



Magnetic polarizabilities of light mesons in $SU(3)$ lattice gauge theory

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Received 17 February 2015; received in revised form 16 June 2015; accepted 12 July 2015

Available online 28 July 2015

Editor: Herman Verlinde

Abstract

We investigate the ground state energies of neutral pseudoscalar and vector meson in $SU(3)$ lattice gauge theory in the strong abelian magnetic field. The energy of ρ^0 meson with zero spin projection $s_z = 0$ on the axis of the external magnetic field decreases, while the energies with non-zero spins $s_z = -1$ and $+1$ increase with the field. The energy of π^0 meson decreases as a function of the magnetic field. We calculate the magnetic polarizabilities of pseudoscalar and vector mesons for lattice volume 18^4 . For ρ^0 with spin $|s_z| = 1$ and π^0 meson the polarizabilities in the continuum limit have been evaluated. We do not observe any evidence in favour of tachyonic mode existence.

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<http://dx.doi.org/10.1016/j.nuclphysb.2015.07.023>

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1. Introduction

Quantum Chromodynamics in abelian magnetic field of hadron scale is a rich area for exploration. The investigation of strongly interacting quark–hadron matter in such field has a fundamental significance. Today it is possible to create a strong magnetic field of $15m_\pi^2 \sim 0.27 \text{ GeV}^2$ [1] in terrestrial laboratories (ALICE, RHIC, NICA, FAIR) during non-central heavy ion collisions. Our studies aim at revealing the effects that can appear in such experiments. The fundamental properties of particles related to their internal structure are also very important for understanding these effects.

Let us mention the most famous results concerning QCD physics in the strong magnetic fields. The asymmetry of emitted charged particles [2–4] in non-central collisions of gold ions at RHIC was explained by chiral magnetic effect [4–6]. Many phenomenological studies have been devoted to understanding of QCD phase diagram in the strong magnetic fields. Chiral perturbation theory predicts the decrease of the transition temperature from the confinement to deconfinement phase with increasing abelian magnetic field [7]. External magnetic fields also strengthen the chiral symmetry breaking [8–12]. It was shown in the framework of Nambu–Jona-Lasinio model that QCD vacuum becomes a superconductor in sufficiently strong magnetic field ($B_c = m_\rho^2/e \simeq 10^{16} \text{ Tl}$) along the direction of the magnetic field [13–17]. The transition to the superconducting phase is accompanied by condensation of charged ρ mesons. The strong magnetic fields can also change the order of the phase transition from the confinement to deconfinement phase [18–22].

According to the calculations on the lattice with two types of valence quarks in QCD the critical temperature of the confinement–deconfinement phase transition slightly increases in the strong magnetic field [24]. Simulations in $N_f = 2 + 1$ theory with dynamical quarks showed that T_c decreases with increasing magnetic field [25]. This temperature behaviour was confirmed in [23] and now this effect is known as inverse magnetic catalysis. Lattice simulations with dynamical overlap fermions in two-flavour lattice QCD also showed the decrease of the critical temperature of the confinement–deconfinement transition when the field strength grows [26].

Numerical simulations in QCD with $N_f = 2$ and $N_f = 2 + 1$ indicate that strongly interacting matter in a large magnetic field possesses paramagnetic properties both in the confinement and deconfinement phases [27–29]. Equation of state of quark–gluon plasma was investigated in [30].

Here we continue our previous work where we studied light mesons in $SU(2)$ lattice gauge theory [31]. We extend this analysis to the $SU(3)$ lattice gauge theory which is more physical and calculate the ground state energies of light mesons depending on their spin as a function of the magnetic field value. Our previous results are in a qualitative agreement with the results of this work. We also calculate several hadron characteristics such as the magnetic polarizabilities of light neutral pseudoscalar and vector mesons. The magnetic polarizability is an important physical quantity which reveals the internal structure of a particle in the external magnetic field. We also made the extrapolation to the zero lattice spacing where it was possible. Our approach is numerically expensive so we do not take into account dynamical quarks.

Several articles are devoted to the study of meson masses in the strong magnetic field. The masses of ρ mesons have been calculated according to the relativistic quark–antiquark model in [32]. Lattice study with dynamical quarks [33] agrees with our data for $eB < 1 \text{ GeV}^2$. Phenomenological study was done in [34]. For the case of non-zero spin our results are in a qualitative agreement with the results of [32–34]. For the zero spin our data agree with these results only for small magnetic fields.

2. Details of calculations

For the generation of $SU(3)$ gauge configurations the tadpole improved Lüscher–Weisz action was used

$$S = \beta_{imp} \sum_{pl} S_{pl} - \frac{\beta_{imp}}{20u_0^2} \sum_{rt} S_{rt}, \tag{1}$$

where $S_{pl,rt} = (1/3) \text{Tr}(1 - U_{pl,rt})$ is the lattice plaquette (denoted as pl) or 1×2 rectangular loop (rt), $u_0 = (W_{1 \times 1})^{1/4} = \langle (1/3) \text{Tr} U_{pl} \rangle^{1/4}$ is the tadpole factor, calculated at the zero temperature [35]. This action suppresses ultraviolet dislocations which lead to the appearance of non-physical near zero-modes of Wilson–Dirac operator.

Next, we solve Dirac equation numerically

$$D\psi_k = i\lambda_k\psi_k, \quad D = \gamma^\mu(\partial_\mu - iA_\mu) \tag{2}$$

and find eigenfunctions ψ_k and eigenvalues λ_k for a test quark in the external gauge field A_μ . We find eigenstates of Dirac operator to calculate the correlators. From the correlators we obtain ground state energies. For the calculation of the fermion spectrum we use the Neuberger overlap operator [36]. This operator allows to investigate the theory in the limit of massless quarks without chiral symmetry breaking and can be written in the following form

$$D_{ov} = \frac{\rho}{a} \left(1 + D_W / \sqrt{D_W^\dagger D_W} \right). \tag{3}$$

