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Judgment scales and consistency measure in AHP

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Abstract

The Analytic Hierarchy Process (AHP) is widely used method in multiple-attribute decision making. In the recent literature many authors used different judgment scales which influenced the results and decisions. In this paper the author reviews and discusses effects of utilization of various judgment scales on priority estimation in AHP. There has been studies that have been concerned with the comparison of judgment scales but there were no studies concerned with consistency measures that are needed. The goal of this paper is to compare and discuss the application of various judgment scales on the results in particular practical example that has been used in previous paper by Saaty (2003). Thus the focus of the paper is to analyze the impact of using different judgment scales on the resulting priorities and consistency to default scale as proposed by Saaty. Results suggest that judgment scales have a profound impact on criteria priorities but not on ranking of criteria. However, the consistency varies among applied judgment scales. Authors calculated the values of random index needed for calculation of the consistency index in AHP for all concerned scales. Based on them the consistency index was computed and compared. Both consistent and inconsistent Saaty matrices were used for comparison.

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Keywords: AHP; analytic hierarchy process; judgement scales; consistency; inconsitency

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1. Introduction

The analytic hierarchy process (AHP) is a decomposition multiple-attribute decision making (MADM) method. It was developed by Saaty (1977), who proposed a method that can represent human decision making process and help to achieve better judgments based on hierarchy, pair-wise comparisons, judgment scales, allocation of criteria weights and selection of the best alternative from a finite number of variants by calculation of their utility functions. Subsequently, there has been a growth of applications and mathematical development to this methodology. These developments were focused on different parts of the method. A significant development has been made by Saaty (1996) who presented a more general approach to AHP which he called the analytic network process (ANP). Other scholars have suggested new judgment scales or they expand the method by fuzzy logic and group decision making (Ishizaka & Labib, 2011).

The goal of this paper is to analyze judgment scales developed for AHP and their influence on results of AHP decision making example. The aim of this approach is to analyze inconsistent and consistent pair-wise comparison matrices to get a better understanding of how changes in scales can influence results and consistency at the same time. Suggestions of how these scales and results of this study can be applied can be found in the discussion and conclusion.

2. Literature review

Since its introduction, AHP has been widely used (Taslicali & Ercan, 2006). AHP is a multi-criteria decision making (MCDM) method that helps decision-maker to face a complex problem with multiple conflicting and subjective criteria (e.g. location or investment selection, projects ranking, etc).

2.1. Summary of the AHP

Basically the method uses following structure: problem modeling, weights valuation, weights aggregation and sensitivity analysis. AHP has the advantage of permitting a hierarchical structure of the criteria, which provides users with a better focus on specific criteria and sub-criteria when allocating the weights. This step is important, because a different structure may lead to a different final ranking. When setting up the AHP hierarchy with a large number of elements, the decision maker should attempt to arrange these elements in clusters so they do not differ in extreme ways.

Psychologists argue that it is easier and more accurate to express one's opinion on only two alternatives than simultaneously on all the alternatives (Ishizaka & Labib, 2011). This also allows consistency check of different pair-wise comparisons. AHP uses a ratio scale, which, contrary to methods using interval scales, requires no units in the comparison. The judgment is a relative value or a quotient *a/b* of two quantities *a* and *b* having the same units (intensity, meters, utility, etc). The decision maker does not need to provide a numerical judgment; instead a relative verbal appreciation is sufficient. The results of paired comparisons for n attributes is organized into positive reciprocal *n x n* matrix Psychologists argue that it is easier and more accurate to express one's opinion on only two alternatives than simultaneously on all the alternatives (Saaty, 1977). This also allows consistency check of different pair-wise comparisons. AHP uses a ratio scale, which, contrary to methods using interval scales, requires no units in the comparison. The judgment is a relative value or a quotient a / b of two quantities a and b having the same units (intensity, meters, utility, etc). The decision maker does not need to provide a numerical judgment; instead a relative verbal appreciation is sufficient. The results of paired comparisons for n attributes are organized into positive reciprocal n x n matrix $\mathbf{S} = \left(s_{ij}\right)$ as follows

$$\mathbf{S} = \begin{pmatrix} 1 & s_{12} & \dots & s_{1n} \\ 1/s_{12} & 1 & \dots & s_{2n} \\ \dots & \dots & \dots & \dots \\ 1/s_{1n} & 1/s_{2n} & \dots & 1 \end{pmatrix}. \tag{1}$$

