d y_R - N_u^T \gamma_2) d \right] \]
\[ \text{where } \nu = \sqrt{v^2 + v^2} = (\sqrt{2} G_F)^{-1/2} \approx 246 \text{ GeV}, \ G_F \text{ is the Fermi coupling constant}, \ y_R = (1 - \gamma_5)/2, \ y_R = (1 + \gamma_5)/2, \ V \text{ stands for the } V_{\text{CKM}} \text{ matrix and } H^0, R \text{ are orthogonal combinations of the fields } \rho_j, \text{ arising when one expands } [21] \text{ the neutral scalar fields around their vevs, } \phi_j^0 = \frac{v_j}{\sqrt{2}} (\psi_j + \rho_j + i n_j). \text{ Similarly, } l \text{ denotes the linear combination of } n_j \text{ orthogonal to the neutral Goldstone boson. The physical neutral Higgs fields are combinations of } H^0, R \text{ and } l. \]

The flavour Changing Neutral Yukawa Couplings (FCNYC) are controlled by the matrices $N_d, N_u$, given by:
\[ N_d = \frac{1}{\sqrt{2}} U_{dL}^\dagger (v_2) \Gamma_1 - v_1 e^{i \alpha} \Gamma_2 U_{dR}, \]
\[ N_u = \frac{1}{\sqrt{2}} U_{uL}^\dagger \delta_1 - v_1 e^{-i \alpha} \Delta_2 U_{uR}. \]

For generic two Higgs doublet models, the coupling matrices $N_d, N_u$ are non-diagonal and arbitrary. We are interested in analysing under what circumstances the flavour structure of $N_d, N_u$ is entirely controlled by the CKM matrix, as required by the MFV paradigm.

For definiteness, let us consider $N_d$, which can be written [22] from Eqs. (7), (8) and (11):
\[ N_d = \frac{v_2}{v_1} D_d - \frac{v_2}{\sqrt{2}} \left( \frac{v_2}{v_1} + \frac{v_1}{v_2} \right) U_{dL} e^{i \alpha} \Gamma_2 U_{dR}. \]

From Eq. (13), one sees that there are two obstacles which one has to surmount in order to have $N_d$ entirely controlled by $V_{\text{CKM}}$:

(i) It is $U_{dL}$ rather than the combination $U_{dL}^\dagger U_{dL}$ corresponding to $V_{\text{CKM}}$ that appears in $N_d$ given by Eq. (13).

(ii) How to get rid of the dependence on $U_{dR}$?

The first difficulty can be solved by means of a flavour symmetry constraining $U_{dL}$ to have mixing only among two generations, for example:
\[ U_{dL} = \begin{bmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 1 \end{bmatrix}. \]

In this case one has:
\[ (V_{\text{CKM}})_3 = (U_{dL})_3. \]

In order to surmount obstacle (i) one has to further require that the above symmetry should also impose that the dependence of the second term of Eq. (13) on $U_{dL}$ be only on elements of its third row, $(U_{dL})_3$. We now turn to question (ii) namely, how to avoid the dependence on $U_{dR}$. Let us assume that the flavour structure of $\Gamma_2$, is such that:
\[ \Gamma_2 \propto PM_d. \]

Where $P$ is a fixed matrix. In this case:
\[ U_{dL}^\dagger \Gamma_2 U_{dR} \propto U_{dL}^\dagger PM_d U_{dR} \propto U_{dL}^\dagger PU_{dL} D_d \]

thus answering question (ii).

Let us now see what should be the flavour structure of $\Gamma_1, \Gamma_2$ so that a fixed matrix $P$ exists, satisfying Eq. (16). One way of achieving this is by having
\[ P \Gamma_2 = k I_{2}, \]
\[ P \Gamma_1 = 0 \]
where $k$ is a constant.