$D_W = M - \rho/a$ is the Wilson–Dirac operator with a negative value of ρ/a , a is the lattice spacing, M is the Wilson term. Fermion fields obey periodical boundary conditions in space and antiperiodic boundary conditions in time. The sign function

$$D_W / \sqrt{(D_W)^\dagger D_W} = \gamma_5 \text{sign}(H_W), \tag{4}$$

is calculated using the minmax polynomial approximation, where $H_W = \gamma_5 D_W$ is hermitian Wilson–Dirac operator. We investigate the behaviour of the ground energy states of mesons in the background gauge field, which is a superposition of non-abelian $SU(3)$ gluon field and $U(1)$ abelian uniform magnetic field. Abelian gauge fields interact only with quarks. In our calculations we have neglected the contribution of dynamical quarks. Therefore we add the magnetic field only to the overlap Dirac operator. For this reason we use the following ansatz:

$$A_{\mu ij} \rightarrow A_{\mu ij} + A_\mu^B \delta_{ij}, \tag{5}$$

where

$$A_\mu^B(x) = \frac{B}{2} (x_1 \delta_{\mu,2} - x_2 \delta_{\mu,1}). \tag{6}$$

In order to make this substitution consistent with the fermion boundary conditions one should use the twisted boundary conditions [37]. Magnetic field is directed along z axis and its value is quantized

$$qB = \frac{2\pi k}{(aL)^2}, \quad k \in \mathbb{Z}, \tag{7}$$

where $q = -1/3 e$ is the elementary quark charge.

The quantization condition implies that the magnetic field has a minimal value $\sqrt{eB_{\min}} = 380$ MeV for the 18^4 lattice volume and spacing $a = 0.125$ fm. We are far from saturation regime, where $k/(L^2)$ is not small because we use k between 0 and 32. For the inversion of overlap Dirac operator we use Gaussian source (with radius $r = 1.0$ in lattice units in space and time direction) and point receiver (the quark position smoothed with Gaussian profile).

Our simulations have been carried out on symmetrical lattices with the lattice volumes 16^4 , 20^4 , lattice spacing 0.115 fm, 0.125 fm and lattice volume 18^4 , $a = 0.084$ fm, 0.095 fm, 0.105 fm, 0.115 fm, 0.125 fm. We use statistically independent configurations of gluon fields, ~ 150 configurations for the volume 16^4 , ~ 200 and ~ 90 configurations for the lattice volumes 18^4 and 20^4 correspondingly. We consider various bare quark masses in the interval $[0.01, 0.06]$.

3. Observables

We calculate the following observables in the coordinate space and background gauge field A

$$\langle \psi^\dagger(x) O_1 \psi(x) \psi^\dagger(y) O_2 \psi(y) \rangle_A, \quad (8)$$

where $O_1, O_2 = \gamma_5, \gamma_\mu$ are Dirac gamma matrices, $\mu, \nu = 1, \dots, 4$ are Lorenz indices, $x = (\mathbf{n}a, n_t a)$ and $y = (\mathbf{n}'a, n'_t a)$ are the lattice coordinates. The spatial lattice coordinate $\mathbf{n}, \mathbf{n}' \in \Lambda_3 = \{(n_1, n_2, n_3) | n_i = 0, 1, \dots, N - 1\}$, n_t, n'_t are the numbers of lattice sites in the time direction. In the Euclidean space $\psi^\dagger = \bar{\psi}$. In order to find the observables (8) we calculate the quark propagators in coordinate space. For the M lowest eigenstates massive Dirac propagator is represented by the following sum

$$D^{-1}(x, y) = \sum_{k < M} \frac{\psi_k(x) \psi_k^\dagger(y)}{i\lambda_k + m}. \quad (9)$$

We use $M = 50$ lowest eigenstates. For the observables (8) the following equation is fulfilled

$$\begin{aligned} \langle \bar{\psi} O_1 \psi \bar{\psi} O_2 \psi \rangle_A &= -\text{Tr}[O_1 D^{-1}(x, y) O_2 D^{-1}(y, x)] \\ &\quad + \text{Tr}[O_1 D^{-1}(x, x)] \text{Tr}[O_2 D^{-1}(y, y)]. \end{aligned} \quad (10)$$

The first term in (10) is the connected part, the second term is the disconnected part. We have checked that in $SU(3)$ theory without dynamical quarks the disconnected part contribution to correlators is zero. We perform Fourier transformation numerically

$$\tilde{\Phi}(\mathbf{p}, t) = \frac{1}{N^{3/2}} \sum_{\mathbf{n} \in \Lambda_3} \Phi(\mathbf{n}, n_t) e^{-i\mathbf{a}\mathbf{n}\mathbf{p}} \quad (11)$$

The momenta \mathbf{p} has the components $p_i = 2\pi k_i / (aN)$, $k_i = -N/2 + 1, \dots, N/2$. For particles with zero momentum their energy is equal to its mass $E_0 = m_0$. As we are interested in the meson ground state energy, we choose $\langle \mathbf{p} \rangle = 0$. To obtain the masses we expand the correlation function to the exponential series

$$\begin{aligned} \tilde{C}(n_t) &= \langle \psi^\dagger(\mathbf{0}, n_t) O_1 \psi(\mathbf{0}, n_t) \psi^\dagger(\mathbf{0}, 0) O_2 \psi(\mathbf{0}, 0) \rangle_A \\ &= \sum_k \langle 0 | O_1 | k \rangle \langle k | O_2^\dagger | 0 \rangle e^{-n_t a E_k}, \end{aligned} \quad (12)$$

$$\tilde{C}(n_t) = A_0 e^{-n_t a E_0} + A_1 e^{-n_t a E_1} + \dots, \quad (13)$$

A_0, A_1 are constants, E_0 is the ground state energy, E_1 is the energy of the first excited state, a is the lattice spacing. From expansion (13) one can see that for a large n_t the main contribution to

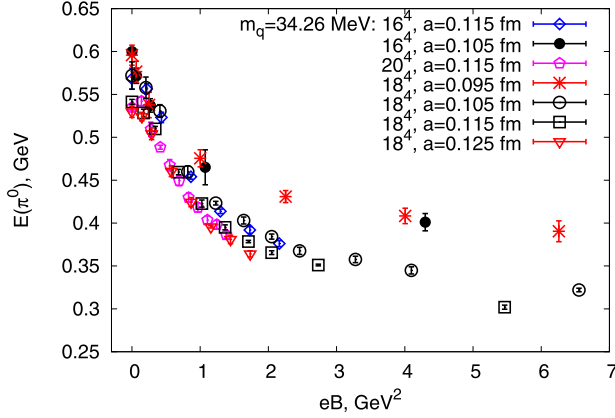


Fig. 1. The energy of the ground state of the pseudoscalar neutral π^0 meson obtained from the cosh fit to the correlator C^{PSPS} as a function of the magnetic field. The data are shown for the bare quark mass $m_q = 34.26$ MeV, lattice volumes 16^4 , 20^4 , lattice spacings 0.115 fm and lattice volume 18^4 , spacings 0.105 fm, 0.115 fm, 0.125 fm.

