# Can the $X(4350)$ narrow structure be a $1^{-+}$exotic state? 

Raphael M. Albuquerque, Jorgivan M. Dias, Marina Nielsen*<br>Instituto de Física, Universidade de São Paulo, C.P. 66318, 05315-970 São Paulo, SP, Brazil

## A R TICLE INFO

## Article history:

Received 19 January 2010
Received in revised form 22 April 2010
Accepted 4 May 2010
Available online 14 May 2010
Editor: W. Haxton

## Keywords:

Non-perturbative methods
Exotic-charmonium states
QCD sum rules


#### Abstract

Using the QCD sum rules we test if the new narrow structure, the $X(4350)$ recently observed by the Belle Collaboration, can be described as a $J^{P C}=1^{-+}$exotic $D_{s}^{*} D_{s 0}^{*}$ molecular state. We consider the contributions of condensates up to dimension eight, we work at leading order in $\alpha_{s}$ and we keep terms which are linear in the strange quark mass $m_{s}$. The mass obtained for such state is $m_{D_{s}^{*} D_{s 0}^{*}}=(5.05 \pm$ $0.19) \mathrm{GeV}$. We also consider a molecular $1^{-+}, D^{*} D_{0}^{*}$ current and we obtain $m_{D^{*} D_{0}^{*}}=(4.92 \pm 0.08) \mathrm{GeV}$. We conclude that it is not possible to describe the $X(4350)$ structure as a $1^{-+} D_{s}^{*} D_{s 0}^{*}$ molecular state.


© 2010 Elsevier B.V. Open access under CC BY license.

In the recent years, many new states were observed by BaBar, Belle and CDF Collaborations. All these states were observed in decays containing a $J / \psi$ or $\psi^{\prime}$ in the final states and their masses are in the charmonium region. Therefore, they certainly contain a $c \bar{c}$ pair in their constituents. Although they are above the threshold for a decay into a pair of open charm mesons they decay into $J / \psi$ or $\psi^{\prime}$ plus pions, which is unusual for $c \bar{c}$ states. Another common feature of these states is the fact that their masses and decay modes are not in agreement with the predictions from potential models. For these reasons they are considered as candidates for exotic states. Some of these new states have their masses very close to the meson-meson threshold, like the $X$ (3872) [1], the $Z^{+}$(4430) [2] and the $Y(4140)$ [3]. Therefore, a molecular interpretation for these states seems natural.

Concerning the $Y(4140)$ structure, it was observed by CDF Collaboration in the decay $B^{+} \rightarrow Y(4140) K^{+} \rightarrow J / \psi \phi K^{+}$. The mass and width of this structure are $M=(4143 \pm 2.9 \pm 1.2) \mathrm{MeV}$, $\Gamma=\left(11.7_{-5.0}^{+8.3} \pm 3.7\right) \mathrm{MeV}$ [3]. Its interpretation as a conventional $c \bar{c}$ state is complicated because it lies well above the threshold for open charm decays and, therefore, a $c \bar{c}$ state with this mass would decay predominantly into an open charm pair with a large total width. This state was interpreted as a $J^{P C}=0^{++}$or $2^{++}$ $D_{s}^{*} \bar{D}_{s}^{*}$ molecular state in different works [4-12]. In particular, using an effective Lagrangian model, the authors of Ref. [7] have suggested that a $D_{s}^{*+} D_{s}^{*-}$ molecular state should be seen in the two-photon process. Following this suggestion the Belle Collabo-

[^0]ration [13] searched for the $Y(4140)$ state in the $\gamma \gamma \rightarrow \phi J / \psi$ process. However, instead of the $Y(4140)$, the Belle Collaboration found evidence for a new narrow structure in the $\phi J / \psi$ mass spectrum at 4.35 GeV . The significance of the peak is 3.2 standard deviations and, if interpreted as a resonance, the mass and width of the state, called $X(4350)$ are $M=\left(4350.6_{-5.1}^{+4.6} \pm 0.7\right) \mathrm{MeV}$ and $\Gamma=\left(13.3_{-9.1}^{+7.9} \pm 4.1\right) \mathrm{MeV}$ [13].

