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Physics Letters B

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PHYSICS LETTERS B

A new approach to evaluate the leading hadronic corrections to the muon *g*-2

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A B S T R A C T

We propose a novel approach to determine the leading hadronic corrections to the muon *g*-2. It consists
in a measurement of the effective electromagnetic coupling in the space-like region extracted from
Bhabha scattering data. We argue that this new method may become feasible at flavor factories, resulting
in an alternative determination potentially competitive with the accuracy of the present results obtained
with the dispersive approach via time-like data.

1. Introduction

The long-standing discrepancy between experiment and the
Standard Model (SM) prediction of a_μ, the muon anomalous mag-
netic moment, has kept the hadronic corrections under close
scrutiny for several years [1–4]. In fact, the hadronic uncertainty
dominates that of the SM value and is comparable with the exper-
imental one. When the new results from the *g*-2 experiments at
Fermilab and J-PARC will reach the unprecedented precision of 0.14
parts per million (or better) [5–7], the uncertainty of the hadronic
corrections will become the main limitation of this formidable test
of the SM.

An intense research program is under way to improve the
evaluation of the leading order (LO) hadronic contribution to a_μ,
due to the hadronic vacuum polarization correction to the one-
loop diagram [8,9], as well as the next-to-leading order (NLO)
hadronic one. The latter is further divided into the $O(\alpha^3)$ con-
tribution of diagrams containing hadronic vacuum polarization
insertions [10], and the leading hadronic light-by-light term, also
of $O(\alpha^3)$ [2,11,12]. Very recently, even the next-to–next-to leading
order (NNLO) hadronic contributions have been studied: inser-

tions of hadronic vacuum polarizations were computed in [13],
while hadronic light-by-light corrections have been estimated
in [14].

The evaluation of the hadronic LO contribution a_μ^{HLO} involves
long-distance QCD for which perturbation theory cannot be em-
ployed. However, using analyticity and unitarity, it was shown long
ago that this term can be computed via a dispersion integral using
the cross section for low-energy hadronic e^+e^- annihilation [15].
At low energy this cross-section is highly fluctuating due to reso-
nances and particle production threshold effects.

As we will show in this paper, an alternative determination
of a_μ^{HLO} can be obtained measuring the effective electromagnetic
coupling in the space-like region extracted from Bhabha ($e^+e^- \to
e^+e^-$) scattering data. A method to determine the running of
the electromagnetic coupling in small-angle Bhabha scattering was
proposed in [16] and applied to LEP data in [17]. As vacuum polar-
ization in the space-like region is a smooth function of the squared
momentum transfer, the accuracy of its determination is only lim-
ited by the statistics and by the control of the systematics of the
experiment. Also, as at flavor factories the Bhabha cross section
is strongly enhanced in the forward region, we will argue that a
space-like determination of a_μ^{HLO} may not be limited by statistics
and, although challenging, may become competitive with stan-
dard results obtained with the dispersive approach via time-like
data.

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2. Theoretical framework

The leading-order hadronic contribution to the muon g-2 is given by the well-known formula [4,15]

$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi^2} \int\limits_0^\infty \frac{ds}{s} \, K(s) \, \text{Im}\Pi_{\text{had}}(s + i\epsilon), \tag{1}$$

where $\Pi_{\text{had}}(s)$ is the hadronic part of the photon vacuum polarization, $\epsilon > 0$,

$$K(s) = \int\limits_0^1 dx \, \frac{x^2(1-x)}{x^2 + (1-x)(s/m_\mu^2)} \tag{2}$$

is a positive kernel function, and m_μ is the muon mass. As the total cross section for hadron production in low-energy e^+e^- annihilations is related to the imaginary part of $\Pi_{\text{had}}(s)$ via the optical theorem, the dispersion integral in Eq. (1) is computed integrating experimental time-like ($s > 0$) data up to a certain value of s [2,18,19]. The high-energy tail of the integral is calculated using perturbative QCD [20].