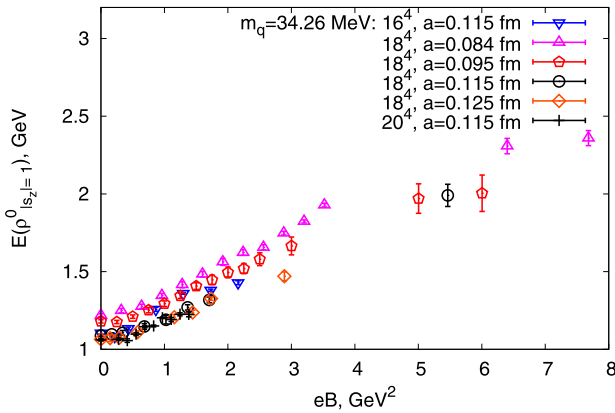


Fig. 2. The energy of the $\rho^0(s_z = \pm 1)$ ground state obtained from the cosh fit to the correlator C^{VV} depending on the magnetic field value. The data are shown for the bare quark mass $m_q = 34.26$ MeV, lattice volumes 16^4 , 20^4 lattice spacing 0.115 fm and lattice volume 18^4 , lattice spacings 0.084 fm, 0.095 fm, 0.115 fm, 0.125 fm.

the correlator comes from the ground state and due to the periodic boundary conditions has the following form

$$\begin{aligned} \tilde{C}_{fit}(n_t) &= A_0 e^{-n_t a E_0} + A_0 e^{-(N_T - n_t) a E_0} \\ &= 2A_0 e^{-N_T a E_0 / 2} \cosh\left(\left(\frac{N_T}{2} - n_t\right) a E_0\right). \end{aligned} \tag{14}$$

The value of the ground state mass can be obtained by fitting the function (14) to the lattice correlator (12). In order to minimize the errors and exclude the contribution of the excited states we take the values of n_t from the interval $6 \leq n_t \leq N_T - 6$. Masses of the ρ meson is obtained from the correlator (8), where $O_1, O_2 = \gamma_\mu$. If $O_1, O_2 = \gamma_5$ we get the pseudoscalar π meson. In our calculations u and d quarks are mass degenerate.

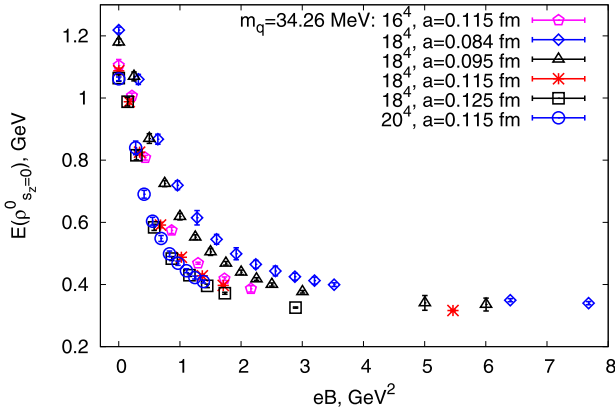


Fig. 3. The energy of the $\rho^0(s_z = 0)$ ground state obtained from the cosh fit to the correlator C^{VV} depending on the magnetic field value. The data are shown for the bare quark mass $m_q = 34.26$ MeV, lattice volumes 16^4 , 20^4 lattice spacing 0.115 fm and lattice volume 18^4 , lattice spacings 0.084 fm, 0.095 fm, 0.115 fm, 0.125 fm.

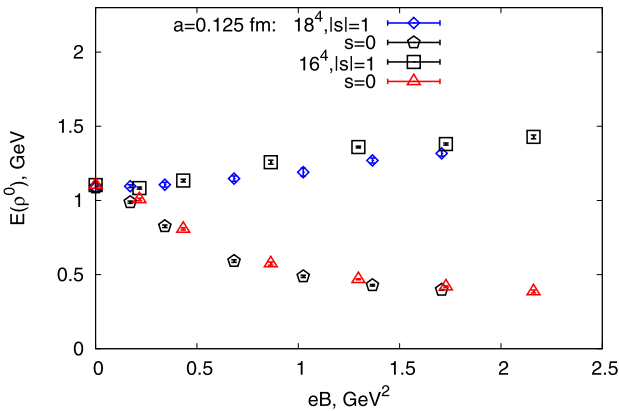


Fig. 4. The energy of the ρ^0 meson for the lattice spacing 0.125 fm, bare quark mass 34.26 MeV, lattice volumes 16^4 and 18^4 for various spin projections on the axis of the magnetic field. The data illustrate lattice volume effects.

4. The ground state energies of mesons in strong magnetic field

Fig. 1 shows the ground state energy of the neutral pion obtained from the correlator $C^{PSPS} = \langle \bar{\psi}(\mathbf{0}, n_t) \gamma_5 \psi(\mathbf{0}, n_t) \bar{\psi}(\mathbf{0}, 0) \gamma_5 \psi(\mathbf{0}, 0) \rangle$. On this plot we present the data for the bare quark mass $m_q = 34.26$ MeV, lattice volumes 18^4 , 16^4 and lattice spacings $a = 0.105$ fm, 0.115 fm, 0.125 fm. The π^0 energy decreases for the all sets of lattice data and slightly depends on the lattice volume and lattice spacing at all values of magnetic field. At large physical volumes the lattice spacing and lattice volume effects are not significant. But for smaller volumes with an increase of the field the lattice effects become more strong, because the wave function of light pion becomes of the order of (or exceeds) the lattice size.

To obtain the energies of neutral vector mesons with various spin projections on the axis of the external magnetic field we use the combinations of the correlators in various spatial dimensions

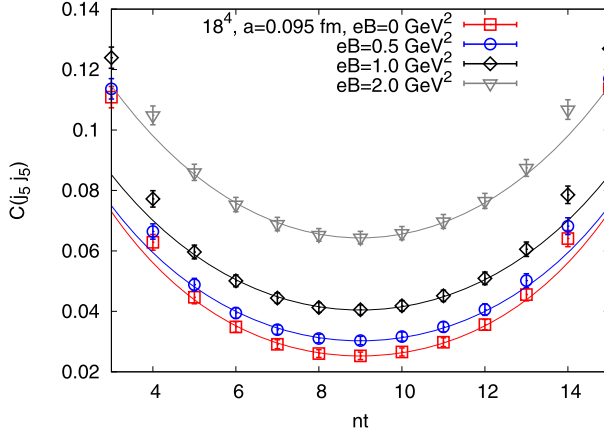


Fig. 5. The correlator of pseudoscalar currents for the lattice spacing 0.095 fm, lattice volume 18^4 and bare quark mass 34.26 MeV for several values of the magnetic field.

