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Gamma-GQM time headway model: endogenous effects in rural two-lane two-way roads

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Abstract

Study of vehicle time headway distributions is essential in many traffic engineering applications, such as capacity and level of service analysis and, in recent years, in the fields of vehicle generation in traffic micro-simulation models and driving simulation applications. This paper presents results from an experimental analysis of vehicle time headway distributions on two-lane two-way rural roads. Analysis focused on estimating a well-known model, the gamma-generalized queuing model (gamma-GQM). A trendless analysis of observed time headways was also carried out. The endogenous traffic parameters considered as affecting time headway distributions were flow rate and flow composition (percentage of heavy vehicles). Exogenous conditions, such as weather and geometric futures, were common to all time periods and cross-sections analysed.
Gamma-GQM pdf appears to be very suitable for representing real headway distributions in all the analysed situations; it fits real-time headway distributions well, despite flow rate range and traffic composition (range of percentage of heavy vehicles).

Keywords: time headways; trendless analysis; gamma-GQM model.

1. Introduction

Time headway (TH) is the time interval between two vehicles passing a cross-section of a road, measured from front bumper to front bumper. Knowledge of headway distributions plays a significant role in several fields in the context of traffic flow analysis and simulation. In particular, we refer to operative analysis of road facilities in interrupted and uninterrupted flow conditions. Several studies have been published on this topic (Branston, 1976; Griffiths & Hunt, 1991; Luttinen, 1996; Hoogendoorn & Botma, 1997; Hoogendoorn & Bovy, 1998; Ha et al, 2011; Ha et al. 2012; Rossi et al. 2012).

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Researchers' attention has also recently focused on the application of vehicle TH distributions in micro-simulation software (Zhang & Owen, 2004), ITS applications (He, Guan, & Ma, 2009; Li, 2009) and driving simulator experiments with regard to vehicle generation (Rossi et al., 2011).

The Transportation Laboratory of the University of Padova is involved in a research project aimed at establishing an information system of traffic flow phenomena. The plan is to collect and manage traffic data on operations concerning interrupted and uninterrupted traffic flow conditions, in order to specify, calibrate and validate mathematical models of observed phenomena.

The first research step identified typical TH probability density functions (pdf's) for two-way two-lane road segments. Lack of information on time headway distribution in Italian contexts (in particular, two-way two-lane roads) was one of the reasons for the study.

In this first step, attention focused on a set of simple models of TH distribution (Rossi et al., 2012). The next step, reported here, used a well-known model, the gamma-generalized queuing model (gamma-GQM) (Ha et al., 2011; Ha et al., 2012). In this study, a trendless analysis (Luttinen, 1996) on the sample of THs was also carried out.

The endogenous traffic parameters considered to affect TH distributions were flow rate (FR) and flow composition (percentage of heavy vehicles, %HV).

Exogenous conditions (Ha et al., 2011), such as weather and geometric futures, were common to all the time periods and cross-sections analysed.

The data used for statistical analysis of headway came from the traffic monitoring system of the Province of Venice (north-east Italy), since such data are commonly used to characterize the operative conditions of segments (starting from traffic observations on cross-sections) and can be systematically used to analyse traffic flow characteristics and simulate them.

The paper is organized as follows. Section 2 gives a brief description of the Gamma-GQM model; section 3 presents the survey method, together with traffic data characteristics and their analysis; detailed results of the gamma-GQM pdf estimation are given for the four analysed road segment cross-sections, together with the main results. Concluding remarks and future research directions are presented in Section 5.

2. Gamma-GQM model. Theoretical background

Previous studies have proved that one of the best theoretical curves to approximate empirical headway data is the gamma-GQM distribution, which consists of a generalized queuing model with a gamma distribution to describe the "following component".

As explained by Branston (1976), the GQM model can take into consideration "two physical factors that prevent traffic in a single lane from behaving as a random phenomenon": constraints imposed by stability and safety considerations, and those imposed by the lack of opportunities for overtaking.

The first phenomenon is modeled by dividing vehicles in two categories - free-flowing and following - and considering two different distributions for them. This concept is clearly explained by Luttinen (1996), thanks to the analogy with a system composed of one or more servers. Introducing the GQM model, Luttinen explains:

"A queuing model with Poisson arrivals and a general time service distribution is called a M/G/1 queuing system. It produces follower headways (T1) as long as the server is busy. With the probability 1-p the server experiences some idleness during the interdeparture period. The length of an interdeparture time with idleness is the sum of the service time (T1) and the idle time (T2)".