Alternatively, if we exchange the x and s integrations in Eq. (1) we obtain [21]

$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi} \int\limits_0^1 dx \, (x-1) \, \overline{\Pi}_{\text{had}}[t(x)], \tag{3}$$

where $\overline{\Pi}_{\text{had}}(t) = \Pi_{\text{had}}(t) - \Pi_{\text{had}}(0)$ and

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0 \tag{4}$$

is a space-like squared four-momentum. If we invert Eq. (4), we get $x = (1-\beta)(t/2m_\mu^2)$, with $\beta = (1 - 4m_\mu^2/t)^{1/2}$, and from Eq. (3) we obtain

$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi} \int\limits_{-\infty}^0 \overline{\Pi}_{\text{had}}(t) \left(\frac{\beta-1}{\beta+1}\right)^2 \frac{dt}{t\beta}. \tag{5}$$

Eq. (5) has been used for lattice QCD calculations of a_μ^{HLO} [22]; while the results are not yet competitive with those obtained with the dispersive approach via time-like data, their errors are expected to decrease significantly in the next few years [23].

The effective fine-structure constant at squared momentum transfer q^2 can be defined by

$$\alpha(q^2) = \frac{\alpha}{1 - \Delta\alpha(q^2)}, \tag{6}$$

where $\Delta\alpha(q^2) = -\text{Re}\,\overline{\Pi}(q^2)$. The purely leptonic part, $\Delta\alpha_{\text{lep}}(q^2)$, can be calculated order-by-order in perturbation theory – it is known up to three loops in QED [24] (and up to four loops in specific q^2 limits [25]). As $\text{Im}\,\overline{\Pi}(q^2) = 0$ for negative q^2, Eq. (3) can be rewritten in the form [26]

$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi} \int\limits_0^1 dx \, (1-x) \, \Delta\alpha_{\text{had}}[t(x)]. \tag{7}$$

Eq. (7), involving the hadronic contribution to the running of the effective fine-structure constant at space-like momenta, can be further formulated in terms of the Adler function [27], defined as the logarithmic derivative of the vacuum polarization, which, in turn, can be calculated via a dispersion relation with time-like

hadroproduction data and perturbative QCD [26,28]. We will proceed differently, proposing to calculate Eq. (7) by measurements of the effective electromagnetic coupling in the space-like region (see also [9]).

3. $\Delta\alpha_{\text{had}}(t)$ from Bhabha scattering data

The hadronic contribution to the running of α in the space-like region, $\Delta\alpha_{\text{had}}(t)$, can be extracted comparing Bhabha scattering data to Monte Carlo (MC) predictions. The LO Bhabha cross section receives contributions from t- and s-channel photon exchange amplitudes. At NLO in QED, it is customary to distinguish corrections with an additional virtual photon or the emission of a real photon (photonic NLO) from those originated by the insertion of the vacuum polarization corrections into the LO photon propagator (VP). The latter goes formally beyond NLO when the Dyson resummed photon propagator is employed, which simply amounts to rescaling the α coupling in the LO s- and t-diagrams by the factor $1/(1 - \Delta\alpha(q^2))$ (see Eq. (6)). In MC codes, e.g. in `BabaYaga` [29], VP corrections are also applied to photonic NLO diagrams, in order to account for a large part of the effect due to VP insertions in the NLO contributions. Beyond NLO accuracy, MC generators consistently include also the exponentiation of (leading-log) QED corrections to provide a more realistic simulation of the process and to improve the theoretical accuracy. We refer the reader to Ref. [30] for an overview of the status of the most recent MC generators employed at flavor factories. We stress that, given the inclusive nature of the measurements, any contribution to vacuum polarization which is not explicitly subtracted by the MC generator will be part of the extracted $\Delta\alpha(q^2)$. This could be the case, for example, of the contribution of hadronic states including photons (which, although of higher order, are conventionally included in a_μ^{HLO}), and that of W bosons or top quark pairs.

Before entering the details of the extraction of $\Delta\alpha_{\text{had}}(t)$ from Bhabha scattering data, let us consider a few simple points. In Fig. 1 (left) we plot the integrand $(1-x)\Delta\alpha_{\text{had}}[t(x)]$ of Eq. (7) using the output of the routine `hadr5n12` [31] (which uses time-like hadroproduction data and perturbative QCD). The range $x \in (0, 1)$ corresponds to $t \in (-\infty, 0)$, with $x = 0$ for $t = 0$. The peak of the integrand occurs at $x_{\text{peak}} \simeq 0.914$ where $t_{\text{peak}} \simeq -0.108$ GeV^2 and $\Delta\alpha_{\text{had}}(t_{\text{peak}}) \simeq 7.86 \times 10^{-4}$ (see Fig. 1 (right)). Such relatively low t values can be explored at e^+e^- colliders with center-of-mass energy $\sqrt{s}$ around or below 10 GeV (the so called "flavor factories") where