If the matrix is perfectly consistent, then the transitivity rule (2) holds for all comparisons:

$$s_{ij} = s_{ik} \cdot s_{kj} . \quad (2)$$

The default Saaty 1–9 scale is based on psychological observations. A minimal consistency is required to derive meaningful priorities, thus, a consistency test must be made (Saaty, 1977). Further details about AHP methodology and approach can be found e.g. in Saaty (1977) or (Franek & Zmeskal, 2013).

2.2. Judgment scales

One of the most prominent features of AHP methodology is to evaluate quantitative as well as qualitative criteria and alternatives on the same preference scale. These can be numerical, verbal or graphical. The use of verbal responses is intuitive. It may also allow some ambiguity in non-trivial comparisons. Due to its pair-wise comparisons AHP needs ratio scales. There are some disputes about scale as the best option of judgment expression but most scholars still prefer this approach. Saaty (1994) states that ratio scales are the only possible measurement if we want to be able to aggregate measurements, as in a weighted sum. Dodd and Donegan (1995) have criticized the absence of a zero in the preference scale but also developed scale using number "1" as well.

In original Saaty's AHP the verbal statements are represented by scale with measures from one to nine. Theoretically there is no reason to be restricted to these numbers and verbal gradation. Although the verbal gradation has been not a concern, several other numerical scales have been proposed, see table 1.

Table 1. Judgement scales used in AHP

Scale type	Mathematical description	Parameters	Approx. scale values
Linear (Saaty, 1977)	$s = x$	$x = \{1, 2, ..., 9\}$	1;2;3;4;5;6;7;8;9
Power (Harker, Vargas, 1987)	$s = x^2$	$x = \{1, 2, ..., 9\}$	1;4;9;16;25;36;49;64;81
Root square (Harker, Vargas, 1987)	$s = \sqrt{x}$	$x = \{1, 2, ..., 9\}$	$1;\sqrt{2};\sqrt{3};2;\sqrt{5};\sqrt{6};\sqrt{7};\sqrt{8};3$
Geometric (Lootsma, 1989)	$s = 2^{x-1}$	$x = \{1, 2, ..., 9\}$	1;2;4;8;16;32;64;128;256
Inverse linear (Ma, Zheng, 1991)	$s = \frac{9}{(10-x)}$	$x = \{1, 2, ..., 9\}$	1;1.13;1.29;1.5;1.8;2.25;3;4.5;9
Asymptotical (Dodd, Donegan, 1995)	$s = \tanh^{-1}\left(\frac{\sqrt{3}(x-1)}{14}\right)$	$x = \{1, 2, ..., 9\}$	0;0.12;0.24;0.36;0.46;0.55;0.63;0.7;0.76
Balanced (Sal Hamalainen, 1997)	$s = \frac{w}{(1-w)}$	$w = \{0.5, 0.55, 0.6, ..., 9\}$	1;1.22;1.5;1.86;2.33;4;5.67;9
Logarithmic (Ishizaka, Balkenborg, Kaplan, 2010)	$s = \log_2(x+1)$	$x = \{1, 2, ..., 9\}$	1;1.58;2;2,.2;2.58;2.81;3;3.17;3.32

Source: elaborated based on Ishizaka and Labib (2011).

Harker and Vargas (1987) have investigated a quadratic and a root square scale in only one simple example and argued in favour of Saaty's 1 to 9 scale. However, one example seems not enough to conclude the superiority of the 1–9 linear scale. Lootsma (1989) argued that the geometric scale is preferable to the 1–9 linear scale. Salo and Hämäläinen (1997) point out that the integers from one to nine yield local weights, which are unequally dispersed, so that there is lack of sensitivity when comparing elements, which are preferentially close to each other. Based on this observation, they proposed a balanced scale where the local weights are evenly dispersed over the weight range

[0.1, 0.9]. Earlier Ma and Zheng (1991) have calculated a scale where the inverse elements x of the scale $1/x$ are linear instead of the x in the Saaty scale. Donegan, Dodd and McMaster (1995) have proposed an asymptotic scale avoiding the boundary problem. The possibility to integrate negative values in the scale has been also investigated (Millet & Schoner, 2005; Saaty & Ozdemir, 2003). The linguistic expression of judgement is used according to original Saaty (1977) setup.