Branco, Grimus and Lavoura have shown [15] that it is possible to find a symmetry which, when imposed to a two Higgs doublet extension of the SM, leads to a structure for $\Gamma_1$ and $\Delta_1$ such that there are scalar FCNC at tree level, with strength completely controlled by $V_{\text{CKM}}$. BGL have imposed the following symmetry $S$ on the Lagrangian:
\[ Q^{13}_L \rightarrow \exp(i \alpha) Q^{13}_L, \quad u^{0}_{R3} \rightarrow \exp(\pm i \alpha) u^{0}_{R3}, \quad \phi_{1,2} \rightarrow \exp(i \alpha) \phi_{1,2}, \]

where $\alpha \neq 0, \pi$, with all other fields transforming trivially under $S$. The most general Yukawa couplings consistent with this symmetry have the following structure:
\[ \Gamma_1 = \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \]
\[ \Delta_1 = \begin{bmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \]

where $\times$ denotes an arbitrary entry while the zeros are imposed by the symmetry $S$.

It is clear that these Yukawa couplings guarantee that Eqs. (14) and (15) are satisfied. They also satisfy Eqs. (16), (18), (19) with
\[ P = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \sqrt{2} \frac{v_2}{v_1} e^{i \alpha} \Gamma_2 = PM_d, \quad k = 1. \]

It follows then that the Yukawa couplings of Eqs. (21) and (22) lead to FCNC at tree level, entirely determined by $V_{\text{CKM}}$. Notice that in this example there are no Higgs mediated FCNC in the $u$ sector, which is due to the fact that the $\Delta_1$ matrices are block diagonal with each one of these matrices having non-zero entries in different blocks. This also automatically leads to a matrix $U_{dL}$ which is block diagonal and therefore of the form given by Eq. (14). The structure of zeros in the matrix $\Gamma_2$ leads to the important relation:
\[ (U_{dL}^\dagger \Gamma_2)_i j = (U_{dL}^\dagger)_i (\Gamma_2)_j j = (V_{\text{CKM}}^\dagger)_i (\Gamma_2)_j j \]

this result together with Eqs. (16), (18), (19) and (13) leads to $N_d$ given by [15]
\[ (N_d)_{ij} = \frac{v_2}{v_1} (D_d)_{ij} - \left( \frac{v_2}{v_1} + \frac{v_1}{v_2} \right) (V_{\text{CKM}}^\dagger)_i (\Gamma_2)_j j \]

whereas
\[ N_u = - \frac{v_1}{v_2} \text{diag}(0, 0, m_t) + \frac{v_2}{v_1} \text{diag}(m_u, m_c, 0). \]

In this example, the Higgs mediated FCNC are suppressed by the third row of the matrix $V_{\text{CKM}}$ and have the structure of Eq. (5). A crucial feature in this example is the fact that each row of $M_d$ only receives contribution from a single Higgs field and the same applies to $M_u$. 

\[ U_{dL}^\dagger \Gamma_2 U_{dR} \propto U_{dL}^\dagger PM_d U_{dR} \propto U_{dL}^\dagger PU_{dL} D_d \]
The example given above corresponds to a class of six different models, as was emphasised in [15]. Three of these models have FCNC only in the down sector, and are obtained from the three different projection matrices of a form similar to that in Eq. (23), the other two cases with the diagonal entry in the other two possible entries. In these additional cases the suppression in the Higgs mediated FCNC is not as large as that of the example given above. Another three models are obtained by exchanging the patterns of zeros of \( \Gamma_2 \) matrices with \( \Delta_1 \) matrices, leading to FCNC in the up sector, and flavour conservation in the down sector.

4. MFV expansion of Yukawa couplings

The neutral Higgs interactions, beyond those present in the SM, i.e., couplings to \( K_1 \) and \( I_1 \), are those that may introduce Higgs mediated FCNC and are given by Eq. (12), where \( N_d \) and \( N_u \) are written in the quark mass eigenstate basis. In a weak basis these couplings are:

\[
N_d^0 = U_d L N_d U_d^T = \frac{1}{\sqrt{2}} (v_2 \Gamma_1 - v_1 e^{i\alpha} \Gamma_2),
\]

\[
N_u^0 = U_u L N_u U_u^T = \frac{1}{\sqrt{2}} (v_2 \Delta_1 - v_1 e^{i\alpha} \Delta_2).
\]