The possible quantum numbers for a state decaying into $J / \psi \phi$ are $J^{P C}=0^{++}, 1^{-+}$and $2^{++}$. At these quantum numbers, $1^{-+}$ is not consistent with the constituent quark model and it is considered exotic [4]. In Ref. [13] it was noted that the mass of the $X(4350)$ is consistent with the prediction for a $\csc \bar{s}$ tetraquark state with $J^{P C}=2^{++}$[14] and a $D_{s}^{*+} \bar{D}_{s 0}^{*-}$ molecular state [15]. However, the state considered in Ref. [15] has $J^{P}=1^{-}$with no definite charge conjugation. A molecular state with a vector and a scalar $D_{s}$ mesons with negative charge conjugation was studied by the first time in Ref. [16], and the obtained mass was ( $4.42 \pm 0.10$ ) GeV , also consistent with the $X$ (4350) mass, but with not consistent quantum numbers. A molecular state with a vector and a scalar $D_{s}$ mesons with positive charge conjugation can be constructed using the combination $D_{s}^{*+} D_{s 0}^{*-}-D_{s}^{*-} D_{s 0}^{*+}$.

There is already some interpretations for this state. In Ref. [17] it was interpreted as an excited $P$-wave charmonium state $\Xi_{c 2}^{\prime \prime}$ and in Ref. [18] it was interpreted as a mixed charmonium- $D_{S}^{*} D_{s}^{*}$ state. In this work, we use the QCD sum rules (QCDSR) [19-21], to study the two-point function based on a $D_{s}^{*} D_{s 0}^{*}$ current with $J^{P C}=1^{-+}$, to test if the new observed resonance structure, $X(4350)$, can be interpreted as such molecular state, as suggested by Belle Collaboration [13].

The QCD sum rule approach is based on the two-point correlation function
$\Pi_{\mu \nu}(q)=i \int d^{4} x e^{i q . x}\langle 0| T\left[j_{\mu}(x) j_{\nu}^{\dagger}(0)\right]|0\rangle$,
where a current that couples with a $J^{P C}=1^{-+} D_{s}^{*} D_{s 0}^{*}$ state is given by:
$j_{\mu}=\frac{1}{\sqrt{2}}\left[\left(\bar{s}_{a} \gamma_{\mu} c_{a}\right)\left(\bar{c}_{b} s_{b}\right)-\left(\bar{c}_{a} \gamma_{\mu} s_{a}\right)\left(\bar{s}_{b} c_{b}\right)\right]$,
where $a$ and $b$ are color indices.
Since the current in Eq. (2) is not conserved, we can write the correlation function in Eq. (1) in terms of two independent Lorentz structures:
$\Pi_{\mu \nu}(q)=-\Pi_{1}\left(q^{2}\right)\left(g_{\mu \nu}-\frac{q_{\mu} q_{\nu}}{q^{2}}\right)+\Pi_{0}\left(q^{2}\right) \frac{q_{\mu} q_{\nu}}{q^{2}}$.
The two invariant functions, $\Pi_{1}$ and $\Pi_{0}$, appearing in Eq. (3), have respectively the quantum numbers of the spin 1 and 0 mesons. Therefore, we choose to work with the Lorentz structure $g_{\mu \nu}$, since it gets contributions only from the $1^{-+}$state.