Among all the proposed scales, the original linear scale has been used by far the most often in applications. Saaty (1994) advocates it as the best scale to represent weight ratios. However, above mentioned scholars dealt with objective measurable alternatives, whereas AHP mainly treats decision processes as subjective issues. Salo and Hämäläinen (1997) demonstrated the superiority of the balanced scale only on comparing two elements. The choice of the appropriate scale is difficult and often discussed problem. Some scholars argue that the choice depends on the person and the decision problem (Harker & Vargas, 1987). But there is no definite manual which scale is better for certain decision making problem, type of alternatives or criteria

3. Methodological fundamentals of consistency in AHP

The evaluation requires a certain level of matrix consistency, i.e. that the elements are linear independent. That can be assessed employing consistency index CI as follows: firstly the λ_{max} (the highest eigenvalue of the matrix) has to be calculated like so (Saaty, 1977):

$$\lambda_{\max} = \sum_{j=1}^{m} \frac{(\mathbf{S} \cdot \mathbf{v})_j}{m \cdot v_j}, \tag{3}$$

where m represents the number of independent rows of the matrix, **S** represents pair-wise comparison matrix and v means the matrix eigenvector. Then the consistency index *(CI)* can be calculated as follows:

$$CI = \frac{\lambda_{\max} - m}{m - 1}. \tag{4}$$

If the matrix is perfectly consistent then $CI=0$.

When dealing with rising number of pair-wise comparisons the possibility of consistency error is also increasing. Thus Saaty (1980) suggested another measure the *CR* (consistency ratio) that can be calculated like so

$$CR = \frac{CI}{RI}, \tag{5}$$

where *RI* is represented by average *CI* values gathered from a random simulation of Saaty pair-wise comparison matrices *CI*s. The suggested value of the CR should be no higher than 0.1 (Saaty, 1980).

The problem of accepting/rejecting matrices has been greatly discussed, especially the relation between the consistency and the scale used to represent the decision maker's judgements. Lane and Verdini (1989) have shown that by using a 9-point scale, Saaty's CR threshold is too restrictive due to the standard deviation of CI for randomly generated matrices being relatively small. However, Salo and Hämäläinen (1997) have found that *CR* threshold depends on the granularity of the scale which is being used.

A historical study of several RIs used and a way of estimating this index can be seen in Alonso and Lamata (2004). At first Saaty (at Wharton) and Uppuluri (at Oak Ridge) simulated the experiment with 500 and 100 runs (Saaty, 1980). In recent study from Alonso and Lamata (2006) the number of simulation ranged from 100 000 to 500 000.The simulation study of paper's authors (Franek & Kresta) was also performed on 500 thousand cases. Both results are compared in the following table 2. In the table it can be seen that from 100 thousand simulations the RIs virtually the same. In the following sections and calculations the RI measures calculated by Franek and Kresta will be used.

On the other hand, it must be emphasized that the table 2 shows the RI only for Saaty scale. Thus, for comparison of judgment scales different RI's were calculated using the same methodology. The results are shown later in this paper.

Table 2. RI values derived from simulations

	Oak Ridge (1980)	Wharton (1980)	Alonso and Lamata (2006)	Alonso and Lamata (2006)	Franek and Kresta (2014)
n	100	500	100 000	500 000	500 000
3	0.38	0.58	0.525	0.525	0.525
4	0.95	0.90	0.880	0.880	0.882
5	1.22	1.12	1.109	1.109	1.110
6	1.03	1.24	1.248	1.248	1.250
7	1.47	1.32	1.342	1.342	1.341
8	1.40	1.41	1.406	1.406	1.404
9	1.35	1.45	1.450	1.450	1.451
10	1.46	1.49	1.485	1.485	1.486
11	1.58	1.51	1.514	1.514	1.514
12	1.48		1.537	1.537	1.536
13	1.56		1.555	1.555	1.555
14	1.57		1.571	1.571	1.570
15	1.59		1.584	1.584	1.584

Source: Alonso and Lamata (2006, p. 450–451) and own calculation.