A headway must therefore be considered as composed of a following component – which always participates - and a non-following component – which only participates with a probability of 1-p. This distinction solves problems of stability and safety. Thanks to the convolution theorem for the addition of random variables, the model is expressed by:

$$f(t)=\theta' g(t)+(1-\theta')\lambda' \exp(-\lambda' t)\int_0^t g(x)\exp(\lambda' x)\,dx \tag{1}$$

where:

- $g(t)$ is the pdf of following THs;
- θ' is the proportion of following vehicles;
- λ' is the parameter of the negative exponential distribution of gaps.

The constraint imposed by the lack of overtaking opportunities is taken into account by estimating parameters θ' and λ' directly in the calibration process. They are not therefore defined *a priori*, but are identified by estimation methods. Branston (1976) in fact states that the time headway distribution differs from expression (1) "if the rate at which vehicles overtake on the section of road is not the same as that at which they catch up". The benefit of this consideration is particularly notable in "two-lane two-way" roads, where overtaking is often forbidden or sometimes not possible due to the flow of traffic in the opposite lane.

Many theoretical models have been proposed for function *g(t)*. After comparing many of these, when referring to the goodness-of-fit test, Ha et al. (2011) concluded that the Gamma distribution is the most appropriate choice for TH modeling, which is:

$$g(t)=\frac{\alpha^{\beta} t^{\beta-1}}{\Gamma(\beta)}\exp(-\alpha\, t) \tag{2}$$

where $\Gamma(\beta)$ is the probability distribution function of the Gamma distribution. The above authors also concluded that best results are obtained by not applying location parameter τ to the Gamma distribution.

The probability distribution function of gamma-GQM is written as:

$$F(t)=\theta G(t)+(1-\theta)\int_0^t G(t-u)\,\lambda \exp(-\lambda u)\;du \tag{3}$$

where $G(t)$ is that function.

In the present study, starting from a literature overview, Gamma-GQM pdf was used to represent TH empirical data, although Inverse Weibull pdf (Rossi et al., 2012) was also estimated, for comparative analysis.

3. Method

This study was carried out in order to estimate the pdf's of vehicle THs observed in some cross-sections of rural roads. The traffic data for this study are the sequences of THs obtained at four Automatic Traffic Recorder (ATR) sites along rural roads in the Province of Venice. The road network consists of two-lane roads. Each ATR monitors directional traffic volumes on a single lane.

In the following sections, we describe the case-study in detail, to illustrate the characteristics of the road segments analysed, the data collected, and the methods used to derive information from them (data analysis and results).

The analysis involved three steps:

- direct observation, collection and coding of data at each cross-section, with reference to a certain time interval;
- identification for each section of time sub-intervals characterized by a trendless condition;
- estimation of probability density functions.

3.1. Survey method

The field data used here came from a continuous survey carried out by the Traffic Monitoring System of the Province of Venice. In particular, we refer to ATR loop detector-based recording THs in both traffic directions, together with estimates of the length and speed of vehicles.

3.2. Characteristics of traffic data

Traffic flow observations at a certain point (cross-section) of a road segment are useful in describing the traffic flow characteristics of the entire segment only if we accept the hypothesis that the segment is homogeneous (in geometric and functional terms) and, on the segment and for a certain time interval, that steady traffic conditions exist (constant traffic volume, regardless of the section position along the road segment, and time-independent traffic density). In our case, the functional and geometric characteristics were homogeneous along the segment considered (for at least 1 Km upstream and downstream of the section).

The selected traffic data came from sections belonging to similar road segments, having carriage width ranging from 6.80 to 7.40 meters; these segments are located in flat terrain and are perfectly straights. Traffic data were collected during good weather and dry surface conditions. In this way, exogenous effects (Ha et al., 2012) were assumed not to affect TH distributions significantly.

Tab. 1 shows the time period and duration of the on-field surveys and the main characteristics of traffic data collected for each cross-section examined.

Vehicles longer than 7.5 meters were considered "heavy" vehicles.

Table 1. Duration of traffic surveys and main characteristics of traffic data.

