Mechanistic Images in Geometric Form: Heinrich Hertz’s *Principles of Mechanics*

Heinrich Hertz published one book during his short life of not quite 37 years; a second one appeared a few months after his death in 1894. The first of these books collected his papers on electrodynamics [Hertz, 1962]. It was accorded a formidably detailed analysis in Jed Buchwald’s *The Creation of Scientific Effects* [Buchwald, 1994]. Buchwald reconstructed just how Hertz took a known phenomenon and a familiar device, gradually transforming it into a detector for a novel effect. This topological transformation, literally a spatial unfolding and material analysis of the Rieß spirals, is exemplary for an instrumental investigation *sui generis* from which emerged an experimental confirmation of Maxwell’s theory.

Jesper Lützen has matched Buchwald’s achievement by analyzing with equal care Hertz’s second book, the *Principles of Mechanics Presented in a New Form*. The new form or context (the German word is “Zusammenhang”) is a differential geometry of systems of points. Lützen shows how this choice of form allowed Hertz to construct a force-free mechanics from the ground up and that, therefore, it is not the elimination of force but the geometrization of mechanics that proved to be Hertz’s most lasting contribution (pp. 159, 217, 288). While Buchwald reconstructs how in his electrodynamic researches Hertz took apart and thus articulated a material device in order to achieve a novel effect [Buchwald, 1994, 219–239], Lützen reconstructs how the need “to derive the correct line element for his basic geometry of configuration space” led Hertz to adopt a particular image of matter (pp. 146–153). He thus shows that Hertz used a geometry of systems of points as a conceptual device that would yield the novel effect of giving a logically purified expression to a forceless mechanics. And just as Hertz’s treatment of the familiar Rieß spiral had no precedent, so it appears that he developed his geometry quite independent even of the mathematicians mentioned in his preface—though these questions of influence by Beltrami, Lipschitz, and Darboux cannot be settled definitively even after Lützen’s consultation of Hertz’s notes and drafts (pp. 261–262, 160).

If this is the core of Lützen’s argument, there is much that is illuminating as he works his way towards and beyond it. Indeed, one could scarcely ask for a more detailed exposition of the *Principles of Mechanics*, a more circumspect chapter-by-chapter development of its main ideas, or a better balancing of contextual considerations and technical detail. It begins by elaborating the physical background regarding principles of mechanics, the mechanization of physics, and the problem of “force.” It then tells how Heinrich Hertz entered the scene by giving a biographical account that focuses on his road to mechanics. From Hertz’s philosophical introduction with its notion of “images of mechanics” that need to be judged according to the criteria of permissibility, correctness, and appropriateness (distinctness and simplicity), Lützen goes on to offer a cautious assessment of Hertz’s apparent Kantianism, especially in the division of *a priori* kinematics and experience-based dynamics. All this prepares the ground for a systematic reconstruction in 14 chapters of Hertz’s mechanics in geometric form: After introducing the three basic concepts of time, space, and mass, Lützen establishes the line element from which Hertz generates his conceptual architecture as well as his geometry that finally allows him to formulate the single fundamental law of mechanics, namely that “every free system persists in its state of rest or of uniform motion in a straightest path” [Hertz, 1956, Sect. 309]. He goes on to further explicate this law. Free and unfree systems are distinguished. Hertz’s definition of force as a Lagrange multiplier is introduced, as well as his conception of concealed masses: Physically and metaphysically less problematic than “forces,” the assumption of hidden “adiabatic cyclical systems” completes his picture of mechanics. Hertz’s coinage of “holonomic” and his consideration of nonholonomic constraints are related to the work of his precursors. Lützen then points to an aspect of Hertz’s mechanics that so far had been appreciated only by a few of his contemporaries, namely Hertz’s “sharp denunciation” (p. 243) of Hamilton’s principle by declaring it invalid for nonholonomic constraints. After this survey of the book’s technical content, Lützen turns to Hertz’s reflections on the applicability of his mechanics and concludes with a review of its reception by physicists and philosophers.

When, shortly before his death, Hertz negotiated his contract for the book, he encouraged his publisher to include as potential buyers “the circle of philosophical readers” (quoted in Fölsing, 1997, 509). Jesper Lützen similarly expresses the hope “that philosophers also will find some of the content of this book interesting” (p. vi). Of course, by treating “all aspects of Hertz’s mechanics be they mathematical, physical or philosophical” (p. 5), he overtaxes any reviewer

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1 Thus grounding his dismissal of the “energetic image” of mechanics (see pp. 117, 77, 240).
who cannot judge all these aspects. Such a reviewer can do no better than note some of the book’s accomplishments and, in this case, also a curious blind spot for the epistemological aspect of Hertz’s mechanics.