4. Judgment scales and consistency measure

This study has been modelled on an example presented by Saaty (2003). There he proposed an approach to reduce and evaluate inconsistency in sample pair-wise matrix. This matrix is presented in table 3. The goal of the decision making is to compare and prioritize criteria for house buyers. The criteria are size, transport, neighbourhood, age, area in square yards, modern, conditions and finance. The judgment scale used is the original Saaty scale (Saaty, 1977).

4.1. Decision making pair-wise comparison example based on Saaty (2003)

Table 3. Decision making about house buying pair-wise comparison matrix for the criteria

Matrix S	Size	Trans.	Nbrhd.	Age	Yard	Modern	Cond.	Finance
Size	1	5	3	7	6	6	1/3	1/4
Transport	1/5	1	1/3	5	3	3	1/5	1/7
Neighbourhood	1/3	3	1	6	3	4	6	1/5
Age	1/7	1/5	1/6	1	1/3	1/4	1/7	1/8
Yard	1/6	1/3	1/6	3	1	1/2	1/5	1/6
Modern	1/6	1/3	1/4	4	2	1	1/5	1/6
Conditions	3	5	1/6	7	5	5	1	1/2
Finance	4	7	5	8	6	6	2	1

Source: Saaty (2003, p 88).

The resulting weights were estimated using Row Geometric Mean Method (RGMM) as proposed by Saaty (1980). The results are presented in table 4. Saaty (2003) used EVM (Eigenvalue method) this explains the difference towards his original paper. But for the comparison the authors have utilized most frequent method of RGMM as presented in (7).

$$w_j = \frac{\left[\prod_{j=1}^{i} s_{ij}\right]^{\frac{1}{i}}}{\sum_{j=1}^{i}\left[\prod_{j=1}^{i} s_{ij}\right]^{\frac{1}{i}}}. \quad (6)$$

where the w_j represents the normalized weight.

Table 4. Results for inconsistent matrix S

Criteria	**Size**	**Trans.**	**Nbrhd.**	**Age**	**Yard**	**Modern**	**Cond.**	**Finance**
Priorities (*w*)	0.1750	0.063	0.149	0.019	0.033	0.042	0.168	0.351
CI=	0.2150							
CR=	0.1525							

Source: own calculations based on Saaty (2003).

Consistency index was calculated according to (4) and consistency ratio according to (5). Then C.I.=0.215, C.R.=0.15. The results show that the matrix is no fully consistent. This can be amended by checking relationships among criteria to find the pair-wise comparison that is not consistent with others in terms of chain of preferences. In this case according to Saaty (2003) it is the item s_{37} and s_{73} of the pair-wise matrix **S** in table 3. There is needed to change the preference from 6 to 1/2 and 1/6 to 2. Then the resulting weights, consistency index and consistency ratio are presented in table 5.

Table 5. Results for consistent matrix S^*

Criteria	**Size**	**Trans.**	**Nbrhd.**	**Age**	**Yard**	**Modern**	**Cond.**	**Finance**
Priorities (*w*)	0.1720	0.062	0.107	0.019	0.032	0.042	0.224	0.343
CI=	0.1065							
CR=	0.0755							

Source: own calculations based on Saaty (2003).

4.2. Decision-making example using various judgment scales

In this part the paper is concerned with application of judgment scales presented on the table 1 in section 2. Based on previous example presented by Saaty (2003) which was set as a default the scales were applied. Further more own simulations of random index R.I. calculation were made. These were applied both in inconsistent matrix **S** as well as in consistent matrix $\mathbf{S}^*$. Resulting priorities were calculated using RGMM method (7), CI and CR were calculated using (5) and (6) respectively. The calculations were made using Matlab software and MS Excel. The results were compared with the default original Saaty scale. The comparison was focused on criteria ranking, values of priorities, and both consistency measures.