Section code	Date	Time Interval	Duration (hours)	Total Vehicles	Heavy Vehicles (%)	Directional Split (%)
SP074-0117	Sat 29/08/2009	8.00 a.m. - 14.00 p.m.	6.00	9,551	1.4	51/49
SP43-0024	Sat 29/08/2009	8.00 a.m. - 14.00 p.m.	6.00	8,839	3.2	55/45
SP18-0025	Fri 22/03/2013	7.00 a.m. - 19.00 p.m.	12.00	4,517	6.3	48/52
SP81-0022	Thu 21/03/2013	7.00 a.m. - 19.00 p.m.	12.00	13,777	8.1	50/50

Starting from the consideration that TH distributions should depend on flow rate and, for the same flow rate, on flow composition, we selected four road sections with differing flow composition: sections SP074-0117 and SP43-0024 are characterized by a low percentage of HV, and sections SP18-0025 and SP81-0022 by a higher percentage.

The traffic conditions represented cover a range of values from 100 to 1,200 pce/lane/hour (Fig. 3). In some peak periods, meta-stable conditions were observed (volumes close to 1,200 pce/lane/hour). These findings match previous data about this kind of road, where capacity values are usually around 1,400 pce/lane/hour.

3.3. Identification of trendless time intervals and TH sampling

In order to create the samples to be used for pdf estimation, the following assumptions were made:

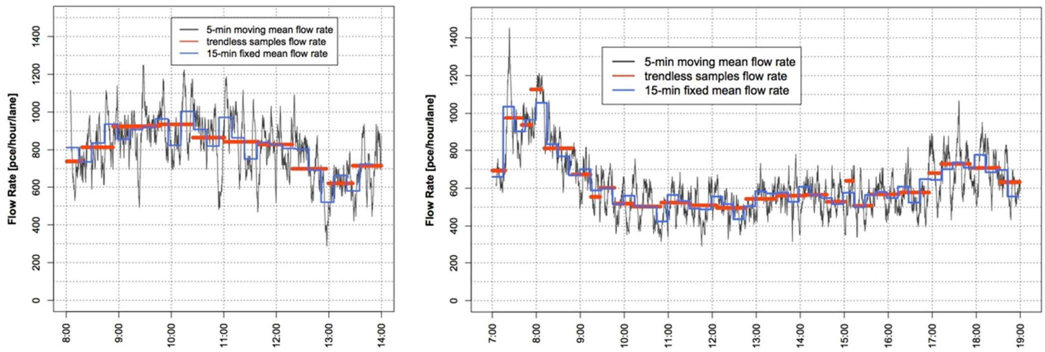
- THs are analysed separately among lanes;
- for each road section, the observation time period is divided into sub-intervals identified by trendless analysis (hypothesis of steady flow conditions);
- the corresponding traffic volumes are converted in pce with an equivalence factor for heavy vehicles of 2.0 pce;

- each sub-interval is classified according to its traffic volume (flow rate in pce) into eight volume ranges: [0-200], (200-400], (400-600], (600-800], (800-1,000] and (1,000-1,200];
- for each section/lane and flow rate range, TH samples were classified according to the %HV, subdivided into three classes: HV1: %HV<=5%, HV2: %HV>5% and <=10%, HV3: %HV>10%) .

Trendless analysis was implemented in R language, following the procedure described by Luttinen (1996). Trend analysis is an essential step in the study of TH, because the real properties of this variable can be inspected only with stationary data. Referring to the calibration process, the distribution parameters must be representative of a specific state: this is why trend analysis which identifies time intervals with the same features must be carried out. In order to obtain stationary data sets, each sample (see Table 1) was submitted to trend analysis by the "exponential ordered scores trend test" proposed by Cox and Lewis (1966) and used by Luttinen (1996). Considering traffic flow as a dynamic process in time and space, and the trend test as a procedure working for a "random process", only data stationary in time can be obtained. The test used by Luttinen was slightly modified here: alterations involved the forward step for evaluating the statistic (1 TH instead of 50) and the extreme stop in the backward run (always 70%). Other conditions were kept, e.g., minimum sample size (100 THs) and sampling period interval (5 to 40 minutes). Following Luttinen (1996), the first observation was discarded if a satisfactory sample was not found.