One major accomplishment consists in elucidating the relation of mathematics and physics. According to Lützen, it does not consist in the application of mathematics but in that “the mathematical formalism suggests entirely new scientific concepts and ways of thinking about the natural phenomenon,” where such new ways of thinking also affect the further development of mathematics (p. 185). This is borne out in the case of Hertz, who begins with a disdain for pure mathematics and the conviction that “the entire new mathematics” is of “no great value to the physicist” in that its “suprasensible” abstractions no longer have “anything in common with reality” (in a letter to his parents quoted by Lützen, p. 130). In the highly mathematized Principles of Mechanics Hertz repeatedly insists that mathematical form and physical content are and should be independent of each other (pp. 160, 247). Lützen finds repeatedly that Hertz foregoes greater mathematical simplicity in order to remain faithful to physical intuitions (see, for example, pp. 90, 139, 154–155). But Hertz also asserts that they are “so suited that they mutually assist one another” [Hertz, 1956, 29]. Lützen elucidates this:

Hertz particularly emphasized that the “essential characteristic of the terminology (the geometry of systems of points) consists in this, that instead of always starting from single points, it from the beginning conceives and considers whole systems of points” [Hertz, 1956, 29]. Something similar can be said about the physical content of Hertz’s mechanics. Indeed, in a presentation of a mechanics without forces, there would be no point in starting with a chapter on the motion of one point mass, as is and was the traditional way to start a mechanics textbook. […] when there are no forces, there is nothing one can say about the motion of one point. (p. 161)

When Hertz takes as his unit of analysis a chosen system of points, this corresponds to certain physical intuitions but at the same time recommends itself for its epistemological virtues.

As opposed to most Hertz scholars before him, Lützen clearly recognizes that Hertz is not a mere conventionalist who is engaged in a comparison of images for their parsimony alone. Instead, Hertz’s “major push for the geometric formalism came from the unique physical content of his image of mechanics” (p. 160) that renders his image of the world different from the Newtonian and energeticist images (p. 261; see pp. 117–118, 234, and also Nordmann, 1998, Sect. 3). By substituting connections for forces, Hertz eliminates time in favor of purely geometrical properties (p. 188; see pp. 191, 199), and along with force he thus eliminates from physics the notions of causality, intentionality, and the like.2 This and the formulation of the fundamental law allow Hertz to treat states of rest as basic or absolute physical states even of systems that in other frames of reference are said to be in motion (p. 113). Depending on our explanatory purposes, we can choose systems without attending to the fact that they are made up of smaller systems, yet allowing us at all times to conceive them as coupled systems and thus as a resultant of forces (compare pp. 140, 154, 224). Thus, Hertz’s program to eliminate distance forces not only in electrodynamics but also in mechanics (pp. 72–73) does not preclude him from considering an object on earth as “being connected to an object on a far-away star” (p. 187). The motions of the solar system are therefore best explained neither in terms of forces acting at a distance nor as resultants of the propagation of innumerable actions among smallest immediately adjoining particles, but as a relatively simple system of connected points. To be sure, such connections would be approximately rigid only:

If a mechanical system, such as a machine is investigated in detail, it will always turn out that the assumed rigid connections between its macroscopic parts are only approximately rigid. “We are compelled to seek the ultimate connections in the world of atoms, and they are unknown to us.” [Hertz, 1956, Sect. 330] (p. 264).

From statements like these, Lützen tends to attribute to Hertz the view that one has to ultimately discover the true story of the really rigid connections. Similarly, he sees in Hertz’s rejection of non-Euclidean geometries the postulate that they cannot correspond to a physical conception of sensible space (p. 130). And most generally, he attributes to Hertz a thoroughgoing reductionism that considers all of “nature as one connected mechanical system” (p. 5; see pp. 30, 38–39, 191).

However, all of these strong metaphysical and ultimately unfounded assumptions can and perhaps should be read in epistemological terms and as cognizant of the limits of physics. Instead of an a priori commitment to Euclidean as

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2 This latter consequence is not discussed by Lützen.
the only really conceivable space, Hertz might express quite simply that he lacks any indication that non-Euclidean geometries could provide conditions for the possibility of scientific experience. The example of Einstein might have persuaded him otherwise. And since we cannot know the ultimate connections in the world of atoms, mechanics provides us with approximate knowledge of approximately rigid connections. Where it solves mechanical problems straightforwardly, his fundamental law appears as a bare experiential fact. Where additional assumptions of concealed motions are required, we have succeeded in regarding the phenomenon as a solvable mechanical problem. Here the fundamental law serves as a permissible, perhaps probable hypothesis. And where there is a “contradiction between the presuppositions of the question and the fundamental law,” these questions “cannot be considered a mechanical problem at all” [Hertz, 1956, Sects. 318, 316–325].3 Where we assume the causal efficacy of an intentional act, for example, we are outside the field of mechanics.4 Thus, instead of reducing all of nature to mechanics, Hertz “allows us to survey the whole domain of mechanics,” showing us “what are the limits of its domain” [Hertz, 1956, 38]. For all we know, these limits are not empirically given but depend on our willingness to transform considerations of animate nature and willful or intentional action into mechanical or purely geometric problems that frame all of nature as inanimate.