All other couplings involving neutral scalars are flavour conserving, therefore they are not relevant for our analysis. The question that we address in this section is how to find a general expansion of \( N_d^0, N_u^0 \) which conforms to the MFV requirements. It is clear that a necessary condition for \( N_d^0, N_u^0 \) to be of the MFV type is that they should be functions of \( M_d, M_u \) and no other flavour dependent couplings. The terms entering in the expansion of \( N_d^0, N_u^0 \) should have the right transformation properties under Weak Basis (WB) transformations, defined by:

\[
Q \rightarrow W_L Q, \quad d_R \rightarrow W_R^d d_R, \quad u_R \rightarrow W_R^u u_R.
\]

Under a WB transformation defined by Eq. (29), the quark mass matrices \( M_d, M_u \) transform as:

\[
M_d \rightarrow W_L^d M_d W_R^d, \quad M_u \rightarrow W_L^u M_u W_R^u.
\]

The matrices \( U_d L, U_d R, U_u L, U_u R \) defined in Eqs. (8), (9) transform under a WB transformation in the following way:

\[
U_d L \rightarrow W_L^d U_d L, \quad U_u L \rightarrow W_L^u U_u L,
\]

\[
U_d R \rightarrow W_R^d U_d R, \quad U_u R \rightarrow W_R^u U_u R.
\]

The Hermitian matrices \( H_d, H_u \) with \( H_{d,u} = (M_{d,u}) (M_{d,u})^\dagger \) transform under a WB transformation as:

\[
H_d \rightarrow W_L^d H_d W_L, \quad H_u \rightarrow W_L^u H_u W_L.
\]

From Eqs. (8), (9) it follows that:

\[
U_d^\dagger H_d U_d = D_d^2
\]

with analogous expression for \( H_u \). It is convenient to write \( H_d, H_u \) in terms of projection operators [23]:

\[
H_d = \sum_i m_d^2 p_{dL}^{iL}
\]

where:

\[
p_{dL} = U_d L P_i U_d^\dagger
\]

with:

\[
(P_i)_{jk} = \delta_{ij} \delta_{ik}.
\]

Obviously, analogous expressions hold for \( H_u \). It is clear that under a WB transformation, \( N_d^0, N_u^0 \) transform as \( M_d, M_u \). A MFV expansion for \( N_d^0, N_u^0 \) with proper transformation properties under a WB transformation can then be built with terms proportional to \( M_d \) (\( M_u \)) respectively, as well as products of terms transforming as \( H_d \) and \( H_u \) multiplying \( M_d \) (\( M_u \)) respectively:

\[
N_d^0 = \lambda_1 M_d + \lambda_2 U_d L P_i U_d^\dagger M_d + \lambda_3 U_u L P_i U_u^\dagger M_u + \cdots
\]

\[
N_u^0 = \tau_1 M_u + \tau_2 U_u L P_i U_u^\dagger M_u + \tau_3 U_d L P_i U_d^\dagger M_d + \cdots
\]

In the quark mass eigenstate basis \( N_d^0, N_u^0 \) become:

\[
N_d = \lambda_1 D_d + \lambda_2 U_d L P_i U_d^\dagger D_d + \cdots
\]

\[
N_u = \tau_1 D_u + \tau_2 U_u L P_i U_u^\dagger D_u + \cdots
\]

which conforms explicitly to the MFV requirement. Terms of the form \( U_d L P_i U_d^\dagger M_d \) and \( U_u L P_i U_u^\dagger M_u \) do not lead to Higgs mediated FCNC, whereas terms of the form \( U_d L P_i U_d^\dagger M_d \) and \( U_d L P_i U_d^\dagger M_u \) do lead to FCNC. At this stage the lambda and tau coefficients of these expansions appear as free parameters. This was to be expected, since the expansions of Eqs. (39), (40), conform to the MFV requirement but have no further restriction. In theories where the MFV requirement results from the imposition of a symmetry on the Lagrangian, the coefficients lambda and tau are constrained.

