1. Introduction

Recently image based systems have gained popularity in the real-world. Now pictographic representations are very common as one can find and use images readily available everywhere. A high dimensional space is used to represent image data, but for computational efficiency it is mapped to a lower dimensional space. This dimensionality reduction actually helps in identifying the key parameters that are sufficient to characterize the data. Also, representing data in low-dimensional space enables the usage of effective data structures for indexing and searching. Principal Component Analysis (PCA) and Linear Discriminant Analysis (LDA) are two commonly used classical techniques to detect important dimensions. These techniques have already proved their applicability in diverse research areas of computer vision, pattern recognition, machine learning, and many more. PCA is unsupervised and works on maximizing the variance in a dataset so as to get good class representations. LDA, on the other hand, is supervised and focuses on maximizing the existing class separation in a dataset.

Both PCA and LDA use Eigen value decomposition and then arrange the eigenvectors by decreasing Eigen values. This process gives higher priority to dimensions with large energy which is good, but not always enough. In this work, another measure of variability ‘range’ is explored and combined with other criteria to acquire a better organization.
of the eigenvectors obtained either with PCA or LDA. Dimensions are prioritized using the proposed method which spreads over two levels. The first level is based on the number of distinct values and the second level uses a combination of correlation and range values to further rank the dimensions which are having same priority at the first level. The final ranking thus achieved is used to arrange the eigenvectors obtained using any of the algorithms. To understand the effect of modified ranking of eigenvectors, a Content Based Image Retrieval (CBIR) system is developed and its performance is examined with the proposed dimension ranking method applied on PCA and LDA.

2. Related Work

Efficiency of image based systems can be improved by using any suitable dimension reduction algorithm. Systems dealing with face images commonly use these algorithms. Luo et al.\textsuperscript{4} used standard deviation of feature vectors to normalize the vectors before applying traditional PCA. The basic idea was to reduce the influence of eigenvectors associated with changes in illumination. An improved performance is observed with the developed system on Yale face database. Fan et al.\textsuperscript{5} modified PCA and developed subspace learning framework using eigenvectors of three similarity matrices (mutual information; angle information; Gaussian kernel). The resulting three sub-spaces were integrated by employing the concept of feature selection approach which was followed by an idea of graph embedding learning. The obtained new similarity subspace contains the most discriminative information of the database. The framework was found to be very suitable for the applications where training samples were less than dimensionality of data. Results presented with various databases for feature selection, face image reconstruction, clustering, and classifications were better than PCA based methods. Ghassabeh et al.\textsuperscript{6} developed a PCA based adaptive face recognition system using a parallel combination of Sangers adaptive and QR decomposition algorithm. Extracted feature vectors were normalized to get a better feature subspace. The effectiveness of the system for real-time face recognition was demonstrated with Yale face database. Also, adaptive nature of the system made it suitable for different real-time face and gesture recognition applications. Chen et al.\textsuperscript{7} developed a LDA-based face recognition system and dealt with major drawback of LDA, i.e., small sample size problem. They also proved that the most expressive within-class scatter matrix vectors in the null space using PCA are equal to the optimal discriminant vectors in original space using LDA. The proposed system showed a significant performance improvement in terms of accuracy, training efficiency and stability. A number of dimension reduction techniques exist and are applicable to a large number of domains. The work in\textsuperscript{3} presented an extensive review of such techniques along with the comparative evaluations.

3. The Developed CBIR System

In this work a simple CBIR system as shown in Fig. 1 is developed. It consists of four modules: feature extraction, dimension reduction, similarity matching, and image retrieval. At first, image features are extracted for all the images in database, then the features are reduced to a lower dimensional space and stored in the image database. At the retrieval time, a query image is submitted to the system; its features are extracted, reduced, and then matched with the stored image features in the database. For the purpose of matching a similarity measure depending upon the used feature vector is employed. Finally, images corresponding to the least distance values are read from the database and returned to the user.