By way of example, with reference to two of the analysed sections, the charts of Fig. 1 show the flow rate computed with traffic volumes observed in trendless time intervals (red line). The extension of the red segments gives the size of each such interval.

This flow rate profile was compared both with the black line representing flow rate (pce/hour/lane) computed as 5-minute moving means, and with the blue line, which represents the flow rate computed using 15-minute sub-intervals.



(a) (b)

Fig. 1. Identification of trendless time intervals and corresponding flow rate (pce/h). (a) Section SP43-0024, lane 1; (b) Section SP81-0022, lane 1

Fig. 2 shows the total number of trendless time intervals for all sections, classified by range of flow rate and percentage of heavy vehicles (numbers in blue). The black rings represent each trendless time interval in the FR-%HV plane and the red dots trendless time intervals of maximum size within each FR/%HV cell.

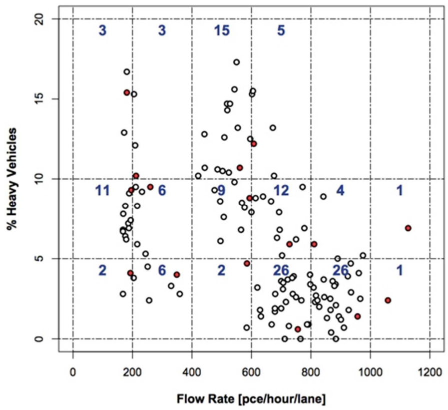


Fig. 2. All sections. Numbers of trendless time intervals by range of FR and %HV

The traffic conditions observed at the four cross-sections during the analysed periods are clearly described by the Flow Rate/Space Mean Speed diagrams of Fig. 3: the set of pairs representing flow rate/space mean speed were estimated both with reference to the trendless time intervals and to 5-minute intervals and then represented in plan *(FR,SMS)*. As a consequence of the second assumption made above, the space mean speed was estimated as the harmonic average of vehicle speeds recorded at the section during the time intervals.

The distribution of points representing the state of the analysed sections shows how the sampled time periods (Tab. 1) cover the whole domain of the flow rate, ranging from free-flow conditions to a value close to the capacity of the road segments (about 1,400 vehicles/lane/hour) at an average speed of 70-90 Km/h.

The samples used to calibrate the TH pdf were then built with the THs belonging to each trendless sub-interval. A total of 132 samples were used in the calibration process (see Fig. 2).

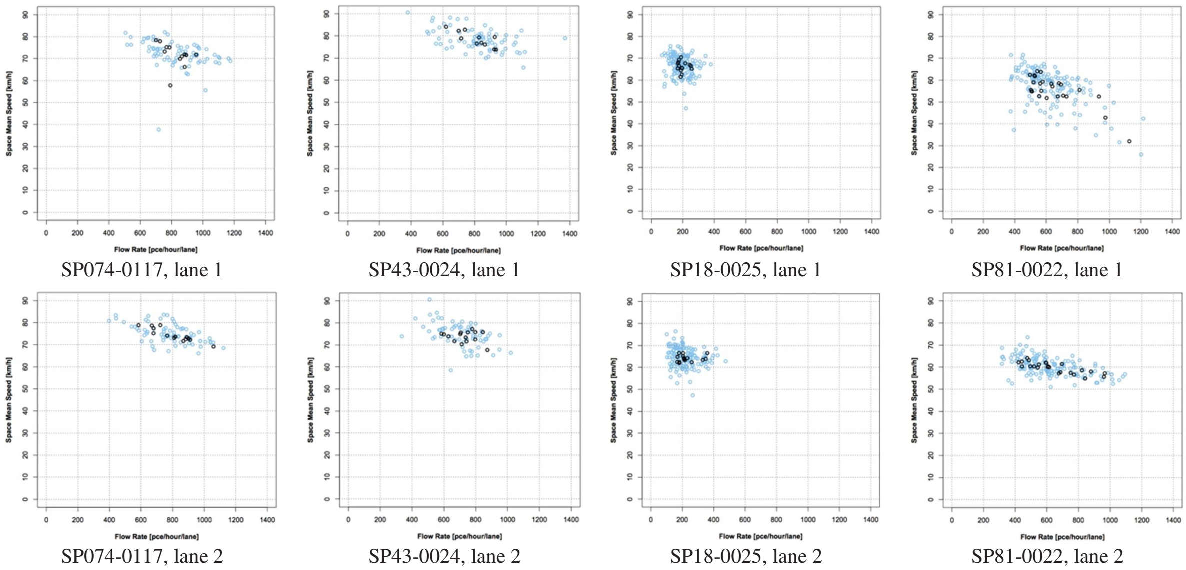


Fig. 3. All sections. Space mean speed/flow rate pairs estimated for each trendless time interval (black rings) and for 5-minute time intervals (blue rings)