Comparing Eqs. (25) and (26) to Eqs. (39) and (40) one realises that the BGL example presented in the previous section corresponds to the following truncation of our MFV expansion:

\[
N_d^0 = \frac{v_2}{v_1} M_d - \left( \frac{v_2}{v_1} + \frac{v_1}{v_2} \right) U_u L P_2 U_u^\dagger M_d.
\]

\[
N_u^0 = \frac{v_2}{v_1} M_u - \left( \frac{v_2}{v_1} + \frac{v_1}{v_2} \right) U_u L P_3 U_u^\dagger M_u.
\]

This result, together with equations:

\[
N_d^0 = \frac{v_2}{v_1} M_d - \frac{v_2}{v_2} \left( \frac{v_2}{v_1} + \frac{v_1}{v_2} \right) e^{i\alpha} \Gamma_2,
\]

\[
N_u^0 = \frac{v_2}{v_1} M_u - \frac{v_2}{v_2} \left( \frac{v_2}{v_1} + \frac{v_1}{v_2} \right) e^{-i\alpha} \Delta_2
\]

implies that the BGL model is fully defined in a covariant way under WB transformations by:

\[
\frac{v_2}{\sqrt{2}} e^{i\alpha} \Gamma_2 = U_u L P_3 U_u^\dagger M_d.
\]

\[
\frac{v_2}{\sqrt{2}} e^{-i\alpha} \Delta_2 = U_u L P_3 U_u^\dagger M_u.
\]

The factors multiplying \( \Gamma_2 \) and \( \Delta_2 \) coincide with the coefficients for these matrices in the expressions of \( M_d \) and \( M_u \). Replacing in these equations the mass matrices written in terms of the Yukawa couplings one obtains:

\[
U_u L P_3 U_u^\dagger \Gamma_2 = \Gamma_2, \quad U_u L P_3 U_u^\dagger \Gamma_1 = 0,
\]

\[
U_u L P_3 U_u^\dagger \Delta_2 = \Delta_2, \quad U_u L P_3 U_u^\dagger \Delta_1 = 0.
\]

These relations are the generalisation to an arbitrary basis of the relations satisfied by the BGL model, namely \( P_2 \Gamma_2 = \Gamma_2, \quad P_3 \Gamma_1 = 0, \quad P_3 \Delta_2 = \Delta_2 \) and \( P_3 \Delta_1 = 0 \) which result from the imposed symmetry. Now, we show that in fact, in this case there is a WB where the matrices \( \Gamma_2, \Delta_1, \Delta_2 \) have the forms given by Eqs. (21) and (22). Starting from a WB where \( M_u \) is real and diagonal, and therefore \( U_u L = 1 \), we may perform a WB transformation by choosing \( W_L \) and \( W_R^d \) block diagonal with mixing in the (12) block only.
As a result, the matrix $M_u$ will also be block diagonal, in this WB. Eq. (45) becomes:
\[
\frac{\sqrt{2}}{\sqrt{2}} e^{i\alpha} \Gamma_2 = W^\dagger_1 (12) P_3 W^\dagger_1 (12) M_d = P_3 M_d
\]
which is exactly the form of $\Gamma_2$ given by Eq. (21). The condition $P_3 \Gamma_1 = 0$ also leads to the $\Gamma_1$ of Eq. (21). For $\Delta_2$ we have:
\[
\frac{\sqrt{2}}{\sqrt{2}} e^{-i\alpha} \Delta_2 = W^\dagger_1 (12) P_3 W^\dagger_1 (12) M_u = P_3 M_u.
\]
In this case, the projector $P_3$ picks the diagonal (33) entry of $M_u$, which together with $P_3 \Delta_1 = 0$ leads to the matrix forms of Eq. (22). The two other models of the same class, with FCNC in the down sector are obtained by taking the two other projectors, $P_1$ and $P_2$ in each case. The three other cases with FCNC in the up sector only, correspond to:
\[
N^0_d = \frac{\sqrt{2}}{\sqrt{2}} M_d - \left( \frac{v_2}{v_1} + \frac{v_1}{v_2} \right) U_d P_1 U^\dagger_{dt} M_d.
\]
\[
N^0_u = \frac{\sqrt{2}}{\sqrt{2}} M_u - \left( \frac{v_2}{v_1} + \frac{v_1}{v_2} \right) U_d P_1 U^\dagger_{dt} M_u.
\]
with $i = 1, 2, 3$ respectively. The special feature of these six different models is the fact that there is no mixing in each other and, as BGL have shown, these models can be implemented by 5-type symmetries.