$$t = -\frac{s}{2}(1 - \cos\theta)\left(1 - \frac{4m_e^2}{s}\right), \tag{8}$$

θ is the electron scattering angle and m_e is the electron mass. Depending on s and θ, the integrand of Eq. (7) can be measured in the range $x \in [x_{\text{min}}, x_{\text{max}}]$, as shown in Fig. 2 (left). Note that to span low x intervals, larger θ ranges are needed as the collider energy decreases. In this respect, $\sqrt{s} \sim 3$ GeV appears to be very convenient, as an x interval $[0.30, 0.98]$ can be measured varying θ between $\sim 2°$ and $28°$. It is also worth remarking that data collected at flavor factories, such as DAΦNE (Frascati), VEPP-2000 (Novosibirsk), BEPC-II (Beijing), PEP-II (SLAC) and SuperKEKB (Tsukuba), and possibly at a future high-energy e^+e^- collider, like FCC-ee (TLEP) [32] or ILC [33], can help to cover different and complementary x regions.

Furthermore, given the smoothness of the integrand, values outside the measured x interval may be interpolated with some theoretical input. In particular, the region below x_{min} will provide a relatively small contribution to a_μ^{HLO}, while the region above x_{max} may be obtained by extrapolating the curve from x_{max} to $x = 1$, where the integrand is null, or using perturbative QCD.
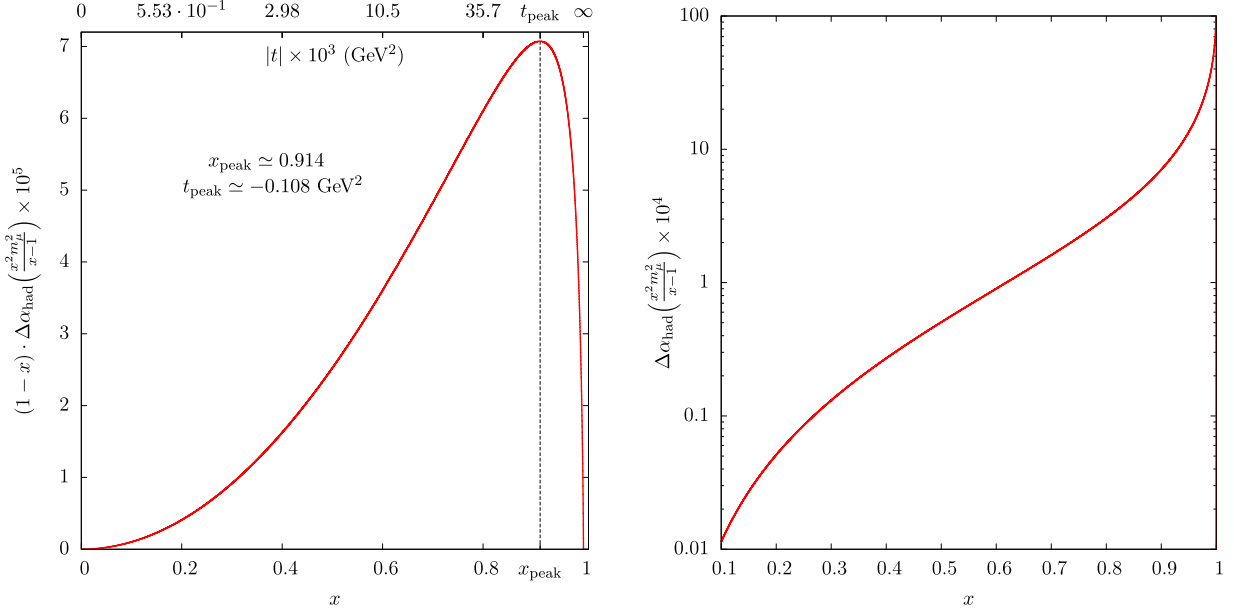
Fig. 1. Left: The integrand $(1-x)\Delta\alpha_{\mathrm{had}}[t(x)] \times 10^5$ as a function of x and t. Right: $\Delta\alpha_{\mathrm{had}}[t(x)] \times 10^4$.
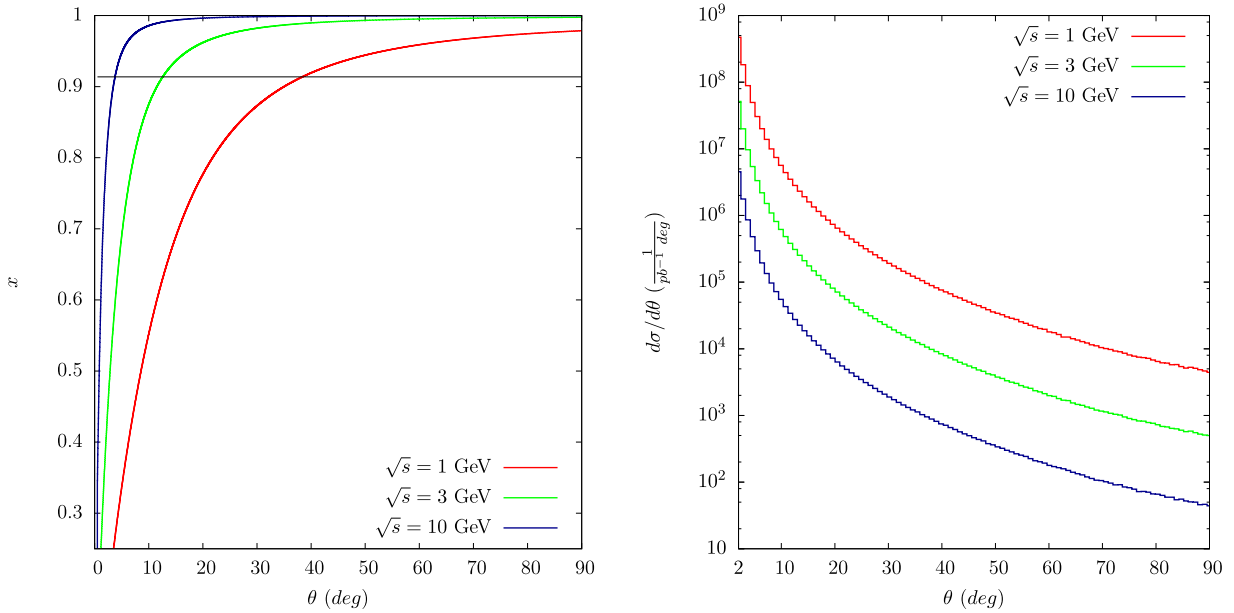