3.4. Estimation of gamma-GQM models

One gamma-GQM model for each one of the 132 trendless time intervals was estimated. The gamma-GQM pdf has 4 parameters to be estimated, as follows:

- α and β, respectively scale and form parameters of the gamma distribution;
- λ and θ, parameters of the general GQM distribution.

Ha et al. (2012) used their own three-step calibration process to estimate these parameters. In the present work, the maximum likelihood method was used; the solution was calculated numerically by a minimization process of the log-likelihood function, implemented in R. The solution was approximated until the goodness-of-fit test statistic showed a non-significant variation and applied a different run-time, depending on sample size (i.e. about 4 minutes for a sample size of about 400 elements).The Kolmogorov-Smirnov test statistic was calculated, together with the associated p-value, to evaluate the theoretical model.

For a general measure of the goodness-of-fit of the estimated gamma-GQMs, the p-values calculated with the K-S test statistic were compared with the corresponding p-values calculated for a simple model of TH distribution. In particular, attention focused on the Inverse Weibull pdf, since it appeared to be the most suitable to represent real headway distributions for most of the flow rate ranges analysed in a previous work (Rossi et al, 2012). By way of example, Fig. 4 shows the results of comparative analysis. Each FR/%HV cell refers to the TH sample corresponding to the trendless time interval with maximum size. The p-value of the gamma-GQM model (first row within each cell) is compared with the corresponding p-value estimated for the Inverse Weibull model (second row within each cell). Green characters indicate the p-value of the K-S test statistic greater than 0.05 (level of significance) and red in the opposite case.

The result is clear-cut: the gamma-GQM fits the observed data better than the Inverse Weibull in the majority of conditions analysed. In most cases, the p-values for the gamma-GQM are very high in absolute terms.

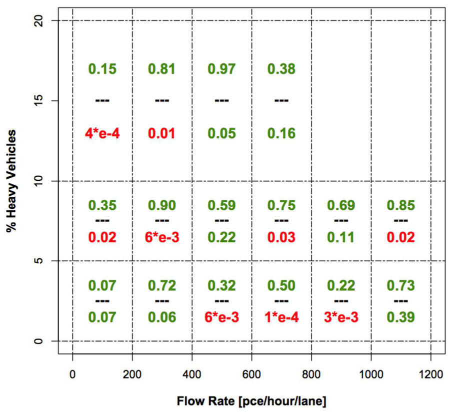
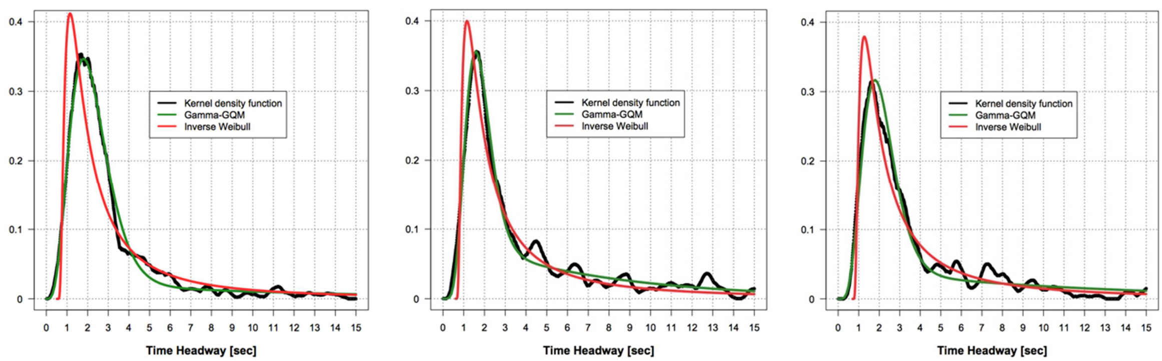