When using different judgment scales it is expected that measures of CI and RI will change dramatically. The higher the variance of the scale the bigger will be the consistency index. However, we can assume that CR will not differ significantly. Thus it is needed to calculate new random indexes and lambda λ (the highest eigenvalue of the matrix) to calculate unique measures for each judgment scale. Then the calculation of new CI and RI for each scale can be made. However, scholars were not fully concerned by these measures. These are presented on the table 6 and 7.

Table 6. Measures of average λ for 500 000 simulations

Average λ								
n	Linear	Power	Root square	Geometric	Inverse linear	Asymptotical	Balanced	Logarithmic
1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	2.000	2.000	2.000	2.000	2.000	0.942	2.000	2.000
3	4.050	10.218	3.228	12.185	3.410	2.225	3.535	3.306
4	6.647	24.961	4.538	31.898	5.000	3.666	5.320	4.724
5	9.440	42.854	5.872	58.287	6.668	5.224	7.201	6.179
6	12.249	61.247	7.214	88.502	8.376	6.789	9.125	7.640
7	15.044	79.429	8.561	120.383	10.099	8.372	11.056	9.106
8	17.831	97.238	9.909	153.001	11.831	9.954	13.002	10.575
9	20.610	114.769	11.258	185.716	13.575	11.543	14.946	12.044
10	23.375	132.105	12.608	218.368	15.307	13.134	16.885	13.513
11	26.135	149.330	13.958	250.576	17.047	14.728	18.829	14.983
12	28.901	166.434	15.308	282.673	18.790	16.315	20.772	16.452
13	31.655	183.527	16.660	314.402	20.526	17.905	22.718	17.924
14	34.416	200.499	18.012	346.001	22.268	19.497	24.657	19.395
15	37.172	217.451	19.364	377.194	24.006	21.084	26.602	20.866

Table 7. Measures of random index (RI) for judgment scales used in AHP

RI (average CI) 500 000 simulations								
n	Linear	Power	Root square	Geometric	Inverse linear	Asymptotical	Balanced	Logarithmic
1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000	−1.058	0.000	0.000
3	0.525	3.609	0.114	4.592	0.205	−0.388	0.267	0.153
4	0.882	6.987	0.179	9.299	0.333	−0.111	0.440	0.241
5	1.110	9.464	0.218	13.322	0.417	0.056	0.550	0.295
6	1.250	11.049	0.243	16.500	0.475	0.158	0.625	0.328
7	1.341	12.071	0.260	18.897	0.517	0.229	0.676	0.351
8	1.404	12.748	0.273	20.714	0.547	0.279	0.715	0.368
9	1.451	13.221	0.282	22.089	0.572	0.318	0.743	0.380
10	1.486	13.567	0.290	23.152	0.590	0.348	0.765	0.390
11	1.514	13.833	0.296	23.958	0.605	0.373	0.783	0.398
12	1.536	14.039	0.301	24.607	0.617	0.392	0.797	0.405

13	1.555	14.211	0.305	25.117	0.627	0.409	0.810	0.410
14	1.570	14.346	0.309	25.539	0.636	0.423	0.820	0.415
15	1.584	14.461	0.312	25.871	0.643	0.435	0.829	0.419

Using the Table 6 and Table 7 above resulting CI and CR were calculated and compared.

5. Results and discussion

Using pair-wise comparison matrix example, **S** from Saaty (2003) as presented in table 3 and its consistent version $\mathbf{S}^*$ following priorities and consistency measures were found (see Table 8 and Table 9 respectively).

Table 8. Priorities and consistency measures for different judgment scales on inconsistent matrix **S**

	Linear	Power	Root square	Geometric	Inverse linear	Asymptotical	Balanced	Logarithmic
Size	0.173	0.125	0.162	0.138	0.173	0.165	0.177	0.164
Transport	0.054	0.014	0.094	0.011	0.083	0.093	0.074	0.087
Neighbourhood	0.188	0.096	0.152	0.082	0.142	0.143	0.145	0.120
Age	0.018	0.001	0.049	0.001	0.038	0.049	0.029	0.040
Yard	0.031	0.003	0.066	0.004	0.067	0.076	0.053	0.071
Modern	0.036	0.006	0.076	0.005	0.071	0.081	0.059	0.076
Conditions	0.167	0.115	0.158	0.106	0.154	0.152	0.159	0.174
Finance	0.333	0.639	0.243	0.654	0.272	0.241	0.304	0.269
lambda λ	9.690	23.441	8.525	21.982	8.352	8.228	8.603	8.170
CI	0.241	2.206	0.075	1.997	0.050	0.033	0.086	0.024
CR	0.170	0.173	0.275	*0.096*	*0.092*	0.117	0.121	*0.087*