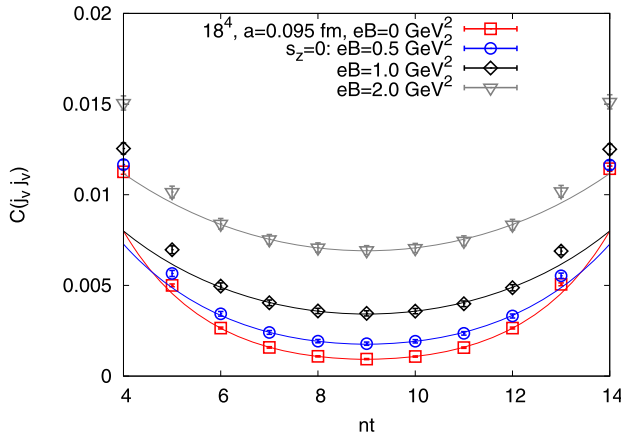


Fig. 6. The correlator of vector currents along ‘z’ direction for non-zero magnetic field and averaged correlator over three spatial directions at $eB = 0$ for the lattice spacing 0.095 fm, lattice volume 18^4 and bare quark mass 34.26 MeV for several values of the magnetic field.

$$C_{xx}^{VV} = \langle \bar{\psi}(\mathbf{0}, n_t) \gamma_1 \psi(\mathbf{0}, n_t) \bar{\psi}(\mathbf{0}, 0) \gamma_1 \psi(\mathbf{0}, 0) \rangle, \tag{15}$$

$$C_{yy}^{VV} = \langle \bar{\psi}(\mathbf{0}, n_t) \gamma_2 \psi(\mathbf{0}, n_t) \bar{\psi}(\mathbf{0}, 0) \gamma_2 \psi(\mathbf{0}, 0) \rangle, \tag{16}$$

$$C_{zz}^{VV} = \langle \bar{\psi}(\mathbf{0}, n_t) \gamma_3 \psi(\mathbf{0}, n_t) \bar{\psi}(\mathbf{0}, 0) \gamma_3 \psi(\mathbf{0}, 0) \rangle. \tag{17}$$

At non-zero magnetic field the mass of ρ^0 meson with the spin projection $s_z = 0$ is obtained from the C_{zz}^{VV} correlator. The combinations of correlators

$$C^{VV}(s_z = \pm 1) = C_{xx}^{VV} + C_{yy}^{VV} \pm i(C_{xy}^{VV} - C_{yx}^{VV}) \tag{18}$$

give the ground state energies of meson with the spins $s_z = +1$ and $s_z = -1$.

Fig. 2 represents the energy of the ρ^0 with non-zero spin projections on the axis of the external magnetic field. The energy increases with the field for the all sets of lattice data. At $|eB| \lesssim$

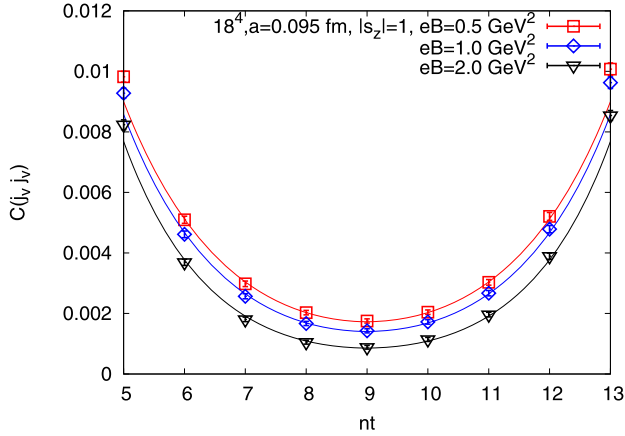


Fig. 7. The combinations of correlators obtained from formula (18) corresponding to the energies of ρ meson with $|s_z| = 1$ for the lattice spacing 0.095 fm, lattice volume 18^4 and bare quark mass 34.26 MeV for several values of the magnetic field.

Table 1

The values of ground state energy of the π^0 meson, its errors and $\chi^2/\text{d.o.f.}$ of cosh fit to the correlator of pseudoscalar currents for the bare quark mass $m_q = 34.26$ MeV, lattice volume 18^4 , lattice spacing $a = 0.095$ fm and several field values.

| eB (GeV ²) | E (GeV) | Error (GeV) | $\chi^2/\text{d.o.f.}$ |
|--------------------------|-----------|-------------|------------------------|
| 0 | 0.596 | 0.012 | 0.058 |
| 0.5 | 0.538 | 0.015 | 0.092 |
| 1.0 | 0.476 | 0.001 | 0.048 |
| 2.0 | 0.408 | 0.009 | 0.047 |

2.5 GeV^2 the energy grows quadratically and at large magnetic fields it is increasing slower. The terms iC_{xy}^{VV} and iC_{yx}^{VV} in (18) are zero for the case of neutral particles, so the ρ^0 masses with $s = -1$ and $s_z = +1$ coincide. This is a manifestation of C -parity. We should note that simple estimates give the following values of magnetic fields corresponding to the lattice spacing cut-off: 2.5 GeV^2 for $a = 0.125$ fm, 2.9 GeV^2 for lattice spacing $a = 0.115$ fm and $eB \sim 3.5 \text{ GeV}^2$ for $a = 0.105$ fm etc.

In Fig. 3 we depict the ground state energy which we obtained from the correlator C_{zz}^{VV} for various lattice spacings and volumes. For the small magnetic fields this energy corresponds to the ρ^0 meson with $s_z = 0$. Also we find the real parts of the non-diagonal terms in correlation matrix equal to zero. At large magnetic fields one cannot neglect the mixing between $\rho^0(s_z = 0)$ and $\pi^0(s = 0)$ because of the large branching of the decay $\rho^0 \rightarrow \pi^0 \gamma$. In this work our main goal was to calculate the polarizabilities of ρ^0 , but not to distinguish ρ^0 and π^0 , this may be a topic for the following work.