3.1 Feature extraction and image retrieval

A good combination of similarity measures and feature vectors is required for a CBIR system to perform well\textsuperscript{8,9}. Six popular features in literature are used to examine the performance of dimension reduction algorithms and their modified versions. These features are color histogram, color moments, Gabor texture, Gray-Level Co-Occurrence Matrix (GLCM), Edge Histogram, and pseudo Zernike moments. A similarity measure is used to prepare a list of distances between a query image and each of the images in database. This work employs suitable similarity measures for each of the six features used for examination and are given in Table 1. At the end, image retrieval module sorts the similarity values in their increasing order and retrieves similar images.
Table 1. Description of features and similarity measures used.

<table>
<thead>
<tr>
<th>Feature Description</th>
<th>Dimension</th>
<th>Similarity Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Color (HSV Space)</td>
<td>68</td>
<td>Vector cosine</td>
</tr>
<tr>
<td>Histogram</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Moments</td>
<td>9</td>
<td>Weighted city-block</td>
</tr>
<tr>
<td>Texture</td>
<td>48</td>
<td>Euclidean</td>
</tr>
<tr>
<td>Gabor</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GLCM</td>
<td>16</td>
<td>Euclidean</td>
</tr>
<tr>
<td>Edge Histogram</td>
<td>80</td>
<td>Euclidean</td>
</tr>
<tr>
<td>Shape</td>
<td>21</td>
<td>Euclidean</td>
</tr>
<tr>
<td>Pseudo Zernike Moments</td>
<td>n = 5 and m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>m ≤ n</td>
<td></td>
</tr>
</tbody>
</table>

3.2 Dimension reduction

Traditionally PCA and LDA are used for dimension reduction. In this work PCA and LDA are modified to prioritize the dimensions according to the proposed method. The steps of these algorithms are given in the following sub-sections.

Let, X is an m × n feature matrix; m is the number of images and each image is represented by an n-dimensional vector. The projection matrix, W, is required to map the n-dimensional vector to a reduced k-dimensional space (k < n). An n × k matrix, W, is computed by choosing top k eigenvectors obtained by a dimension reduction algorithm. Mathematically,

\[ Y = W^T X \]  

Before applying any of the dimension reduction algorithms, we first arrange the dimensions in the decreasing order of their priorities. The steps of the proposed dimension ranking method are as given in Section 3.2.1.

3.2.1 Dimension ranking method

1. Calculate \( distVal_i, i = 1, 2, 3, \ldots, n \). \( distVal_i \) is the number of distinct values for \( i^{th} \) dimension. Also, compute \( \text{dimRange}_i, i = 1, 2, 3, \ldots, n \). \( \text{dimRange}_i \) is the range of \( i^{th} \) dimension and is calculated as \( \text{dimRange}_i = \max(X(img, i)) - \min(X(img, i)) \). \( img = 1, 2, 3, \ldots, m \).
2. Sort dimensions in decreasing order of \( distVal \) values. If \( distVal_i = distVal_j \), then dimensions \( i \) and \( j \) have same priority at this moment.
3. For each priority identified at Step 2
   (a) Let \( \{i, j, k, \ldots, l\} \) be a set of dimensions at the same priority. \( D = X(\text{img}, d) \). \( \text{img} = 1, 2, 3, \ldots, m \); and \( d = i, j, k, \ldots, l \).
   (b) Calculate correlation \( R \) among all the dimensions in \( D \). \( R = \text{corr}(D) \).
   (c) Update correlation matrix \( R \) as \( R(i, j) = \frac{R(i, j)}{\text{dimRangi} \times \text{dimRangj}} \).
   (d) Rearrange the dimensions \( \{i, j, k, \ldots, l\} \) in the increasing order of \( R \) values.

4. Concatenate the ordered list of dimensions obtained after each execution of Step 3 to get the final ranking of \( n \) dimensions.

3.2.2 PCA with the proposed dimension ranking method

Apply the traditional PCA algorithm on \( m \times n \) feature matrix, \( X \), and get the eigenvectors corresponding to each of the \( n \) dimensions. Arrange these vectors according to the order obtained by the proposed dimension ranking method given in Section 3.2.1.