Fig. 4. TH sample corresponding to trendless time interval with maximum size for each FR/%HV cell. P-value calculated for gamma-GQM model (first row within each cell) and Inverse Weibull model (first row within each cell)

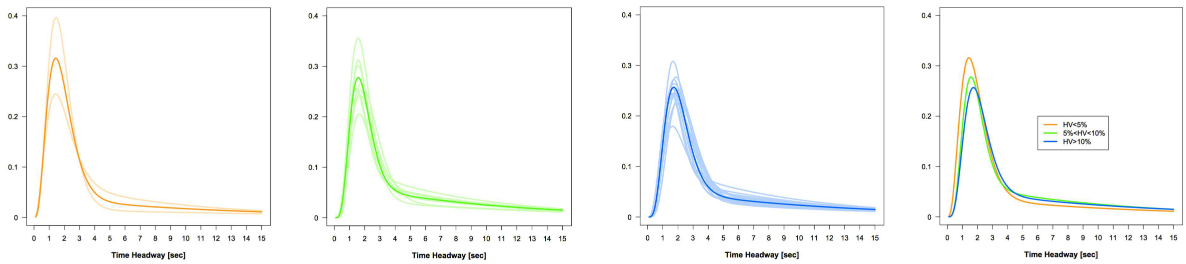
This result is also strengthened from the qualitative point of view when we look at Fig. 5, which compares the gamma-GQM model, Inverse Weibull model and Kernel density estimation of the observed THs. The data refer to TH samples corresponding to the trendless time interval with maximum size for each FR/%HV cell. As an example, only the case of FR belonging to the (600, 800] interval is shown.

For a clear-cut interpretation of results, the gamma-GQM fitted curves are shown in Figs. 6 and 7, respectively for flow rate ranges (400,600] and (600,800]; the thick lines correspond to curves plotted with the average values of the estimated parameters. With reference to each figure, the last diagram compares these curves.



%HV≤5% 5%<%HV≤10% %HV>10%

Fig. 5. TH sample corresponding to trendless time interval with maximum size for each FR/%HV cell, 600≥FR>800. Kernel density estimation, gamma-GQM and Inverse Weibull pdf's depend on percentage of heavy vehicles

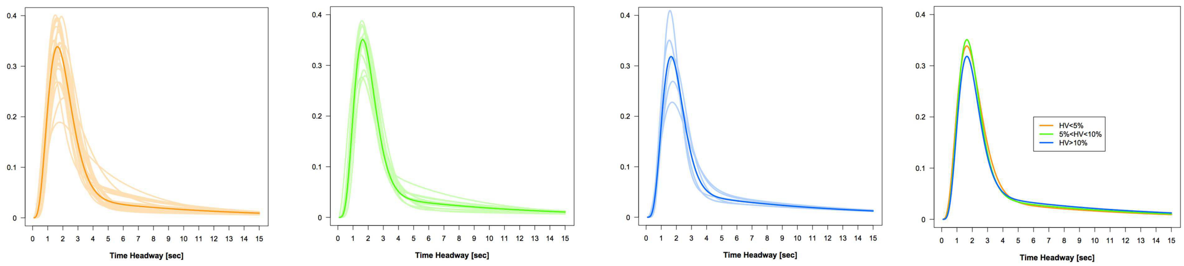


%HV≤5% 5%<%HV≤10% %HV>10% all %HV

Fig. 6. All sections. Flow rate range (400, 600]. Gamma-GQM models estimated by range of percentage of heavy vehicles

Analysis of the estimated pdf's highlights a trend: regardless of flow rate range, the values of the gamma-GQM curves are lower with higher percentages of heavy vehicles.

Another interesting output is the comparison (Fig. 8) among the curves obtained using the average values of the 4 parameters of the Gamma-GMQ computed over the FR classes. The attention has been focused on the extreme classes of HV% (less than 5% and over than 10%). Parameters of the calibrated distributions together with mean, variance and mode are reported in Tab. 1.