In Fig. 4 we show for comparison the energy of ρ^0 meson at $a = 0.125$ fm for 16^4 and 18^4 lattices with various spins. This picture illustrates that the lattice volume effects are not large.

To discuss the quality of the fermion correlators we present the relative plots. In Fig. 5 the correlator of pseudoscalar currents is depicted for several values of the magnetic field for lattice volume 18^4 and lattice spacing 0.095 fm. The solid lines are the fits do these data by formula (14). The fits describe the data very well at $n_t \geq 6$. In Table 1 the values of π^0 energy, its error and

Table 2

The values of ground state energy of the ρ^0 meson with spin projection $s = 0$ and $\chi^2/\text{d.o.f.}$ of cosh fit for the bare quark mass $m_q = 34.26$ MeV, lattice volume 18^4 , lattice spacing $a = 0.095$ fm and several field values.

| eB , (GeV ²) | E (GeV) | Error (GeV) | $\chi^2/\text{d.o.f.}$ |
|----------------------------|-----------|-------------|------------------------|
| 0 | 1.179 | 0.008 | 0.174 |
| 0.5 | 0.870 | 0.016 | 0.209 |
| 1.0 | 0.618 | 0.010 | 0.081 |
| 2.0 | 0.439 | 0.007 | 0.027 |

Table 3

The values of ground state energy of the ρ^0 meson with spin projections $s = \pm 1$ and $\chi^2/\text{d.o.f.}$ of cosh fit for the bare quark mass $m_q = 34.26$ MeV, lattice volume 18^4 , lattice spacing $a = 0.095$ fm and several field values.

| eB , (GeV ²) | E (GeV) | Error (GeV) | $\chi^2/\text{d.o.f.}$ |
|----------------------------|-----------|-------------|------------------------|
| 0 | 1.179 | 0.008 | 0.174 |
| 0.5 | 1.210 | 0.011 | 0.227 |
| 1.0 | 1.294 | 0.018 | 0.503 |
| 2.0 | 1.495 | 0.035 | 1.145 |

$\chi^2/\text{d.o.f.}$ of the fits are shown. In Fig. 6 we present the correlator of vector currents along the axis of the external magnetic field for non-zero eB . At zero magnetic field we average the correlator over x , y and z because they are equivalent, this diminishes the errors in the determination of ρ^0 meson mass. Fig. 7 shows the combinations of correlators obtained from formula (18) corresponding to the energies of ρ^0 meson with $|s_z| = 1$ for the lattice spacing 0.095 fm and lattice volume 18^4 . The parameters of the fits are collected in Tables 2 and 3.

In Fig. 8 we show the ρ^0 energy averaged over three spin components $E(\rho^0) = (E(\rho_{s_z=0}^0) + E(\rho_{s_z=-1}^0) + E(\rho_{s_z=+1}^0))/3$ corresponding to the energy of unpolarized vector meson, we observe it is approximately a constant value. For the magnetic fields $eB < 2.5$ GeV² the meson energy may deviates from a constant value because of the lattice spacing effects. With the diminishing of the lattice spacing the energy of unpolarized meson becomes closer to a constant value, so it confirms the supposition that a mixing between $\rho^0(s_z = 0)$ and $\pi^0(s = 0)$ states may be weak at $eB < 2.5$ GeV².

From the observation of constant energy of unpolarized meson and the behavior of non-zero spin components (Fig. 2) one can draw some conclusions that there is no tachyonic mode for the explored range of the magnetic fields, i.e. the energy of $\rho^0(s = 0)$ doesn't turn to zero. We see the lattice volume effects are small and do not change this conclusion at large magnetic fields. The decrease of the energy may be compensated by higher powers of eB for its values larger than 1 GeV or so. These effects can be preventing, in particular, from energy turning to zero and possible emergence of tachyonic mode. The same will be true also for the energies of all spin states of charged ρ mesons [38]. Still, the case of neutral mesons demands further investigation and study of mixing and lattice spacing effects.

Therefore our calculations show that there is the splitting of ground state energy of neutral vector meson in a strong abelian magnetic field which represents an interesting physical effect.

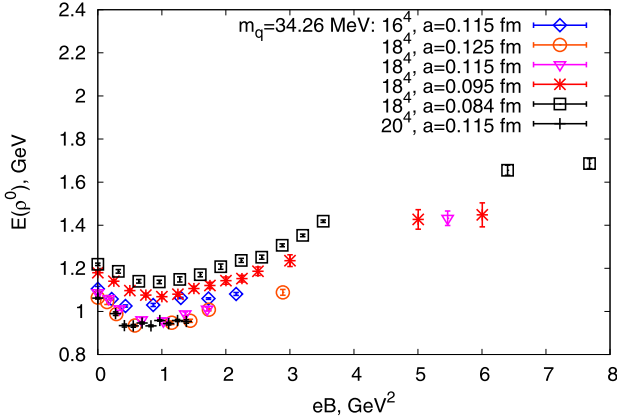


Fig. 8. The energy of the ground state of unpolarized vector neutral ρ^0 meson obtained from the cosh fits to the correlators as a function of the magnetic field. The data are shown for lattice volumes 16^4 , 20^4 lattice spacing 0.115 fm, lattice volume 18^4 , lattice spacings 0.084 fm, 0.095 fm, 0.115 fm, 0.125 fm and the bare quark mass $m_q = 34.26$ MeV.

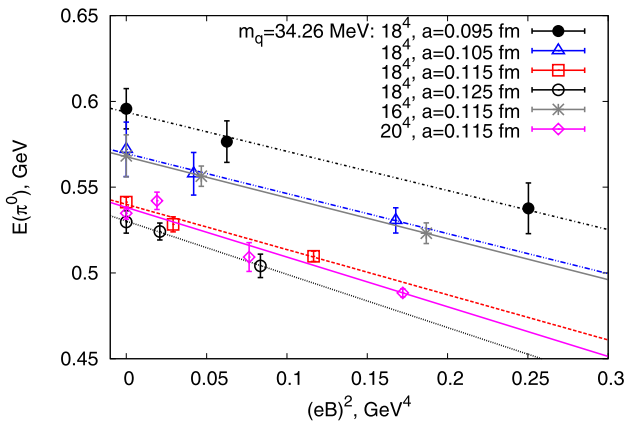


Fig. 9. The energy of the ground state of π^0 meson as a function of the squared magnetic field. The data are shown for the bare quark mass $m_q = 34.26$ MeV, lattice volumes 16^4 , 20^4 , lattice spacing 0.115 fm and lattice volume 18^4 , lattice spacings 0.095 fm, 0.105 fm, 0.115 fm, 0.125 fm.