One could continue and elaborate on many points of detail this juxtaposition of Lützen’s rather more physicalist reading, according to which the Principles of Mechanics afford a comprehensive theory of nature, and the suggested epistemological reading that focuses on the construction of mechanical representations and explanations [Nordmann, 1998]. It may be to the credit of Hertz’s work and its philosophical importance that it is open to continued philosophical debate. But it also draws attention to a difficult question that asks to be resolved, namely how Hertz’s apparent conventionalism can be reconciled with his fallibilism. He leaves open whether failure even to regard a physical phenomenon as a mechanical problem and to represent it with appeal to concealed masses would constitute a refutation of his mechanics or would indicate merely the limit of its domain. Accordingly, it is difficult to understand just what Hertz has in mind when he states that “that which is derived from experience can again be annulled by experience” [Hertz, 1956, 9; Lützen, 2005, 87].

Hertz states that only the Fundamental Law is derived from experience in the Principles of Mechanics. He seems to suggest that it can only be annulled by future experience and by an experience of a peculiar kind. It does not concern the ability of his or any other image of mechanics to regard and solve problems as mechanical problems. It concerns instead a kind of meta-issue, namely how an image of mechanics explains, that is, where it yields a straightforwardly correct and where only an approximate result (pp. 117–118, 234). To annul Hertz’s Fundamental Law5 one would have to experimentally determine that instead of fixed relations between positions and only approximate relative accelerations as in Hertz’s geometric image of mechanics, accelerations are really the “final constant elements” of nature with relative positions only approximately fixed (as in the Newtonian image of mechanics):

If only we could study the motions of nature with sufficient precision, we would know right away, whether the relative acceleration or the relative positions of masses or both are only approximately invariable in these motions. We would then also know immediately which one of our assumptions is false or whether both are false, for they cannot both be simultaneously correct. [Hertz, 1956, 41]

But clearly, this study of motion involves more than carrying measurement precision to an unachievable standard error of zero; it also requires a neutral vantage point from which to independently represent phenomena of motion. Lützen is right that Hertz does not actually explain here how his image could be annulled by experience (p. 87). What we see is only how Hertz’s physicalist fallibilism runs up against his epistemological conventionalism. And what we get from this are conflicting philosophical interpretations of Lützen’s masterful reconstruction of the interplay in Hertz’s mechanics between physical content and mathematical form.

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3 In his discussion of these paragraphs, Lützen (pp. 264–271) does not dwell on how they define “mechanical problems.”
4 To be sure, to the extent that the actions by animate systems can be replaced by those of robots and the question thus reframed, this does not entail a contradiction to a purely mechanical account (see Lützen, pp. 116, 191–192, 268–272).
5 I.e., free systems in a geometry of systems of points persist in the straightest path.
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References


Alfred Nordmann
Institut für Philosophie,
Technische Universität Darmstadt,
D-64283 Darmstadt, Germany
E-mail address: nordmann@phil.tu-darmstadt.de

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Hausdorff on Ordered Sets

This volume contains an English edition of the papers on order structures written by Felix Hausdorff (1868–1942). Its main part contains a complete collection of Hausdorff’s publications on set theory that appeared before his “magnum opus” Grundzüge der Mengenlehre [Hausdorff, 1914]. They were published between 1901 and 1909 and are presently only accessible in the original journals (in German). They will be contained in a future volume of the ongoing edition of Hausdorff’s Collected Works [Hausdorff, 2001ff., Vol. I]. All of these papers have commentaries by the editor, with the translations undertaken jointly with Marion Scheepers. In the Appendix, an additional translation of a later Hausdorff paper on Sums of $\aleph_1$ sets (1936) is given without a further commentary.

During the first decade of the last century Felix Hausdorff became one of the leading figures of the “second generation of Cantorians,” as Jacob Plotkin rightly comments (p. 181). Hausdorff entered the new field in 1901 by generalizing Cantor’s investigations of order structures from well-orders to what he called “graded” order types, because he hoped for a possible relationship to the continuum problem. The latter had been raised to a first-rank problem of research mathematics by Hilbert’s talk at Paris in 1900. But Hausdorff’s hope did not come true. “Graded types quickly passed from the scene,” as the editor remarks (p. 6).

The next contribution, the Hausdorff recursion formula for transfinite cardinal numbers (denomination due to A. Tarski), arose from a dense and irritating discussion at the 1904 International Congress of Mathematicians at Heidelberg and after. Because of an incorrect formula for transfinite cardinals, due to F. Bernstein, J. König came