%HV≤5% 5%<%HV≤10% %HV>10% all %HV

Fig. 7. All sections. Flow rate range (600, 800]. Gamma-GQM models estimated by range of percentage of heavy vehicles

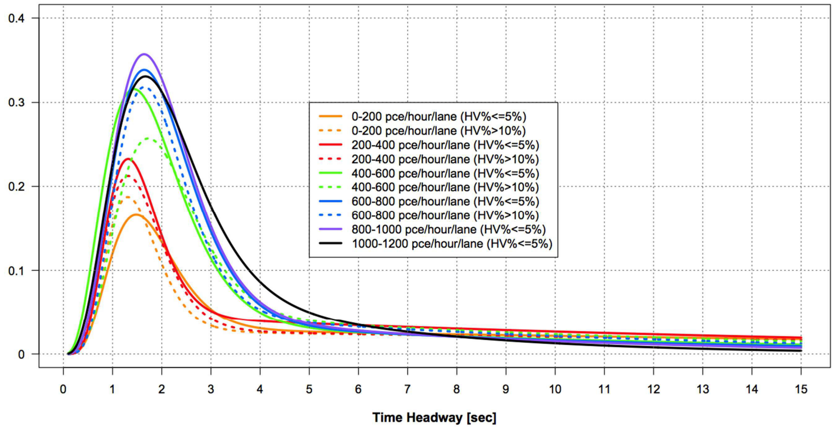


Fig. 8. All sections. All flow rate ranges. Gamma-GQM models estimated by range of percentage of heavy vehicles (only HV%≤5% and HV%>10%)

Also in this case the analysis highlights a trend: as the FR arises the curves tend to move to the right and the corresponding modes became higher.

Looking at the values of mean and variance (Tab. 2), as expected the tendency sounds consistent with the increase of traffic density related to the increase of FR (in the stable branch of the flow-density diagram):
the greater is the density the lower is the mean time headway, the greater is the frequency of headways of similar size due to an increase of vehicles mutual conditioning and the lower is the variance.

Moreover for the same FR class, the function estimated for low percentage of heavy vehicles tends to be greater than the function estimated for high percentage of heavy vehicles. This findings is particularly true for medium to high FR.

Table 2. All sections. All flow rate ranges (pce/hour/lane). Gamma-GQM models parameters by range of percentage of heavy vehicles (only HV%≤5% and HV%>10%).

Conditions		Gamma-GQM parameters						
FR range	HV%	α	β	λ	θ	Mean	Mode	Variance
0-200	≤5	5.93	3.44	0.0399	0.254	20.42	0.17	469.09
0-200	>10	7.20	4.85	0.0355	0.230	23.17	0.19	611.30
200-400	≤5	6.98	4.69	0.0634	0.284	12.78	0.23	178.45
200-400	>10	6.12	3.99	0.0398	0.291	19.35	0.21	447.97
400-600	≤5	4.33	2.38	0.0901	0.597	6.29	0.32	50.41
400-600	>10	5.69	2.81	0.0908	0.469	7.87	0.26	65.13
600-800	≤5	5.26	2.65	0.1130	0.645	5.13	0.34	28.55
600-800	>10	6.33	3.32	0.1054	0.530	6.37	0.32	42.88
800-1,000	≤5	5.34	2.71	0.1454	0.667	4.26	0.36	16.48
1,000-1,200	≤5	4.58	2.25	0.2405	0.652	3.48	0.33	6.92

4. Concluding remarks and future directions

The main aim of the analysis presented here was to identify headway pdf's representing empirical time headway distributions usually observed on cross-sections along two-way two-lane rural roads with different flow rate ranges and different traffic composition (endogenous effects). This aim was directly connected to the need to simulate these phenomena, both in micro-simulation models and driving simulator virtual environments.

At this stage of our research, we report traffic observations on four cross-sections in the rural road network of the Province of Venice having similar geometric features but different traffic compositions.

Exogenous conditions, such as weather and geometric futures, were common to all time periods and cross-sections analysed.

With reference to all the road sections anlaysed, the results allow us to draw the following conclusions:

- Gamma-GQM pdf appears to be highly suitable for representing real headway distributions for all analysed situations; it fits real TH distributions very well, despite various flow rate ranges and traffic compositions (range of percentage of heavy vehicles).

Future research will focus on the following issues:

- extension of field observations to other sections in different contexts (urban and rural areas) in order to generalize results;
- application of results to driving simulator experiments (e.g., gap-acceptance analysis) and micro-simulation models.

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