3.2.3 LDA with the proposed dimension ranking method

LDA maximizes the inter-class scatter matrix and minimizes the intra-class scatter matrix. In this case the proposed method is first used to rank the dimensions according to inter-class scatter matrix. In the next step, the proposed method is again executed to rank the dimensions for intra-class scatter matrix. The temporary rank lists thus obtained are combined using CombSUM fusion\(^{15}\) to get the final ordered list of dimensions. Similar to PCA, LDA is applied to get the eigenvectors corresponding to each of the \( n \) dimensions. These eigenvectors are arranged as per the final order list obtained earlier after applying the CombSUM fusion.

4. Experimental Results

A system with Core 2 Quad processor, 8GB RAM and 500GB hard disk is used to evaluate the effect of the modified ranking of eigenvectors on PCA and LDA. All experiments are performed on WANG and ZuBuD databases. WANG consists of 1000 images from 10 categories: Africa, Beach, Bus, Dinosaur, Elephant, Flower, Food, Horse, Monument,
and Natural Scene. On the other hand ZuBuD contains 1005 images for 201 different Zurich city buildings only, i.e., 5 images for each building. The interface of the developed CBIR system is shown in Fig. 2.

In the experiments, six features are explored and reduced by PCA, LDA, and their proposed modified versions (referred as PCA Modified, LDA Modified hereafter). For each $n$-dimensional feature, number of dimensions in reduced space varies from 2, 3, 4, ..., $n$. The performance of the system is analyzed in terms of precision values. For WANG, precision is calculated for top 100 retrieved images, but in case of ZuBuD precision is computed for top 5 retrieved images. These values are chosen according to the total number of relevant images in the respective databases, and therefore the recall values will remain same as the precision values. Precision and Recall are defined as in Eq. 2 and Eq. 3 respectively.

\[
\text{Precision} = \frac{\text{Number of relevant images retrieved}}{\text{Total number of images retrieved}} \tag{2}
\]

\[
\text{Recall} = \frac{\text{Number of relevant images retrieved}}{\text{Total number of relevant images in the database}} \tag{3}
\]

The graphs shown in Fig. 3 and Fig. 4 are plotted for the precision values obtained with increasing number of dimensions for the four forms of dimension reduction algorithms: PCA, LDA, PCA Modified, and LDA Modified on
WANG and ZuBuD databases. It is clear from the graphs that the modified versions are performing well for all six features considered in this work.

In case of WANG database as shown in Fig. 3, for color moments and GLCM, the modified LDA seems to perform a little better than LDA, but the modified PCA outperforms traditional PCA. The precision values obtained for color histogram feature are better for both the modified versions. In case of Gabor and edge histogram features, modified LDA performs better than LDA. For the same features traditional PCA has achieved better precision values than the modified ones. Similar scenario is observed with the sixth feature, i.e., pseudo Zernike moments. Overall it can be concluded that modified LDA performs well for all six features. In case of PCA, the modified version shows better performance for two color features, i.e., color moments and color histogram, and one texture feature, i.e., GLCM. But for Gabor texture and shape features the traditional PCA shows good results with reduced dimensions.

Similarly, observing the performance of the system using ZuBuD database (Fig. 4), the Modified LDA seems to perform better than LDA for all the features except color moments. For color moments, the results obtained with both the versions of LDA are more or less same. Differing from these observations, the performances of PCA and Modified PCA are almost similar to each other in all the cases. But for pseudo Zernike moments, the proposed modification performs a little better than the traditional algorithm. Here also, modified LDA performs well for all the features.

Fig. 4. The graphs for six features on ZuBuD obtained with various dimension reduction methods.
5. Conclusion

In this paper, a new way to arrange the eigenvectors obtained through PCA and LDA is presented and termed as Dimension Ranking Method. A CBIR system is developed to evaluate the performance of the proposed method. Precision values obtained with WANG and ZuBuD databases show the effectiveness of the proposed modification. Although for lesser number of dimensions the method does not perform that well but with increasing values of dimensions the improvement shown in precision values is acceptable. Also, it is observed that the proposed algorithm outperforms in some cases. This further proves that the algorithm can be used as a substitute to traditional methods. Although its acceptability depends upon the domain being dealt by the implemented CBIR system. The inclusion of the proposed method with other dimension reduction methods may be explored in future.

References