The QCD sum rule is obtained by evaluating the correlation function in Eq. (1) in two ways: in the OPE side, we calculate the correlation function at the quark level in terms of quark and gluon fields. We work at leading order in $\alpha_{s}$ in the operators, we consider the contributions from condensates up to dimension eight and we keep terms which are linear in the strange quark mass $m_{s}$. In the phenomenological side, the correlation function is calculated by inserting intermediate states for the $D_{s}^{*} \bar{D}_{s}^{*}$ molecular scalar state. Parametrizing the coupling of the exotic state, $X=D_{s}^{*} D_{s 0}^{*}$, to the current, $j_{\mu}$, in Eq. (2) in terms of the parameter $\lambda$ :
$\langle 0| j_{\mu}|X\rangle=\lambda \varepsilon_{\mu}$,
the phenomenological side of Eq. (1), in the $g_{\mu \nu}$ structure, can be written as
$\Pi_{1}^{\text {phen }}\left(q^{2}\right)=\frac{\lambda^{2}}{M_{X}^{2}-q^{2}}+\int_{0}^{\infty} d s \frac{\rho^{\text {cont }}(s)}{s-q^{2}}$,
where the second term in the RHS of Eq. (5) denotes higher resonance contributions.

The correlation function in the OPE side can be written as a dispersion relation:
$\Pi_{1}^{O P E}\left(q^{2}\right)=\int_{4 m_{c}^{2}}^{\infty} d s \frac{\rho^{O P E}(s)}{s-q^{2}}$,
where $\rho^{\text {OPE }}(s)$ is given by the imaginary part of the correlation function: $\pi \rho^{O P E}(s)=\operatorname{Im}\left[-\Pi_{1}^{O P E}(s)\right]$.

As usual in the QCD sum rules method, it is assumed that the continuum contribution to the spectral density, $\rho^{\text {cont }}(s)$ in Eq. (5), vanishes bellow a certain continuum threshold $s_{0}$. Above this threshold, it is given by the result obtained with the OPE. Therefore, one uses the ansatz [22]
$\rho^{\text {cont }}(s)=\rho^{O P E}(s) \Theta\left(s-s_{0}\right)$.
To improve the matching between the two sides of the sum rule, we perform a Borel transform. After transferring the continuum contribution to the OPE side, the sum rules for the exotic meson, described by a $1^{-+} D_{s}^{*} D_{s 0}^{*}$ molecular current, up to dimension-eight condensates, using factorization hypothesis, can be written as:
$\lambda^{2} e^{-m_{D_{s}^{*} D_{s 0}^{*}}^{2} / M^{2}}=\int_{4 m_{c}^{2}}^{s_{0}} d s e^{-s / M^{2}} \rho^{O P E}(s)$,
where

$$
\begin{align*}
\rho^{O P E}(s)= & \rho^{\text {pert }}(s)+\rho^{\langle\bar{s} s\rangle}(s)+\rho^{\left\langle G^{2}\right\rangle}(s)+\rho^{m i x}(s) \\
& +\rho^{\langle\bar{s} s\rangle^{2}}(s)+\rho^{\langle 8\rangle}(s), \tag{9}
\end{align*}
$$

with

$$
\begin{aligned}
\rho^{p e r t}(s)= & \frac{1}{2^{12} \pi^{6}} \int_{\alpha_{\min }}^{\alpha_{\max }} \frac{d \alpha}{\alpha^{3}} \int_{\beta_{\min }}^{1-\alpha} \frac{d \beta}{\beta^{3}}(1-\alpha-\beta) F^{3}(\alpha, \beta) \\
& \times\left[3(1+\alpha+\beta) F(\alpha, \beta)+2 m_{c}^{2}(1-\alpha-\beta)^{2}\right], \\
\rho^{m_{s}}(s)= & -\frac{3 m_{s} m_{c}}{2^{9} \pi^{6}} \int_{\alpha_{\min }}^{\alpha_{\max }} \frac{d \alpha}{\alpha^{3}} \int_{\beta_{\min }}^{1-\alpha} \frac{d \beta}{\beta^{2}}(1-\alpha-\beta)^{2} F^{3}(\alpha, \beta), \\
\rho^{\langle\bar{s}\rangle\rangle}(s)= & -\frac{3 m_{c}\langle\bar{s} s\rangle}{2^{6} \pi^{4}} \int_{\alpha_{\min }}^{\alpha_{\max }} \frac{d \alpha}{\alpha^{2}} \int_{\beta_{\min }}^{1-\alpha} \frac{d \beta}{\beta}(1-\alpha-\beta) F^{2}(\alpha, \beta), \\
\rho^{\left.m_{s} \cdot\langle\bar{s}\rangle\right\rangle}(s)= & -\frac{3 m_{s}\langle\bar{s} s\rangle}{2^{7} \pi^{4}}\left[\int_{\alpha_{\min }}^{\alpha_{\max }} \frac{d \alpha}{\alpha} \int_{\beta_{\min }}^{1-\alpha} \frac{d \beta}{\beta} m_{c}^{2}(3+\alpha+\beta) F(\alpha, \beta)\right. \\
& \left.-\int_{\alpha_{\min }}^{\alpha_{\max }} \frac{d \alpha}{\alpha(1-\alpha)} H^{2}(\alpha)\right],
\end{aligned}
$$