5. Magnetic polarizabilities

The polarizability of meson is an important physical quantity for the understanding of its internal structure. The magnetic polarizability shows how the distribution of fermion currents responds to the external magnetic field. In this section we discuss the magnetic polarizabilities of pseudoscalar π^0 and vector ρ^0 mesons.

In Fig. 9 we show the ground state energy of pion as a function of squared magnetic field $(eB)^2$ for the bare quark mass 34.26 MeV, various lattice volumes and spacings in a large scale at small magnetic fields. We fit the data at $(eB)^2 \in [0, 0.3 \text{ GeV}^4]$ by the function

$$E = E(B = 0) - 2\pi\beta_m(eB)^2, \quad (19)$$

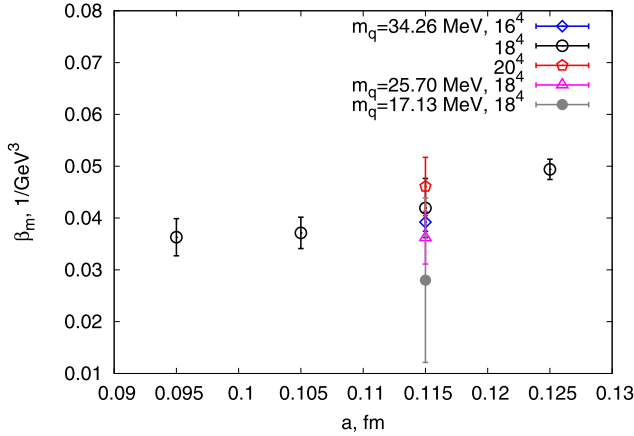


Fig. 10. The magnetic polarizability of π^0 meson versus the lattice spacing for the lattice volumes 16^4 , 18^4 , 20^4 and various bare quark mass.

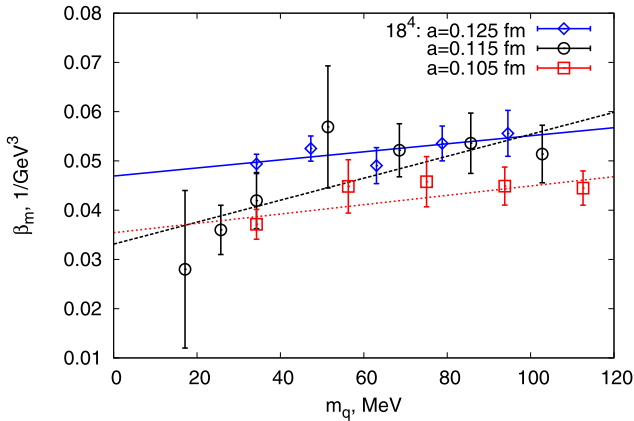


Fig. 11. The magnetic polarizability of pseudoscalar neutral meson π^0 versus the bare lattice quark mass for lattice volume 18^4 and various lattice spacings.

where we use “natural” units $\hbar = c = 1$, but $e^2 = 1/137$ in Gaussian units. $E(B = 0)$ and β_m are the parameters which we find from the fit. We choose this interval for the fit because the term $\sim (eB)^4$ gives a small contribution to the pion energy at such magnetic fields.

The dashed–dotted line is for the lattice volume 18^4 and lattice spacing $a = 0.095$ fm, the solid lines corresponds to the lattice volumes 16^4 , 20^4 and lattice spacing $a = 0.115$ fm, the dashed line is for 18^4 and $a = 0.115$ fm and dotted one corresponds to the case of $a = 0.125$ fm.

The polarizabilities obtained for various lattices are summarized in Table 4. We do not observe any functional dependence of the magnetic polarizability on the lattice spacing, the results are presented in Fig. 10. However there is a dependence of the magnetic polarizability on the quark mass. Fig. 11 shows that the value of the magnetic polarizability decreases with the quark mass. The magnetic polarizability $(2.14 \pm 1.22) \cdot 10^{-4} \text{ fm}^3$ calculated at the lowest quark mass $m_q = 17.13$ MeV and lattice spacing $a = 0.115$ fm agrees with the prediction of the chiral perturbation theory $(1.5 \pm 0.3) \cdot 10^{-4} \text{ fm}^3$ [39]. The precise determination of $\beta(\pi^0)$ value requires a careful

Table 4

The values of magnetic polarizability of the π^0 meson for the various quark masses, lattice volumes 18^4 and 20^4 and various lattice spacings.

| V_{latt} | m_q (MeV) | a (fm) | β_m (GeV^{-3}) | Error (GeV^{-3}) | $\chi^2/\text{d.o.f.}$ |
|------------|-------------|----------|---------------------------------|-----------------------------|------------------------|
| 18^4 | 34.26 | 0.095 | 0.036 | 0.004 | 0.092 |
| 18^4 | 34.26 | 0.105 | 0.037 | 0.003 | 0.050 |
| 18^4 | 34.26 | 0.115 | 0.042 | 0.006 | 1.034 |
| 18^4 | 34.26 | 0.125 | 0.049 | 0.002 | 0.0131 |
| 18^4 | 25.70 | 0.115 | 0.036 | 0.005 | 1.105 |
| 18^4 | 17.13 | 0.115 | 0.028 | 0.016 | 1.552 |
| 20^4 | 34.26 | 0.115 | 0.046 | 0.006 | 2.502 |

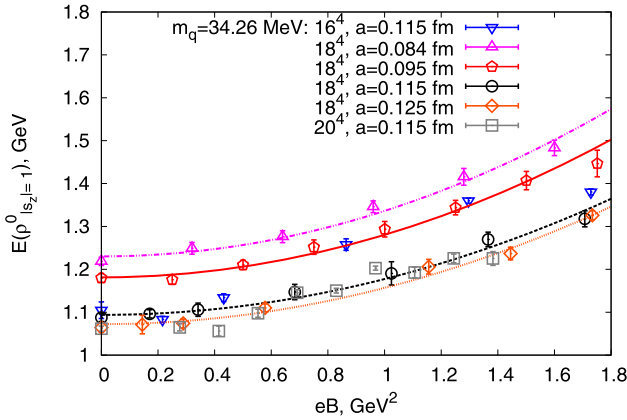


Fig. 12. The ρ^0 meson with the fits $E = E_0(B=0) - 2\pi\beta_m^{|s|=1}(eB)^2$ to the data on the interval $eB \in [0, 1.8 \text{ GeV}^2]$. All the fits correspond to the 18^4 lattice, the dashed-dotted is for the lattice spacing 0.084 fm, the solid line corresponds to the $a = 0.095$ fm, the dashed one is for the $a = 0.115$ fm and dotted line corresponds to the case of $a = 0.125$ fm.

study of its dependence on the quark mass, large statistics and lattice volumes and undoubtedly deserves an attention. At $m_q = 17.13$ MeV the pion mass is still sufficiently high and equals to 396 MeV.