The results in the Table 8 show distinctive properties of used judgment scales. In the inconsistent matrix scales other than Linear (Saaty) are influencing the ranking of the second most preferred criterion. In the Linear scale the neighborhood comes second, while in the others is ranked as the fourth and the conditions comes third and size comes second. In the case of the Logarithmic scale conditions precede size. The biggest change can be seen in values of particular priorities. The Power and Geometric scales use larger values and thus also the value of the most important criterion is higher, though the other criteria values have not changed that much. On the other hand the Asymptotical and Root square scales affected the dominance of the most important criterion more than the other criteria. As for the consistency the Geometric, Inverse linear and Logarithmic scales tolerate the inconsistency more that other scales. The least tolerant or inconsistency more sensitive seems to be the Root square scale.

Table 9. Priorities and consistency measures for different judgment scales on consistent matrix $\mathbf{S}^*$

	Linear	Power	Root square	Geometric	Inverse linear	Asymptotical	Balanced	Logarithmic
Size	0.175	0.116	0.161	0.129	0.172	0.164	0.176	0.162
Transport	0.062	0.013	0.094	0.010	0.083	0.093	0.073	0.089
Neighbourhood	0.103	0.042	0.124	0.038	0.120	0.124	0.115	0.122
Age	0.019	0.001	0.048	0.001	0.038	0.049	0.029	0.042
Yard	0.034	0.003	0.066	0.003	0.066	0.076	0.053	0.058
Modern	0.041	0.006	0.075	0.005	0.071	0.081	0.058	0.069

Conditions	0.221	0.227	0.190	0.199	0.181	0.174	0.196	0.199
Finance	0.345	0.592	0.242	0.614	0.270	0.240	0.300	0.260
lambda λ	8.810	14.671	8.341	10.638	8.105	8.073	8.187	8.532
CI	0.116	0.953	0.049	0.377	0.015	0.010	0.027	0.076
CR	*0.083*	*0.075*	0.179	*0.018*	*0.027*	*0.037*	*0.037*	0.207

In the case of Saaty (2003) consistent matrix in the Table 9 it can be seen that ranking of criteria is uniform for all scales. Differences remain in allocation of priorities values and consistency. Again the Power and Geometric scales influence the value of the most important criterion. And on the other hand Asymptotical and Root square enable to allocate values of priorities more equally. Consistency wise results show that most of the matrices are consistent with exception of Root Square and Logarithmic. Though, the Root square seems to be very sensitive towards inconsistency, the Logarithmic behaves in different way and show more inconsistency in this particular example.

Thus to summarize based on the evidence from this example it is worth to investigate behavior of various judgment scales under conditions of inconsistent and consistent matrices. Based on results from this study it can be suggested to classify judgment scales into groups using the characteristic of allocation of priority values and consistency sensitivity. The default measure has to be the Saaty scale and it serves as a benchmark. The following Table 10 summarizes the classification.

Table 10. Classification of judgment scales based on consistency and allocation of priorities

Dimension / Characteristic	Consistency sensitivity	Variance of allocation of priorities' values
High	Root square, Logarithmic	Power, Geometric
Moderate	Linear (Saaty), Power	Linear (Saaty) , Balanced
Low	Geometric, Inverse linear, Asymptotical and Balanced	Root square, Inverse linear, Asymptotic

6. Conclusion

In this paper authors investigate application and characteristics of different judgment scales developed by scholars for use in AHP. Results and their comparison show that judgement scales play significant role in AHP decision making. Thus it should be recommended to pay more attention to their use. Based on results it is likely to classify judgement scales to three groups based on consistency and allocation of priorities. The consistency is measured by consistency ration using own calculations random index (RI). The Saaty original 9 point linear scale is set as benchmark for comparison of other judgment scales. Using simple decision making problem an insonsitent and consistent matrix is used to investigate changes in consistency. Based on evidence from this AHP example judgment scales can be classified in to three groups: highly sensitive, moderate sensitive and low sensitive.