$$
\rho^{\left\langle G^{2}\right\rangle}(s)=-\frac{\left\langle g^{2} G^{2}\right\rangle}{32^{12} \pi^{6}} \int_{\alpha_{\min }}^{\alpha_{\max }} \frac{d \alpha}{\alpha^{3}} \int_{\beta_{\min }}^{1-\alpha} \frac{d \beta}{\beta}\left[6 \alpha(1-2 \alpha-2 \beta) F^{2}(\alpha, \beta)\right.
$$

$$
-3 m_{c}^{2}(1-\alpha-\beta)\{1+\alpha(1-2 \alpha)+\beta(\alpha+3 \beta)\}
$$

$$
\left.\times F(\alpha, \beta)-m_{c}^{4} \beta(1-\alpha-\beta)^{3}\right]
$$

$$
\rho^{m i x}(s)=\frac{3 m_{c}\langle\bar{s} g \sigma . G s\rangle}{2^{8} \pi^{4}} \int_{\alpha_{\min }}^{\alpha_{\max }} \frac{d \alpha}{\alpha^{2}} \int_{\beta_{\min }}^{1-\alpha} \frac{d \beta}{\beta}
$$

$$
\times[\alpha(1-\alpha)-\beta(5 \alpha+2 \beta)] F(\alpha, \beta)
$$

$$
\rho^{m s \cdot m i x}(s)=\frac{m_{s}\langle\bar{s} g \sigma \cdot G s\rangle}{2^{8} \pi^{4}}\left[\int_{\alpha_{\min }}^{\alpha_{\max }} \frac{d \alpha}{\alpha} \int_{\beta_{\min }}^{1-\alpha} d \beta\right.
$$

$$
\times\left\{m_{c}^{2}(3+5 \alpha+4 \beta)-\alpha \beta s\right\}
$$

$$
\left.-\int_{\alpha_{\min }}^{\alpha_{\max }} \frac{d \alpha}{\alpha}\left\{m_{c}^{2}(2+\alpha)-\alpha(1-\alpha) s(2-7 \alpha)\right\}\right]
$$

$$
\rho^{\langle\bar{s} s\rangle^{2}}(s)=-\frac{\langle\bar{s} s\rangle^{2}}{2^{6} 3 \pi^{2}}\left(8 m_{c}^{2}+s\right) \sqrt{1-4 m_{c}^{2} / s},
$$

$$
\rho^{m_{s} \cdot\langle\bar{s} s\rangle^{2}}(s)=-\frac{m_{c} m_{s}\langle\bar{s} s\rangle^{2}}{2^{5} \pi^{2}} \sqrt{1-4 m_{c}^{2} / s}
$$

$$
\rho^{\langle 8\rangle}(s)=\frac{m_{c}^{2}\langle\bar{s} s\rangle\langle\bar{s} g \sigma . G s\rangle}{2^{6} \pi^{2}} \frac{\sqrt{1-4 m_{c}^{2} / s}}{s}
$$