In Fig. 12 in the large scale we depict the energy of the ρ^0 ground state with non-zero spin depending on the magnetic field for the bare quark mass $m_q = 34.26$ MeV, lattice volumes 16^4 , 18^4 , 20^4 and various lattice spacings. To obtain the magnetic polarizability we fit our data to the function $E = E(B=0) - 2\pi\beta_m^{|s|=1}(eB)^2$ at magnetic fields $0 \leq eB < 1.8 \text{ GeV}^2$, where the data are well described by quadratic law. $E(B=0)$ and $\beta_m^{|s|=1}$ are the unknown parameters which were found from the fitting procedure. The values of $\beta_m^{|s|=1}$ for the ρ^0 meson with the spin $|s_z| = 1$, the errors and lattice parameters are summarized in Table 5 and shown in Fig. 13.

We see the strong dependence of the results on the lattice spacing so we perform an extrapolation to the continuum limit. The extrapolation gives the polarizability value $\beta_m^{|s|=1}(\rho^0) = (-0.0235 \pm 0.0023) \text{ GeV}^{-3}$ for the lattice volume 18^4 and bare quark mass $m_q = 34$ MeV. The magnetic polarizability does not depend on the sign of the ρ^0 spin projection, so it has a scalar nature.

In Fig. 14 the mass of ρ^0 meson with zero spin is depicted for the small magnetic fields. We observe the linear in $(eB)^2$ behaviour only for $(eB)^2 \in [0, 0.1 \text{ GeV}^4]$ and get the value

Table 5

The values of magnetic polarizability of the vector ρ^0 meson with non-zero spin for the bare quark mass $m_q = 34.26$ MeV, lattice volume 18^4 and various lattice spacings. The last two rows correspond to the polarizability after the chiral extrapolation.

| V_{latt} | a (fm) | $\beta_m^{mq} = 34 \text{ MeV (GeV}^{-3}\text{)}$ | Error (GeV $^{-3}$) | $\chi^2/\text{d.o.f.}$ |
|---|---------------|---|----------------------|------------------------|
| 18^4 | 0.084 | -0.0169 | 0.0012 | 1.235 |
| 18^4 | 0.095 | -0.0158 | 0.0009 | 0.730 |
| 18^4 | 0.115 | -0.0133 | 0.0008 | 0.754 |
| 18^4 | 0.125 | -0.0135 | 0.0006 | 0.832 |
| 18^4 | $a = 0$ extr. | -0.0235 | 0.0023 | 0.561 |
| $\beta_m^{ch.extr} \text{ (GeV}^{-3}\text{)}$ | | | | |
| 18^4 | 0.115 | -0.0138 | 0.0005 | 2.648 |
| 18^4 | 0.125 | -0.0161 | 0.0025 | 23.862 |

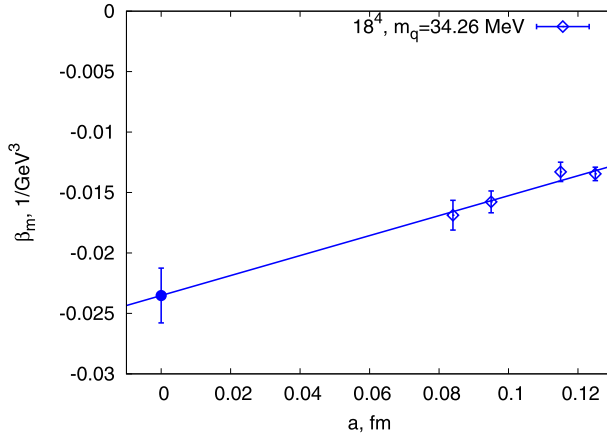


Fig. 13. The magnetic polarizability of ρ^0 meson and its extrapolation by a linear function to zero lattice spacing for the lattice volume 18^4 and bare quark mass $m_q = 34.26$ MeV.

$\beta_m^{s=0}(\rho^0) = (0.47 \pm 0.03) \text{ GeV}^{-3}$ for the lattice 18^4 and spacing $a = 0.125$ fm. It is approximately in 35 times larger and opposite in sign compared to the magnetic polarizability for non-zero spin case $\beta_m^{|s|=1}(\rho^0) = (-0.0135 \pm 0.0006) \text{ GeV}^{-3}$. Unfortunately we cannot consider smaller fields to study effects of lattice spacing.

From the sign of the magnetic polarizability we can conclude about the behaviour of the meson wave function depending on its spin and quantum numbers. Since $\beta > 0$ the magnetic field extends the wave function of ρ^0 meson with zero spin. In the case of non-zero spin the magnetic field squeezes it and the magnetic polarizability has a negative value.

The small value of the magnetic polarizability is not surprising, because we have to apply a very strong magnetic field to observe the response of the internal structure of neutral mesons composed of charged quarks having spin.

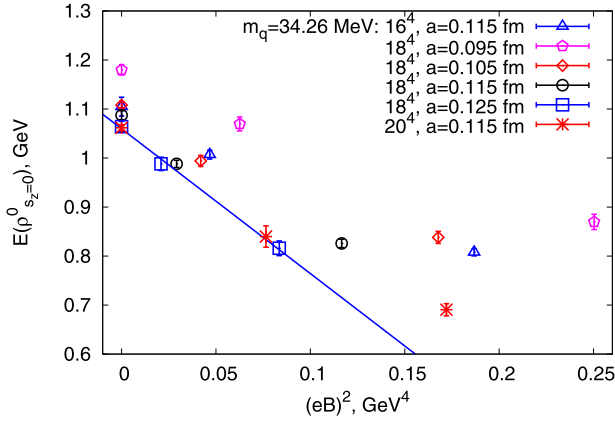


Fig. 14. The energy of the ground state of the vector ρ^0 meson with $s_z = 0$ depending on the squared magnetic field value. The data are shown for the bare quark mass $m_q = 34.26$ MeV, lattice volumes 16^4 , 20^4 , lattice spacing 0.115 fm and lattice volume 18^4 , lattice spacings 0.095 fm, 0.105 fm, 0.115 fm, 0.125 fm.

