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Damage tolerance and reliability assessment under random Markovian loads

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Abstract

Accounting for load uncertainties plays an important role in the design of safe structural components of aircrafts under damage tolerance requirements. The purpose of this paper is to develop a reliability assessment technique for cracked structures submitted to non-stationary random fatigue loads modeled by first-order Markov chains with discrete state space and identified from in-flight measurements. The strategy based on a multi-level version of the cross-entropy method consists in progressively updating the transition probability matrix in order to generate load sequences of increasing severity which are likely to cause failure. The proposed method is applied to a cracked M(T) specimen under the defined random fatigue loads. Load cycle interactions and retardation effects are accounted for by means of the PREFFAS crack closure model. The efficiency of the proposed approach in terms of computational cost is clearly observed for rare failure events in comparison with direct Monte Carlo simulations. In addition to the failure probability estimate, the multi-level cross-entropy method provides the analyst with information on the most probable load sequences at failure.

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Keywords: damage tolerance ; reliability assessment ; importance sampling ; cross-entropy method ; first-order Markov chains

1. Introduction

The damage tolerance design of aircraft structural components is mainly based on the assumption of a deterministic approach of the crack growth process. Real environmental conditions however exhibit various and significant sources of uncertainty which need to be accounted for. The main sources are known to come from material properties, length/location/orientation of initial cracks and applied loads. The work presented here focuses on the latter source of uncertainty of real importance for structures subjected to significantly scattered loads and when interaction effects between cycles along the random loading sequence need to be assessed in order to avoid overconservative designs. Non-stationary variable amplitude fatigue load sequences are here modeled by means of first-order Markov chains.
with discrete state space. Their parameters are identified from in-flight measurements recorded on a fleet of fighter aircrafts. The constructed random fatigue load model is then used for the reliability assessment of a simplified damage tolerance problem. For this purpose, an importance sampling strategy based on the Cross-Entropy (CE) method [1] is developed for an efficient and accurate estimation of low failure probabilities. The algorithm consists in updating the transition matrix of the Markov chain in a multi-level approach so that the fatigue load sequences generated according to the new transition matrix progressively lead to an increasing number of failures. The proposed method is applied to the reliability assessment of a cracked M(T) specimen under random fatigue loads. Results obtained by the proposed approach are compared both in terms of accuracy and efficiency with those obtained with a crude Monte Carlo simulation (MCS) considered as reference results.

The paper is organized as follows. Section 2 briefly recalls the probabilistic model identified from the recorded data, based on previous works of the authors [2, 3]. The reliability problem is formulated in Section 3 and the solving strategy based on the CE method is detailed in Section 4. The application example is presented in Section 5, where the strengths of the CE method are highlighted. A conclusion is drawn in Section 6.

2. Probabilistic model of the fatigue loading

A recourse to continuous time processes for modeling random load sequences has been a common practice in probabilistic fracture mechanics. Several works were carried out in the fields of fatigue initiation [4, 5, 6] and fatigue crack propagation [7, 8, 9, 10, 11, 12, 13, 14, 15]. A stationary Gaussian random process is most often conveniently assumed and its parameters (e.g. the shape of the power spectral density function) are selected or varied regardless of any real load data. Conversely, when load data are available, Markov processes have been considered as interesting alternatives [16, 17, 6]. This type of models has been investigated by the present authors in previous works [2, 3] for the following reasons:

- the identification of a suitable continuous time process was not straightforward due to the form of the recorded data (variable-length time steps selected according to the aircraft activity, variable flight durations),
- the load time series recorded on a fleet of fighter aircraft were obviously non-stationary,
- Markov processes are convenient for a direct modeling of sequences of max-min pairs expected for the purpose of crack propagation (with continuous-time processes, an additional filtering stage is required for obtaining such sequence from time trajectories),
- the recorded data were provided in a sufficient amount for the identification of a Markov process.

It is worth mentioning that the main objective here is the identification of a suitable process or a few processes from the data, without any intention to model the physical and complex loading phenomena involved during the flights. In this paper and for the sake of simplicity, we will focus on the randomness of fatigue loads observed in a single flight for a selected type of mission of the aircraft.