$$
\Pi^{\langle 8\rangle}\left(M^{2}\right)=\frac{m_{c}^{2}\langle\bar{s} s\rangle\langle\bar{s} g \sigma . G s\rangle}{2^{6} \pi^{2}} \int_{0}^{1} \frac{d \alpha}{(1-\alpha)} e^{-\frac{m_{c}^{2}}{\alpha(1-\alpha) M^{2}}}
$$

$$
\times\left[1-3 \alpha+\frac{2 m_{c}^{2}}{\alpha M^{2}}\right]
$$



Fig. 1. The OPE convergence for the $J^{P C}=1^{-+}, D_{s}^{*} D_{s 0}^{*}$ molecule in the region $2.8 \leqslant$ $M^{2} \leqslant 4.8 \mathrm{GeV}^{2}$ for $\sqrt{s_{0}}=5.3 \mathrm{GeV}$. We plot the relative contributions starting with the perturbative contribution plus de $m_{s}$ correction (long-dashed line), and each other line represents the relative contribution after adding of one extra condensate in the expansion: $+\langle\bar{s} s\rangle+m_{s}\langle\bar{s} s\rangle$ (dashed line), $+\left\langle g^{2} G^{2}\right\rangle$ (dotted line), $+\langle\bar{s} g \sigma . G s\rangle+$ $m_{s}\langle\bar{s} g \sigma . G s\rangle$ (dot-dashed line), $+\langle\bar{s} s\rangle^{2}+m_{s}\langle\bar{s} s\rangle^{2}$ (line with circles), $+\langle 8\rangle+m_{s}\langle 8\rangle$ (line with squares).

$$
\begin{aligned}
\Pi^{m_{s} \cdot\langle 8\rangle}\left(M^{2}\right)= & \frac{m_{s} m_{c}\langle\bar{s} s\rangle\langle\bar{s} g \sigma \cdot G s\rangle}{32^{7} \pi^{2}} \int_{0}^{1} \frac{d \alpha}{\alpha} e^{-\frac{m_{c}^{2}}{\alpha(1-\alpha) M^{2}}} \\
& \times\left[\frac{m_{c}^{2}}{M^{2}} \frac{\left(6-4 \alpha-10 \alpha^{2}\right)}{\alpha(1-\alpha)}-\left(6-13 \alpha+20 \alpha^{2}\right)\right]
\end{aligned}
$$

where we use the following definitions:
$F(\alpha, \beta)=m_{c}^{2}(\alpha+\beta)-\alpha \beta s$,
$H(\alpha)=m_{c}^{2}-\alpha(1-\alpha) s$.
The integration limits are given by $\alpha_{\min }=\left(1-\sqrt{1-4 m_{c}^{2} / s}\right) / 2$, $\alpha_{\max }=\left(1+\sqrt{1-4 m_{c}^{2} / s}\right) / 2, \beta_{\min }=\alpha m_{c}^{2} /\left(s \alpha-m_{c}^{2}\right)$. We have neglected the contribution of the dimension-six condensate $\left\langle g^{3} G^{3}\right\rangle$, since it is assumed to be suppressed by the loop factor $1 / 16 \pi^{2}$.

To extract the mass $m_{D_{s}^{*} D_{s 0}^{*}}$ we take the derivative of Eq. (8) with respect to $1 / M^{2}$, and divide the result by Eq. (8).

For a consistent comparison with the results obtained for the other molecular states using the QCDSR approach, we have considered here the same values used for the quark masses and condensates as in Refs. [16,23-29]: $m_{c}\left(m_{c}\right)=(1.23 \pm 0.05) \mathrm{GeV}, m_{s}=$ $(0.13 \pm 0.03) \mathrm{GeV},\langle\bar{q} q\rangle=-(0.23 \pm 0.03)^{3} \mathrm{GeV}^{3},\langle\bar{s} s\rangle=0.8\langle\bar{q} q\rangle$, $\langle\bar{s} g \sigma . G s\rangle=m_{0}^{2}\langle\bar{s} s\rangle$ with $m_{0}^{2}=0.8 \mathrm{GeV}^{2},\left\langle g^{2} G^{2}\right\rangle=0.88 \mathrm{GeV}^{4}$.