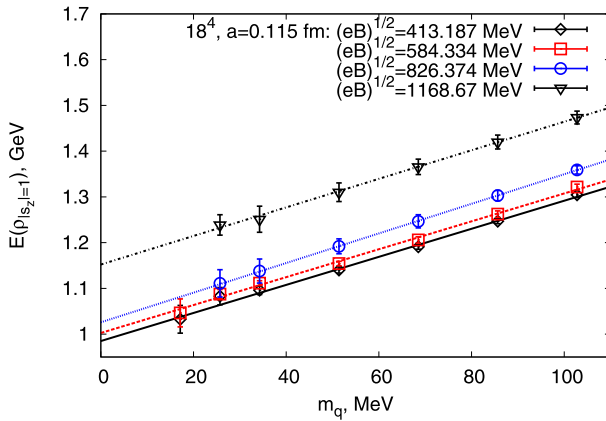


Fig. 15. The ground state energy of the neutral vector ρ meson spin $|s_z| = 1$ for lattice volume 18^4 , lattice spacing $a = 0.115$ fm, various quark masses and several values of magnetic fields. The extrapolation was done by the fit (20) to the chiral limit.

6. Quark mass extrapolations

In Fig. 15 we show the ρ^0 meson with the spin $|s_z| = 1$ as a function of quark mass for lattice volume 18^4 , lattice spacing $a = 0.115$ fm and various magnetic fields. The mass of ρ^0 meson was calculated for several m_q values in the interval $m_q a \in [0.01, 0.06]$. Calculations at small quark mass requires large statistics are computational expensive, so we carry out the chiral extrapolation. Then we perform a fit by a linear function

$$m_\rho = a_0 + a_1 m_q \tag{20}$$

and find the coefficient a_0 and a_1 with errors by χ^2 method. Then we extrapolate $m_\rho(m_q)$ to the limit of zero quark mass $m_q = 0$, so-called chiral limit. The result of this extrapolation is presented in Fig. 16 for the lattice volume 18^4 and lattice spacings $a = 0.115$ fm, 0.125 fm.

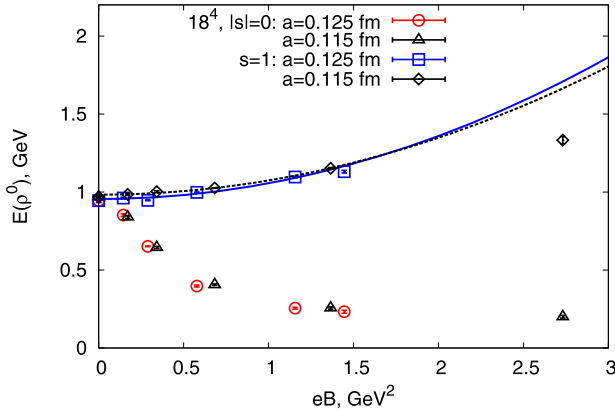


Fig. 16. The mass dependence of the neutral vector ρ^0 meson with various spins on the value of external magnetic field for the lattice volume 18^4 and lattice spacings $a = 0.115$ fm, 0.125 fm after chiral extrapolation. The solid curve is for the fit to the data at 0.125 fm lattice spacing and the dashed curve corresponds to the data at 0.115 fm.

The masses of ρ^0 with zero spin smoothly decrease with magnetic field while the energies with non-zero spin increase with the field value. In Table 5 in the last two rows we show the values of magnetic polarizability obtained from the fits to the data after the chiral extrapolation. The values of $\beta_m^{|s|=1}$ after the extrapolation agree with the values of $\beta_m^{|s|=1}$ obtained for the bare quark mass $m_q = 34$ MeV within the errors for $a = 0.115$ fm. For $a = 0.125$ fm the agreement is not so good, because the extrapolation needs rich data.

7. Conclusions

In this work we explore the ground state energies of π^0 and ρ^0 mesons. The mass of pseudoscalar meson diminishes with the magnetic field value. We obtain the magnetic polarizability of π^0 meson. For the smallest bare quark mass 17 MeV it equals to (0.028 ± 0.016) GeV^{-3} for the lattice spacing $a = 0.115$ fm and agrees with the prediction of chiral perturbation theory.

We observe the splitting of ground state energy of the neutral vector ρ meson depending on its spin projection on the axis of the external magnetic field. The energies of ρ^0 with the spins $s_z = +1$ and $s_z = -1$ coincide and increase with the magnetic field value. The magnetic polarizability at non-zero spin is equal to $\beta_m^{|s|=1}(\rho^0) = (-0.0235 \pm 0.0023)$ GeV^{-3} after extrapolation to zero lattice spacing.

The energy of ρ^0 with the spin $s_z = 0$ decreases with the field value. The magnetic polarizability at $s_z = 0$ differs at least in sign from the case of non-zero spin $|s_z| = 1$. We consider this phenomena to be the result of the anisotropy created by the magnetic field. The coincidence of vector meson energies with spins $s_z = +1$ and $s_z = -1$ is the consequence of C-parity.

We suppose the mixing between π^0 and $\rho^0(s = 0)$ states is not strong at $eB < 2$ GeV^2 , this is the subject for the further detailed investigation.

We do not observe any evidence in favour of tachyonic mode existence.

Acknowledgements

The authors are grateful to FAIR-ITEP supercomputer center where these numerical calculations were performed. This work was carried out with the financial support of Grant of President

MK-6264.2014.2 and FAIR-Russia Research Centre. E.V. Luschevskaya is partially supported by RFBR grants 14-02-00395-a and 15-32-21117. O.T. is supported by RFBR grant 14-01-00647 and in part by Heisenberg–Landau Program. O.S. is partially supported by Grant of President MD-3215.2014.2 and RFBR grant 15-02-07596, O.K. is partially supported by RFBR grant 13-02-01387-a.

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