Another characteristic is allocation of priorities. There based on used AHP example judgment scales can be divided into three groups as well: high variance of priorities' values, moderate and low. Results can be found in the Table 10.

Decision-maker can face selection the most suitable scale for his problem. According to presented results the Linear (Saaty scale) is still a favorable option. However, if the decision maker prefers better higher consistency then Root square or Logarithmic scales can be selected as well. On the other hand concerning priorities' values and selecting the most important criterion the decision maker can select Power or Geometric scale to clearly highlight the most preferred criterion.

Further research should be focused on use of different scales on different decision making problems. The best approach would be mathematical programming and simulation with comparison of results for all judgement scales. Further research should be also concerned with consistency index for whole AHP with different judgement scales.

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References

Alonso J. A., Lamata, M. T. 2004. Estimation of the Random Index in the Analytic Hierarchy Process . Proceedings of Information Processing and Management of Uncertainty in Knowledge-Based Systems. pp 317–322.

Alonso, J., & Lamata, T. (2006). Consistency in the Analytic Hierarchy Process: a New Approach. International Journal of Uncertainty, Fuzziness and Knowledge – Based Systems, 14, 445–459.

Dodd, F. J., Donegan H. A. 1995. Comparison of priotization techniques using interhierarchy mappings. Journal of the Operational Research Society, 46(4), 492–498.

Franek, J., Zmeškal, Z. 2013. A Model of Strategic Decision Making Using Decomposition SWOT-ANP Method. In: Financial Management of Firms and Financial Institutions 9th International Scientific Conference Proceedings (Part I-III). Ostrava: VŠB – Technical University of Ostrava, pp 172–180.

Harker, P., Vargas, L. 1987. The Theory of Ratio Scale Estimation: Saaty's Analytic Hierarchy Process. Management Science. 33(11), 1383–1403.

Ishizaka, A., Balkenborg, D., Kaplan T. 2010. Influence of aggregation and measurement scale on ranking a compromise alternative in AHP. Journal of the Operational Research Society, 62(4), 700–710.

Ishizaka, A., Labib, A., 2011. Review of the main developments in the analytic hierarchy process. Expert Systems with Applications, 38(11), 14336–14345.

Lane, E. F., Verdini, W. A. 1989. A consistency test for AHP decision makers, Decision Science 20, 575–590

Lootsma, F. 1989. Conflict Resolution via Pairwise Comparison of Concessions. European Journal of Operational Research. Elsevier: Amsterdam. 40(1), 109–116.

Ma, D., Zheng, X. 1991. 9/9-9/1 Scale Method of AHP. In 2nd Int. Symposium on AHP (Vol. 1,). University of Pittsburgh: Pittsburgh, pp. 197–202.

Saaty, T. L., 1977. A scaling method for priorities in hierarchical structures. Journal of Mathematical Psychology, 15(3), 234–281.

Saaty, T. L. 1980. The Analytic Hierarchy Process. New York.

Saaty, T. L. 1994. Fundamentals of Decision Making and Priority Theory with the Analytic Hierarchy Process. RWS Publications, Pittsburgh .

Saaty, T. L. 1996. Decision Making With Dependence And Feedback: The Analytic Network Process. Pittsburgh: RWS Publications.

Saaty, T. L. 2003. Decision-making with the AHP: Why is the principal eigenvector necessary. European Journal of Operational Research, 145, 85–91.

Salo, A., Hamalainen, R. 1997. On the Measurement of Preference in the Analytic Hierarchy Process. Journal of Multi-Criteria Decision Analysis. 6(6), 309–319.

Taslicali, A. K., Ercan, S. 2006. The analytic hierarchy & the analytic network processes in multicriteria decision making: a comparative study. Journal Of Aeronautics And Space Technologies, 2(4), 55–65.