It is also possible to build simple models of MFV type with Higgs mediated FCNC in both sectors, like the one defined by the following equations:
\[
N^0_d = \frac{\sqrt{2}}{\sqrt{2}} M_d - \left( \frac{v_2}{v_1} + \frac{v_1}{v_2} \right) U_d P_1 U^\dagger_{dt} M_d.
\]
\[
N^0_u = \frac{\sqrt{2}}{\sqrt{2}} M_u - \left( \frac{v_2}{v_1} + \frac{v_1}{v_2} \right) U_d P_1 U^\dagger_{dt} M_u.
\]
It is also possible to have MFV models beyond standard NFC [3] but without FCNC, like
\[
N^0_d = \frac{\sqrt{2}}{\sqrt{2}} M_d - \left( \frac{v_2}{v_1} + \frac{v_1}{v_2} \right) U_d P_1 U^\dagger_{dt} M_d.
\]
\[
N^0_u = \frac{\sqrt{2}}{\sqrt{2}} M_u - \left( \frac{v_2}{v_1} + \frac{v_1}{v_2} \right) U_d P_1 U^\dagger_{dt} M_u.
\]
One can also construct more involved MFV models of the BGL type:
\[
N^0_d = \frac{\sqrt{2}}{\sqrt{2}} M_d - \left( \frac{v_2}{v_1} + \frac{v_1}{v_2} \right) U_d P_1 U^\dagger_{dt} M_d.
\]
\[
N^0_u = \frac{\sqrt{2}}{\sqrt{2}} M_u - \left( \frac{v_2}{v_1} + \frac{v_1}{v_2} \right) U_d P_1 U^\dagger_{dt} M_u.
\]
with $i \neq j$. In all cases the $\Gamma$ and $\Delta$ matrices obey relations of the same type as those written in Eqs. (47) and (48). However, the zero texture structure of these models is more involved than in the BGL case and the question of assuring its loop stability, through the introduction of symmetries, is not obvious [24].

5. Models with three Higgs doublets

Let us now consider the case of three Higgs doublets in the context of MFV, where the analogous to Eq. (6) includes the Yukawa terms of the third Higgs doublet.