In the sequel we consider a \( N \)-cycle load sequence \( (X_1, \ldots, X_N, \ldots, X_N) \) where \( X_n \) is taken here as a cycle \( X_n = (M_n, m_n) \) and \( M_n \) (resp. \( m_n \)) stands for the peak (resp. trough) stress value of the \( n \)th cycle. We assume a first-order dependence in terms of load cycles, which implies a greater dependence in terms of stress levels. The First-order Markov Chain (FMC) with discrete state space \( E \) therefore satisfies the following property, for all \( n \in \mathbb{N}^* \):

\[
P(X_{n+1} = x_{n+1} | X_n = x_n, \ldots, X_1 = x_1) = P(X_{n+1} = x_{n+1} | X_n = x_n)
\]

The state space \( E \) is composed of a finite number \( K \) of load cycle states:

\[
E = \{ e_k = (s_i, s_j), s_i > s_j \text{ and } i, j \in \{1, \ldots, K_c\} \}
\]

where \( s_i, s_j \) represent two given stress levels taken from a set of \( K_c \) levels. These \( K_c \) levels are selected according to the shape of the empirical distribution of the measured stress levels, see Fig. 1 (a). The state space \( E \) encompasses \( K = K_c (K_c - 1)/2 \) cycle states such that a valley \( m_n \) is systematically followed by a peak \( M_n \). The present work is based on group “B” data of reference [3]: \( s_i, s_j \in \{0.039, 0.113, 0.248, 0.507, 0.840\} \) and we therefore have \( K_c = 5 \) and \( K = 10 \).
The probabilities of moving from a given load cycle to the next one are gathered in a $K \times K$ transition probability matrix $P$ supposed constant over time $n$ (time-homogeneous Markov chain), represented in Fig. 1 and which satisfies the following conditions:

$$\begin{cases}
0 \leq p_{i,j} \leq 1 & \text{for } i, j \in \{1, \ldots, K\} \\
\sum_{j=1}^{K} p_{i,j} = 1 & \text{for } i \in \{1, \ldots, K\}
\end{cases}$$

(3)

where $p_{i,j} = P(X_{n+1} = e_j | X_n = e_i) = P(X_2 = e_j | X_1 = e_i)$, for all $n \in \mathbb{N}^*$ and any $(e_i, e_j) \in E \times E$.

The FMC is therefore completely defined by its transition matrix $P$, its initial distribution $X_1$ and its length $N$ (see Fig. 1).

3. Formulation of the reliability problem

We assume that failure is caused by a crack growth instability, when the stress intensity factor $K$ in opening mode (mode I) exceeds the fracture toughness $K_c$ at a reference stress level $\sigma_l$ as defined in Eq. 4. This stress level $\sigma_l$ could be seen here as representative of the limit load of the aircraft as defined by aviation authorities.

$$K(a, \sigma_l) \geq K_c$$

(4)

The crack size $a$ in Eq. 4 represents the damage accumulation from an initial crack size $a_0$ under a given trajectory of the previously defined FMC, which consists of a variable amplitude load sequence. The crack extension is evaluated by means of a crack growth prediction model. Two models are used in the present work: the Paris-Erdogan law and the PREFFAS crack closure model [18]. This latter model presents the major advantage to account for potential retardation or acceleration effects on the crack growth process due to overloads/underloads in the variable amplitude load sequences. Such effects are important in the damage tolerance design of structural aircraft components and they need to be assessed in order to avoid overconservative designs.
For a reliability problem of a reduced complexity, we will assume that all load sequences (or flights) are composed of a given deterministic number of cycles \( N \) and that all FMC start from the same initial cycle \( x_1 = e_2 = (s_3, s_1) = (0.248, 0.039) \). The limit-state function \( g \) is expressed as follows, for \( x \in E^N \):

\[
g(x) = g(x_{1 \leq n \leq N}) = a_c - a(x_{1 \leq n \leq N})
\]

where \( a_c \) is the critical crack length solution of \( K(a_c, \sigma_l) = K_c \) and \( a(x_{1 \leq n \leq N}) \) is the crack length resulting from the propagation under the \( N \)-cycle load sequence \( x_{1 \leq n \leq N} \).