Fig. 2. Left: Ranges of x values as a function of the electron scattering angle θ for three different center-of-mass energies. The horizontal line corresponds to $x = x_{\mathrm{peak}} \simeq 0.914$. Right: Bhabha differential cross section obtained with `BabaYaga` [29] as a function of θ for the same three values of $\sqrt{s}$ in the angular range $2° < \theta < 90°$. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

The analytic dependence of the MC Bhabha predictions on $\alpha(t)$ (and, in turn, on $\Delta\alpha_{\mathrm{had}}(t)$) is not trivial, and a numerical procedure has to be devised to extract it from the data.[1] In formulae, we have to find a function $\alpha(t)$ such that

$$\frac{d\sigma}{dt}\bigg|_{\mathrm{data}} = \frac{d\sigma}{dt}\Big(\alpha(t), \alpha(s)\Big)\bigg|_{\mathrm{MC}}, \qquad (9)$$

where we explicitly kept apart the dependence on the time-like VP $\alpha(s)$ because we are only interested in $\alpha(t)$. We emphasize that, in our analysis, $\alpha(s)$ is an input parameter. Being the Bhabha

cross section in the forward region dominated by the t-channel exchange diagram, we checked that the present $\alpha(s)$ uncertainty induces in this region a relative error on the θ distribution of less than $\sim 10^{-4}$ (which is part of the systematic error).

We propose to perform the numerical extraction of $\Delta\alpha_{\mathrm{had}}(t)$ from the Bhabha distribution of the t Mandelstam variable. The idea is to let $\alpha(t)$ vary in the MC sample around a reference value and choose, bin by bin in the t distribution, the value that minimizes the difference with data. The procedure can be sketched as follows:

1. choose a reference function returning the value of $\Delta\alpha_{\mathrm{had}}(t)$ (and hence $\alpha(t)$) to be used in the MC sample, we call it $\bar{\alpha}(t)$;

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[1] This was not the case for example in [16,17]: there $\alpha(t)$ was extracted from Bhabha data in the very forward region at LEP, where the t channel diagrams are by far dominant and $\alpha(t)$ factorizes.