After spontaneous symmetry breakdown the Higgs doublets can be decomposed as:
\[
\Phi_j = e^{i\alpha_j} \left( \frac{1}{\sqrt{2}} (\phi_j^+ + \phi_j^0 + i\eta_j) \right), \quad j = 1, 2, 3
\]
with real scalar fields $\rho_j$, $\eta_j$. We perform the following transformation:
\[
\begin{pmatrix} H_0^0 \\ R \end{pmatrix} = O \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \end{pmatrix}, \quad \begin{pmatrix} C_0 \\ I' \end{pmatrix} = O \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}
\]
with the matrix $O$ given by:
\[
O = \begin{bmatrix} \frac{v_1}{\sqrt{2}} & \frac{v_2}{\sqrt{2}} & \frac{v_3}{\sqrt{2}} \\ \frac{v_1}{\sqrt{2}} & -\frac{v_2}{\sqrt{2}} & 0 \\ \frac{v_3}{\sqrt{2}} & \frac{v_2}{\sqrt{2}} & -(v_1^2 + v_2^2)/v_3 \end{bmatrix}
\]
where $v = \sqrt{v_1^2 + v_2^2 + v_3^2}$, $v' = \sqrt{v_1^2 + v_2^2 + v_3^2}$ and $v'' = \sqrt{v_1^2 + v_2^2 + v_3^2}$. The orthogonal matrix $O$ singles out $H_0^0$ and the neutral pseudo-Goldstone boson $G$. $H_0^0$ has couplings to the quarks which are proportional to the mass matrices. In general, flavour changing neutral currents arise from the couplings to the remaining four neutral Higgs fields. The diagonalisation of the quark mass matrices gives rise to the following neutral Higgs interactions of the physical quarks:
\[
L_Y (\text{neutral}) = -\frac{v}{\sqrt{2}} H_0^0 \left( \bar{d}_l D_d d_R + \bar{u}_L U d_R \right)
\]
\[
- \bar{d}_l \frac{1}{\sqrt{2}} N_2 (R + iI) d_R - \bar{u}_L \frac{1}{\sqrt{2}} N_2 (R - iI) u_R
\]
\[
- \bar{d}_l \frac{v_3}{\sqrt{2}} N_2' (R' + iI') d_R
\]
\[
- \bar{u}_L \frac{1}{\sqrt{2}} N_2' (R' - iI') u_R + \text{h.c.}
\]
with
\[
N_2 = \frac{1}{\sqrt{2}} U^\dagger_{dl} (v_2 e^{i\alpha_1} \Gamma_1 - v_1 e^{i\alpha_2} \Gamma_2) U_{dl}.
\]
\[
N_2' = \frac{1}{\sqrt{2}} U^\dagger_{ul} (v_2 e^{-i\alpha_1} \Delta_1 - v_1 e^{-i\alpha_2} \Delta_2) U_{ul}.
\]
\[
N_2'' = \frac{1}{\sqrt{2}} U^\dagger_{ul} (v_1 e^{-i\alpha_2} \Delta_1 + v_2 e^{-i\alpha_2} \Delta_2 + v_3 e^{-i\alpha_2} \Delta_3) U_{ul}.
\]
where $x = -(v_1^2 + v_2^2)/v_3$.

For definiteness, let us consider $N^a_d$ and $N^a_u$, which can be written:
\[
N^a_d = \frac{v^2}{\sqrt{2}} D_d - \frac{v_2}{\sqrt{2}} \left( \frac{v_1}{v_2} + \frac{v_1}{v_2} \right) U^\dagger_{dt} e^{i\alpha_2} \Gamma_2 U_{dt}.
\]
\[
N^a_d = \frac{v^2}{\sqrt{2}} D_d - \frac{v_2}{\sqrt{2}} \left( \frac{v_1}{v_2} + \frac{v_1}{v_2} \right) U^\dagger_{dt} e^{i\alpha_2} \Gamma_2 U_{dt}.
\]

Imposing the following symmetry on the Lagrangian:
\[
Q_{11}^0 \rightarrow \omega Q_{11}^0, \quad Q_{12}^0 \rightarrow \omega^2 Q_{12}^0, \quad Q_{13}^0 \rightarrow \omega^4 Q_{13}^0,
\]
\[
\Phi_1 \rightarrow \omega \Phi_1, \quad \Phi_2 \rightarrow \omega^2 \Phi_2, \quad \Phi_3 \rightarrow \omega^4 \Phi_3,
\]
\[
u_{R1}^0 \rightarrow \omega^2 \nu_{R1}^0, \quad \nu_{R2}^0 \rightarrow \omega \nu_{R2}^0, \quad \nu_{R3}^0 \rightarrow \omega^4 \nu_{R3}^0,
\]
\[
\nu_{Rj}^0 \rightarrow \omega \nu_{Rj}^0.
\]
with \( \omega = \exp(i\pi/4) \), restricts the Yukawa coupling matrices to have the following structure:

\[
\begin{align*}
\Gamma_1 &= \begin{bmatrix} * & * & * \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, & \Gamma_2 &= \begin{bmatrix} 0 & 0 & 0 \\ * & * & * \\ 0 & 0 & 0 \end{bmatrix}, \\
\Gamma_3 &= \begin{bmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{bmatrix}, & \Delta_1 &= \begin{bmatrix} * & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, & \Delta_2 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{bmatrix}, \\
\Delta_3 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},
\end{align*}
\]

where \( * \) denotes an arbitrary entry while the zeros are imposed by the above symmetry.