The failure probability is given by the following expression:

\[
P_f = \int_{D_{fN}} p_X(dx) = \int_{D_{fN}} p_X(x) \, dx
\]

where \( p_X \) denotes the probability measure over the observable space \( E^N \), \( p_X \) refers to the probability density function (pdf) of the Markov chain \( X \) and \( D_{fN} = \{ x \in E^N : g(x) \leq 0 \} \) represents the failure domain.

According to the first-order Markov property, the probability of the homogeneous FMC \( X \) reads, by successive conditioning:

\[
P(X) = P(X_1 = x_1) \, \prod_{n=1}^{N-1} P(X_{n+1} = x_{n+1} | X_n = x_n)
\]

The pdf of the FMC \( p_X \) can therefore be written as:

\[
p_X(x) = p_X(x, \mathbf{P}) = \prod_{i,j=1}^{K} p_{i,j}^{n_{i,j}(x)}
\]

where \( n_{i,j}(X) \) denotes the number of transitions of the Markov chain \( X = (X_1, \ldots, X_N) \) from cycle \( e_i \) to cycle \( e_j \). It is assumed that \( P(X_1 = x_1) = 1 \), i.e. all trajectories of the first-order Markov chain \( X \) start from a unique cycle state \( x_1 \).

4. Solution based on the CE method

The probability of failure \( P_f \) solution of Eq. 6 is now expressed as the following expectation:

\[
P_f = \int_{D_{fN}} p_X(x) \, dx = \int_{E^N} \mathbb{1}_{D_{fN}}(x) \, p_X(x) \, dx = \mathbb{E}_{p_X} \left[ \mathbb{1}_{D_{fN}}(X) \right]
\]

where \( \mathbb{1}_{D_{fN}} \) is the indicator function of the failure domain \( D_{fN} \) such that \( \mathbb{1}_{D_{fN}}(x) = 1 \) if \( x \in D_{fN} \) and \( \mathbb{1}_{D_{fN}}(x) = 0 \) otherwise.

A direct estimation of the failure probability by a crude Monte Carlo Simulation (MCS) from Eq. 9 is known to be too costly in the case of rare failure events. Importance Sampling (IS) constitutes an efficient alternative method for such a purpose. With IS, the reliability problem is rewritten in the following form:

\[
P_f = \int_{E^N} \mathbb{1}_{D_{fN}}(x) \, \frac{p_X(x)}{q_X(x)} \, q_X(x) \, dx = \mathbb{E}_{q_X} \left[ \mathbb{1}_{D_{fN}}(X) \, W(X) \right]
\]

where \( q_X \) is an instrumental pdf which must dominate \( \mathbb{1}_{D_{fN}} p_X \) and \( W(x) = p_X(x)/q_X(x) \) is called the likelihood ratio.

The corresponding IS statistical estimator of \( P_f \) from a set of \( N_s \) samples is given by:

\[
\hat{P}_f^{IS} = \frac{1}{N_s} \sum_{k=1}^{N_s} \mathbb{1}_{D_{fN}}(X^{(k)}) \, W(X^{(k)})
\]

where \( X^{(1)}, \ldots, X^{(N_s)} \) are \( N_s \) i.i.d. copies of \( X \) with pdf \( q_X \).

The pdf which leads to a zero-variance of \( \hat{P}_f^{IS} \) is denoted by \( q_X^* \). This optimal solution has the following expression:

\[
q_X^*(x) = \frac{\mathbb{1}_{D_{fN}}(x) \, p_X(x)}{P_f}
\]
The CE method introduced by Rubinstein [1] and used in the present work consists in selecting the importance sampling density \( q_X \) which is the closest to the optimal density \( q^*_X \) within the family of the nominal density \( \{ p_X (\cdot, Q) \} \) indexed by the parameter transition matrix \( Q \). The CE problem therefore consists in choosing the optimal parameter matrix \( Q^* \) such that the Kullback-Leibler distance between the densities \( q_X^* \) and \( p_X (\cdot, Q) \) is minimal:

\[
Q^* = \arg\min_Q \mathcal{D} ( q_X^*, p_X (\cdot, Q) ) = \arg\min_Q \mathbb{E}_{q_X^*} \left[ \log \left( \frac{q_X^*(X)}{p_X(X, Q)} \right) \right]
\]