2. for each generated event, calculate N MC weights by rescaling $\bar{\alpha}(t) \rightarrow \bar{\alpha}(t) + \frac{i}{N}\delta(t)$, where $i \in [-N, N]$ and $\delta(t)$ is for example the error induced on $\bar{\alpha}(t)$ by the error on $\Delta\alpha_{\text{had}}(t)$. Being done on an event by event basis, the full dependence on $\alpha(t)$ of the MC differential cross section can be kept;

3. for each bin j of the t distribution, compare the experimental differential cross section with the MC predictions and choose the i_j which minimizes the difference;

4. $\bar{\alpha}(t_j) + \frac{i_j}{N}\delta(t_j)$ will be the extracted value of $\alpha(t_j)$ from data in the jth bin. $\Delta\alpha_{\text{had}}(t_j)$ can then be obtained through the relation between $\alpha(t)$ and $\Delta\alpha_{\text{had}}(t)$.

We finally find, for each bin j of the t distribution,

$$\frac{d\sigma}{dt}\Big|_{j,\text{data}} = \frac{d\sigma}{dt}\left(\bar{\alpha}(t) + \frac{i_j}{N}\delta(t), \alpha(s)\right)\Big|_{j,\text{MC}}. \qquad (10)$$

We remark that the algorithm does not assume any simple dependence of the cross section on $\alpha(t)$, which can in fact be general, mixing s, t channels and higher order radiative corrections, relevant (or not) in different t domains.

In order to test our procedure, we perform a pseudo-experiment: we generate pseudo-data using the parameterization $\Delta\alpha_{\text{had}}^I(t)$ of Refs. [19,34] and check if we can recover it by inserting in the MC the (independent) parameterization $\Delta\alpha_{\text{had}}^{II}(t)$ (corresponding to $\bar{\alpha}(t)$ of Eq. (10)) of Ref. [31] by means of the method described above. For this exercise, we use the generator BabaYaga in its most complete setup, generating events at $\sqrt{s} = 1.02$ GeV, requiring $10° < \theta_\pm < 170°$, $E_\pm > 0.4$ GeV and an acollinearity cut of $15°$. We choose $\delta(t)$ to be the error induced on $\alpha(t)$ by the 1-σ error on $\Delta\alpha_{\text{had}}(t)$, which is returned by the routine of Ref. [31], we set $N = 150$, and we produce distributions with 200 bins. We note that in the present exercise $\alpha(s)$ and all the radiative corrections both in the pseudo-data and in the MC samples are exactly the same, because we are interested in testing the algorithm rather than assessing the achievable accuracy, at least at this stage.

In Fig. 3, $\Delta\alpha_{\text{had}}^{\text{extr}}$ is the result extracted with our algorithm, corresponding to the minimizing set of i_j: the figure shows that our method is capable of recovering the underlying function $\Delta\alpha_{\text{had}}(t)$ inserted into the "data". As the difference between $\Delta\alpha_{\text{had}}^I$ and $\Delta\alpha_{\text{had}}^{\text{extr}}$ is hardly visible on an absolute scale, in Fig. 3 all the functions have been divided by $\Delta\alpha_{\text{had}}^{II}$ to display better the comparison between $\Delta\alpha_{\text{had}}^I$ and $\Delta\alpha_{\text{had}}^{\text{extr}}$.

In order to assess the achievable accuracy on $\Delta\alpha_{\text{had}}(t)$ with the proposed method, we remark that the LO contribution to the cross section is quadratic in $\alpha(t)$, thus we have (see Eq. (6))

$$\frac{1}{2}\frac{\delta\sigma}{\sigma} \simeq \frac{\delta\alpha}{\alpha} \simeq \delta\Delta\alpha_{\text{had}}. \qquad (11)$$

Eq. (11) relates the *absolute* error on $\Delta\alpha_{\text{had}}$ with the *relative* error on the Bhabha cross section. From the theoretical point of view, the present accuracy of the MC predictions [30] is at the level of about 0.5‰, which implies that the precision that our method can, at best, set on $\Delta\alpha_{\text{had}}(t)$ is $\delta\Delta\alpha_{\text{had}}(t) \simeq 2 \cdot 10^{-4}$. Any further improvement requires the inclusion of the NNLO QED corrections into the MC codes, which are at present not available (although not out of reach) [30].