It can be readily verified that in this case there are Higgs mediated FCNC only in the down sector, with \( N_d \) and \( N'_d \) given by:

\[
(N_d)_{ij} = \frac{v_2}{v_1}(D_{d})_{ij} - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right)(V_{CKM}^\dagger i_2)(V_{CKM} j_2)(D_{d})_{ij} \nonumber \]

\[
- \frac{v_3}{v_1}(V_{CKM}^\dagger i_3)(V_{CKM} j_3)(D_{d})_{ij}. 
\]

(71)

In this case the couplings \( N_d \) include terms that violate flavour proportional to \( (V_{CKM}^\dagger i_2)(V_{CKM} j_3)(D_{d})_{ij} \) together with terms proportional to \( (V_{CKM}^\dagger i_3)(V_{CKM} j_3)(D_{d})_{ij} \). The couplings \( N'_d \) only include terms that violate flavour proportional to \( (V_{CKM}^\dagger i_3)(V_{CKM} j_3)(D_{d})_{ij} \). It is clear that all Higgs mediated neutral couplings are only a function of \( V_{CKM} \) and therefore the symmetry of Eq. (69) leads to a MFV structure in the context of a three Higgs-doublet model. From a phenomenological point of view, there is an important difference between the scalar FCNC in this MFV three Higgs doublet model and those encountered in the MFV two Higgs doublet model considered in the previous sections. In the case of two Higgs doublet models, there is one variant of the BGL models where the tree level Higgs mediated \( \Delta S = 2 \) amplitude is naturally suppressed by terms proportional to \( (V_{CKM}^\dagger i_2)(V_{CKM} j_3)^2 \). This very strong suppression opens the possibility of having neutral Higgs relatively light of order \( 10^2 \) GeV, without entering in conflict with the size of the \( K_L - K_S \) mass difference or the strength of CP violation in the kaon sector. In the case of the MFV three Higgs doublet model \( N_d \) includes FCNC terms where the suppression factor in \( \Delta S = 2 \) transitions is only \( (V_{CKM}^\dagger i_2)(V_{CKM} j_3)^2 \), which then requires quite heavy neutral Higgs, with mass of order TeV.

### 6. Conclusions

We have analysed how to extend the MFV concept to general multi-Higgs models without NFC in the Higgs sector. We have studied in special detail the case of two Higgs doublet models, analysing the requirements which have to be satisfied in order that the neutral Higgs couplings to quarks be only functions of \( V_{CKM} \), with no other flavour dependent parameters. The Branco–Grimus–Lavoura (BGL) models proposed some time ago are an example where the MFV constraints are satisfied as the result of a symmetry of the Lagrangian. We have proposed a general MFV expansion of the neutral Higgs couplings to quarks and have shown that the BGL models correspond to specific values of the coefficients of the proposed MFV expansion and, in addition, we have shown that the values of these coefficients are fixed by the symmetry.

Multi-Higgs models with Higgs mediated FCNC have a rich phenomenology and some of its aspects have been recently analysed in the literature [25]. A detailed phenomenological analysis of multi-Higgs MFV models without NFC, is beyond the scope of this Letter and will be left to a separate work [26].

### Acknowledgements

This work was partially supported by Fundação para a Ciência e a Tecnologia (FCT, Portugal) through the projects CERN/FP/83503/2008 and CFTP-FCT Unit 777 which are partially funded through POCTI (FEDER), by Marie Curie RTN MRTN-CT-2006-035505, by Accion Complementaria Luso-Espanhola PORT2008-03 and EPA-2008-04002-PORTU, by European FEDER, Spanish MICINN under grant FPA2008–02878. G.C.B. and M.N.R. are very grateful for the hospitality of Universitat de València during their visits. F.J.B. is very grateful for the hospitality of CFTP/IST Lisbon during his visit.

### References