The Borel window is determined by analyzing the OPE convergence, the Borel stability and the pole contribution. To determine the minimum value of the Borel mass we impose that the contribution of the dimension- 8 condensate should be smaller than $20 \%$ of the total contribution.

In Fig. 1 we show the contribution of all the terms in the OPE side of the sum rule. From this figure we see that for $M^{2} \geqslant$ $2.8 \mathrm{GeV}^{2}$ the contribution of the dimension-8 condensate is less than $10 \%$ of the total contribution, which indicates a good Borel convergence. However, from Fig. 2 we see that the Borel stability is good only for $M^{2} \geqslant 3.2 \mathrm{GeV}^{2}$. Therefore, we fix the lower value of $M^{2}$ in the sum rule window as $M_{\text {min }}^{2}=3.2 \mathrm{GeV}^{2}$.


Fig. 2. The exotic meson mass, described with a $D_{s}^{*} D_{s 0}^{*}$ molecular current, as a function of the sum rule parameter $\left(M^{2}\right)$ for $\sqrt{s_{0}}=5.3 \mathrm{GeV}$ (solid line), $\sqrt{s_{0}}=5.4 \mathrm{GeV}$ (dotted line), $\sqrt{s_{0}}=5.5 \mathrm{GeV}$ (dot-dashed line), and $\sqrt{s_{0}}=5.6 \mathrm{GeV}$ (dashed line). The crosses indicate the upper and lower limits in the Borel region.


Fig. 3. The dashed line shows the relative pole contribution (the pole contribution divided by the total, pole plus continuum, contribution) and the solid line shows the relative continuum contribution for $\sqrt{s_{0}}=5.3 \mathrm{GeV}$.

To be able to extract, from the sum rule, information about the low-lying resonance, the pole contribution to the sum rule should be bigger than, or at least equal to, the continuum contribution. Since the continuum contribution increases with $M^{2}$, due to the dominance of the perturbative contribution, we fix the maximum value of the Borel mass to be the one for which the pole contribution is equal to the continuum contribution.

From Fig. 3 we see that for $\sqrt{s_{0}}=5.3 \mathrm{GeV}$, the pole contribution is bigger than the continuum contribution for $M^{2} \leqslant$ $3.74 \mathrm{GeV}^{2}$. We show in Table 1 the values of $M_{\max }$ for other values of $\sqrt{s_{0}}$. For $\sqrt{s_{0}} \leqslant 5.1 \mathrm{GeV}$ there is no allowed Borel window.

Using the Borel window, for each value of $s_{0}$, to evaluate the mass of the exotic meson and then varying the value of the continuum threshold in the range $5.3 \leqslant \sqrt{s_{0}} \leqslant 5.6 \mathrm{GeV}$, we get $m_{D_{s}^{*} D_{s 0}^{*}}=(5.04 \pm 0.09) \mathrm{GeV}$.

Table 1
Upper limits in the Borel window for the $!^{-+}, D_{s}^{*} D_{s 0}^{*}$ current obtained from the sum rule for different values of $\sqrt{s_{0}}$.

| $\sqrt{s_{0}}(\mathrm{GeV})$ | $M_{\max }^{2}\left(\mathrm{GeV}^{2}\right)$ |
| :--- | :--- |
| 5.2 | 3.42 |
| 5.3 | 3.74 |
| 5.4 | 3.95 |
| 5.5 | 4.26 |
| 5.6 | 4.47 |