(13)

After some straightforward calculations which make use of the expression of \( q_X^* \) defined at Eq. (12), the optimal solution \( Q^* \) takes the following form:

\[
Q^* = \arg\max_Q \mathbb{E}_{p_X (\cdot, P)} \left[ \mathbb{I}_{D_{fN}} (X) \log p_X (X, Q) \right]
\]

\[
\text{s.t. } \begin{cases}
0 \leq q_{i,j} \leq 1 & \text{for } i, j \in \{1, \ldots, K\} \\
\sum_{j=1}^K q_{i,j} = 1 & \text{for } i \in \{1, \ldots, K\}
\end{cases}
\]

(14)

where the constraints are added so that \( Q \) is a transition matrix.

In the case of rare failure events, it is expected that most of the realizations of the indicator function \( \mathbb{I}_{D_{fN}} \) are zeros when \( X \) is sampled from the original probability distribution \( p_X (\cdot, P) \), which leads to poor estimations of \( Q^* \). A multi-level CE procedure has been developed to circumvent this issue [19, 20]. The key idea consists in building a sequence of reference parameters \( \{ \hat{Q}_t, t \geq 0 \} \) and a sequence of thresholds \( \{ \hat{\gamma}_t, t \geq 1 \} \) which are adaptively updated. At each level \( t \), a CE solution is derived by IS. Assuming some arbitrary proposal density \( p_X (\cdot, R) \) for IS, Eq. (14) rewrites as follows:

\[
Q^* = \arg\max_Q \mathbb{E}_{p_X (\cdot, R)} \left[ \mathbb{I}_{D_{fN}} (X) W (X, P, R) \log p_X (X, Q) \right]
\]

\[
\text{s.t. } \begin{cases}
0 \leq q_{i,j} \leq 1 & \text{for } i, j \in \{1, 2, \ldots, K\} \\
\sum_{j=1}^K q_{i,j} = 1 & \text{for } i \in \{1, 2, \ldots, K\}
\end{cases}
\]

(15)

where the likelihood ratio takes the form \( W (x, P, R) = p_X (x, P) / p_X (x, R) = \prod_{j=1}^K \left( \frac{p_{i,j}}{r_{i,j}} \right)^{n_{i,j}(x)} \).

The statistical estimator \( \hat{Q}^* \) is easily derived from the solution of the maximization problem given in Eq. 15:

\[
\hat{q}_{i,j}^t = \frac{1}{N_s} \sum_{k=1}^{N_s} \mathbb{I}_{D_{fN}} (X^{(k)}) \left( \prod_{j=1}^K \left( \frac{p_{i,j}}{r_{i,j}} \right)^{n_{i,j}(X^{(k)})} \right) n_{i,j}(X^{(k)})
\]

\[
\frac{1}{N_s} \sum_{k=1}^{N_s} \mathbb{I}_{D_{fN}} (X^{(k)}) \left( \prod_{j=1}^K \left( \frac{p_{i,j}}{r_{i,j}} \right)^{n_{i,j}(X^{(k)})} \right) n_i(X^{(k)})
\]

(16)

where \( X^{(1)}, \ldots, X^{(N_s)} \) are sampled from \( p_X (\cdot, R) \) distribution and \( n_i(X) = \sum_{j=1}^K n_{i,j}(X) \) represents the number of transitions of the Markov chain \( X \) starting from state \( e_i \).

The multi-level CE algorithm is described below:

1. Set \( \hat{Q}_0 = P \) and \( t = 1 \).
2. Generate \( N_s \) samples \( x^{(1)}, \ldots, x^{(k)}, \ldots, x^{(N_s)} \) from \( p_X (\cdot, \hat{Q}_{t-1}) \) and evaluate the corresponding limit-state values \( g(x^{(k)}) \). Define \( \hat{\gamma}_t \) as the sample \( \rho \)-quantile of \( g(x^{(k)}) \) where \( \rho \) is a not very small chosen parameter (\( \rho = 0.1 \) in the present work). 0
3. Use the same samples $x^{(1)}, \ldots, x^{(N_s)}$ to estimate $\hat{q}_{i,j}$ from Eq. (16) with $r_{i,j} = \hat{q}_{i-1,j}$ for $i, j \in \{1, \ldots, K\}$. The indicator function $\mathbb{I}_{D_{j,N}}(x^{(k)})$ is replaced by $\mathbb{I}_{D_{j,N}(t)}(x^{(k)})$ in Eq. (16), where $D_{f,N}(t) = \{ x \in E^N : g(x) \leq \hat{y} \}$. Denote $\hat{q}_{i,j}$ the solution of Eq. (16).