From the experimental point of view, we remark that a measurement of a_μ^{HLO} from space-like data competitive with the current time-like evaluations would require an $\mathcal{O}(1\%)$ accuracy. Statistical considerations show that a 3% fractional accuracy on the a_μ^{HLO} integral can be obtained by sampling the integrand $(1 - x)\Delta\alpha_{\text{had}}[t(x)]$ in ~ 10 points around the x peak with a fractional accuracy of
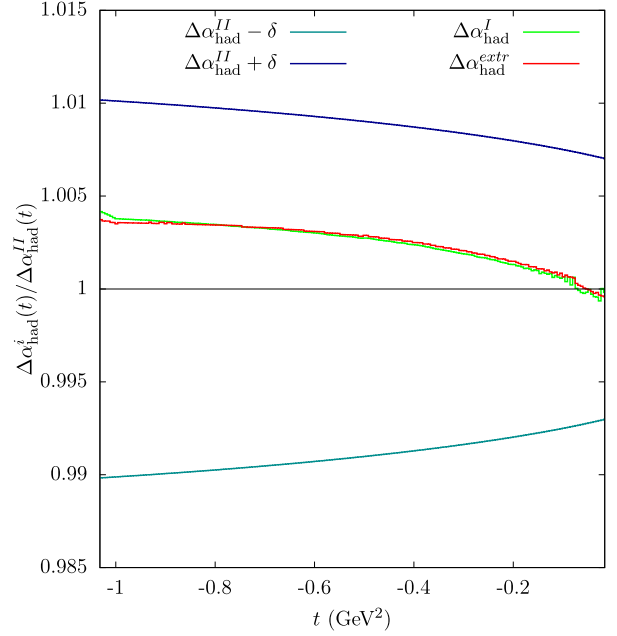

Fig. 3. The extracted function $\Delta\alpha_{\text{had}}^{\text{extr}}(t)$ compared to the function $\Delta\alpha_{\text{had}}^I(t)$ used in the pseudo-data (see text). The functions $\Delta\alpha_{\text{had}}^{II}(t) \pm \delta(t)$ are shown to display the range spanned by the MC samples. All functions have been divided by $\Delta\alpha_{\text{had}}^{II}(t)$. The tiny difference between $\Delta\alpha_{\text{had}}^I$ and $\Delta\alpha_{\text{had}}^{\text{extr}}$ is due to the binning discretization. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

10%. Given the value of $\mathcal{O}(10^{-3})$ for $\Delta\alpha_{\text{had}}$ at $x = x_{\text{peak}}$, this implies that the cross section must be known with relative accuracy of $\sim 2 \times 10^{-4}$. Such a statistical accuracy, although challenging, can be obtained at flavor factories, as shown in Fig. 2 (right). With an integrated luminosity of $\mathcal{O}(1)$, $\mathcal{O}(10)$, $\mathcal{O}(100)$ fb^{-1} at $\sqrt{s} = 1$, 3 and 10 GeV, respectively, the angular region of interest can be covered with a 0.01% accuracy per degree. The experimental systematic error must match the same level of accuracy.

A fraction of the experimental systematic error comes from the knowledge of the machine luminosity, which is normalized by calculating a theoretical cross section in principle not depending on $\Delta\alpha_{\text{had}}$. We devise two possible options for the normalization process:

1. using the $e^+e^- \rightarrow \gamma\gamma$ process, which has no dependence on $\Delta\alpha_{\text{had}}$, at least up to NNLO order;
2. using the Bhabha process at $t \sim 10^{-3}$ GeV^2 ($x \sim 0.3$), where the dependence on $\Delta\alpha_{\text{had}}$ is of $\mathcal{O}(10^{-5})$ and can be safely neglected.

Both processes have advantages and disadvantages; a dedicated study of the optimal choice goes beyond the scope of this paper and will be considered in a future detailed analysis.

4. Conclusions

We presented a novel approach to determine the leading hadronic correction to the muon g-2 using measurements of the running of $\alpha(t)$ in the space-like region from Bhabha scattering data. Although challenging, we argued that this alternative determination may become feasible using data collected at present flavor factories and possibly also at a future high-energy e^+e^- collider. The proposed determination can become competitive with the accuracy of the present results obtained with the dispersive approach via time-like data.

Acknowledgements

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