4. If $\hat{y} \leq 0$, stop the algorithm and proceed with step 5, set $t = t + 1$ otherwise and reiterate from step 2.

5. Estimate the failure probability by IS:

$$\hat{P}_f = \frac{1}{N_s} \sum_{k=1}^{N_s} q_{i,j} W(x^{(k)}, P, \hat{Q}_t)$$

where $t$ denotes here the final number of iterations (i.e. number of levels used).

For an insufficient number of samples $N_s$, some transitions $e_i \to e_j$ may appear unobserved at some iteration of the algorithm despite non-zero (but small) transition probabilities. The corresponding estimated transition probabilities $\hat{q}_{i,j}$ then remain equal to zero until the end of the multi-level CE algorithm, which results in a transition matrix $\hat{Q}_t$ obtained at the last level potentially far from the true optimal solution. Weighting the solution obtained at each level with either the nominal probabilities $P$ or the solution of the previous level $\hat{Q}_{t-1}$ has been proposed to address such an issue [20]. This latter solution is used in the subsequent application treated in Section 5, with a smoothing parameter $\alpha$ set to 0.6:

$$\hat{q}_{i,j} = (1 - \alpha)\hat{q}_{i-1,j} + \alpha \frac{\sum_{k=1}^{N_s} \mathbb{I}_{D_{j,N}(t)}(X^{(k)}) W(X^{(k)}, P, \hat{Q}_t) n_{i,j}(X^{(k)})}{\sum_{k=1}^{N_s} \mathbb{I}_{D_{j,N}(t)}(X^{(k)}) W(X^{(k)}, P, \hat{Q}_t) n_j(X^{(k)})}$$

5. Application example

The reliability analysis is applied to the crack propagation of a M(T) specimen 150 mm width, 2 mm thickness, made of a 2024-T351 aluminum alloy and submitted to the random fatigue loading modeled by FMC as described in Section 2. Two crack growth models are investigated: the Paris-Erdogan law (case 1 and 2) and the PREFFAS crack closure model (case 3), see Table 1. The following parameters are used: $C = 2.417 \cdot 10^{-13}$ and $m = 3.42$ in the Paris law (SIF range $\Delta K$ consistent with MPa$\sqrt{\text{mm}}$ and crack growth rate $da/dN$ with mm/cycle), $A = 0.45$ and $B = 0.55$ for Elber constants in the PREFFAS model. An initial crack size $a_0 = 5$ mm is assumed for all three cases. The critical crack length $a_c$ is arbitrary fixed for sufficiently low failure probabilities. For case 3, the whole load sequence is composed of a randomly generated 525-cycle load subsequence repeated 100 times in order to meet the stationary spectra assumption of the PREFFAS model [18]. Works are underway to improve the PREFFAS model and relax such an assumption in order to assess the propagation of crack under fully random load sequences.

<table>
<thead>
<tr>
<th>Crack growth model</th>
<th>Number of applied cycles $N$</th>
<th>Initial crack length $a_c$ (mm)</th>
<th>Critical crack length $a_c$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1 Paris law</td>
<td>500</td>
<td>5</td>
<td>5.091</td>
</tr>
<tr>
<td>Case 2 Paris law</td>
<td>102000</td>
<td>5</td>
<td>25.2</td>
</tr>
<tr>
<td>Case 3 PREFFAS</td>
<td>52500</td>
<td>5</td>
<td>9.34</td>
</tr>
</tbody>
</table>

Results obtained with the multi-level CE algorithm and crude MCS are listed in Table 2. For the CE method, the coefficient of variation (c.o.v.) is obtained empirically from 30 independent runs of the algorithm. The CE method clearly outperforms crude MCS. For an equivalent accuracy given in terms of c.o.v., the CE method requires a number of calls to the limit-state function which is several orders or magnitude less than the one of a crude MCS. The gain increases with low failure probabilities. The averaged value of the 30 estimates of the failure probability obtained with the CE method is close to the single MCS estimate, which allows us to conclude that the CE method has no significant bias.
Table 2. Reliability results.

<table>
<thead>
<tr>
<th>Case</th>
<th>Method</th>
<th>Total number of samples</th>
<th>Failure probability $P_f$</th>
<th>c.o.v. of $P_f$ (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>MCS</td>
<td>$10^7$</td>
<td>$8.39 \times 10^{-6}$</td>
<td>10.9</td>
</tr>
<tr>
<td></td>
<td>CE($N_i = 1000$; 4 levels)</td>
<td>$4 \times 10^3$</td>
<td>$7.69 \times 10^{-6}$ (*)</td>
<td>26.2</td>
</tr>
<tr>
<td></td>
<td>CE($N_i = 5000$; 4 levels)</td>
<td>$2 \times 10^4$</td>
<td>$7.71 \times 10^{-6}$ (*)</td>
<td>5.82</td>
</tr>
<tr>
<td>2</td>
<td>MCS</td>
<td>$10^5$</td>
<td>$1.10 \times 10^{-3}$</td>
<td>9.53</td>
</tr>
<tr>
<td></td>
<td>CE($N_i = 1000$; 3 levels)</td>
<td>$3 \times 10^3$</td>
<td>$1.11 \times 10^{-3}$ (*)</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>CE($N_i = 5000$; 3 levels)</td>
<td>$1.5 \times 10^4$</td>
<td>$1.11 \times 10^{-3}$ (*)</td>
<td>4.77</td>
</tr>
<tr>
<td>3</td>
<td>MCS</td>
<td>$10^5$</td>
<td>$5.31 \times 10^{-4}$</td>
<td>13.7</td>
</tr>
<tr>
<td></td>
<td>CE($N_i = 1000$; 3 levels)</td>
<td>$3 \times 10^3$</td>
<td>$5.06 \times 10^{-4}$ (*)</td>
<td>17.1</td>
</tr>
<tr>
<td></td>
<td>CE($N_i = 5000$; 3 levels)</td>
<td>$1.5 \times 10^4$</td>
<td>$4.84 \times 10^{-4}$ (*)</td>
<td>4.31</td>
</tr>
</tbody>
</table>

(*): averaged value over 30 independent runs of the multi-level CE algorithm

The biased transition matrix $\hat{Q}_t$ at the last iteration of the multi-level CE algorithm brings also some additional information. The load sequences simulated with this matrix induce severe crack growths which are likely to cause failure. The evolution of $\hat{Q}_t$ with iterations is represented in Fig. 2 for case 3 which involves the PREFFAS model (plotting of the difference between $\hat{Q}_t$ and $P$). With the PREFFAS model, the crack growth is especially sensitive to the maximum stress of the loading sequence, which is repeatedly applied as pointed out in [3]. In this model, the maximum stress is responsible for the overall amount of retardation but it also contributes more than the other maxima to the crack growth when it is applied. Transition probabilities which increase at failure are therefore those involving the maximum stress, here $s_5$, as naturally expected.

6. Conclusion

This paper presents a method for assessing failure probabilities of cracked structures submitted to random loads modeled by first-order Markov chains with discrete state space. This problem is of real interest for a safe design of structural components of aircrafts under damage tolerance requirements, for which it is of importance to account for fatigue load uncertainties and interactions effects between cycles. The reliability problem is solved by means of the CE method and its multi-level algorithm. It is worth pointing that the problem tackled here differs from those addressed in existing works, most of them in the field of communication networks. Failure is obtained here from a complex mechanical model whose input is a Markov chain whereas failure is directly given by the state of a Markov chain in other works of the literature. The results obtained clearly demonstrate the accuracy and efficiency of the CE method. Beside the failure probability estimate, this method also provides us with the most probable load sequences at failure which brings further details about failure. The application of this method to Markov chains with continuous state space will be the subject of a forthcoming paper.
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References