Up to now we have kept the values of the quark masses and condensates fixed. To check the dependence of our results with these values we fix $\sqrt{s_{0}}=5.45 \mathrm{GeV}$ and vary the other parameters in the ranges: $m_{c}=(1.23 \pm 0.05) \mathrm{GeV}, m_{s}=(0.13 \pm$ $0.03) \mathrm{GeV},\langle\bar{q} q\rangle=-(0.23 \pm 0.03)^{3} \mathrm{GeV}^{3}, m_{0}^{2}=(0.8 \pm 0.1) \mathrm{GeV}^{2}$. In our calculation we have assumed the factorization hypothesis. However, it is important to check how a violation of the factorization hypothesis would modify our results. For this reason we multiply $\langle\bar{s} s\rangle^{2}$ and $\langle\bar{s} s\rangle\langle\bar{s} g \sigma . G s\rangle$ in Eq. (10) by a factor $K$ and we vary $K$ in the range $0.5 \leqslant K \leqslant 2$. We notice that the results are more sensitive to the variations on the values of $\langle\bar{q} q\rangle$ and $K$.

Taking into account the uncertainties given above we get
$m_{D_{s}^{*} D_{s 0}^{*}}=(5.05 \pm 0.19) \mathrm{GeV}$.
For a more conservative prediction, here we enlarge the range of the $m_{c}$ values according to PDG [30]: $1.16 \leqslant m_{c} \leqslant 1.34 \mathrm{GeV}$. Considering this range we get $m_{D_{s}^{*} D_{s 0}^{*}}=(5.11 \pm 0.18) \mathrm{GeV}$, with the central value even bigger than the result in Eq. (12). It is important to mention that the continuum contribution increases with $m_{c}$ and for $m_{c} \geqslant 1.42 \mathrm{GeV}$ there is no allowed Borel window.

The value given in Eq. (12) is not compatible with the mass of the narrow structure $X(4350)$ observed by Belle. It is, however, very interesting to notice that the mass obtained for a state described with a $1^{--}, D_{s}^{*} D_{s 0}^{*}$ molecular current is $m_{1^{--}}=(4.42 \pm$ $0.10) \mathrm{GeV}$, much smaller than what we have obtained with the $1^{-+}, D_{s}^{*} D_{s 0}^{*}$ molecular current. This may be interpreted as an indication that it is easier to form molecular states with not exotic quantum numbers.

From the above study it is very easy to get results for the $D^{*} D_{0}^{*}$ molecular type current with $J^{P C}=1^{-+}$. For this we only have to take $m_{s}=0$ and $\langle\bar{s} s\rangle=\langle\bar{q} q\rangle$ in Eq. (10). The OPE convergence in this case is very similar to the preliminary case, and we also get a good Borel stability only for $M^{2} \geqslant 3.2 \mathrm{GeV}^{2}$. Fixing $M_{\text {min }}^{2}=3.2 \mathrm{GeV}^{2}$, the minimum allowed value for the continuum, threshold is $\sqrt{s_{0}}=5.2 \mathrm{GeV}$. We show, in Fig. 4, the result for the mass of such state using different values of the continuum threshold, with the upper and lower limits in the Borel region indicated.

Using the values of the continuum threshold in the range $5.2 \leqslant$ $\sqrt{s_{0}} \leqslant 5.5 \mathrm{GeV}$ we get for the state described with a $1^{-+}, D^{*} D_{0}^{*}$ molecular current: $m_{D^{*} D_{0}^{*}}=(4.92 \pm 0.08) \mathrm{GeV}$. Approximately one hundred MeV bellow the value obtained for the similar strange state. In the case of the $D^{*} D_{0}^{*}$ molecular current with $J^{P C}=1^{--}$, the mass obtained was [16]: $m_{1^{--}}=(4.27 \pm 0.10) \mathrm{GeV}$, again much smaller than for the exotic case.

In conclusion, we have presented a QCDSR analysis of the twopoint function based on $D_{s}^{*} D_{s 0}^{*}$ and $D^{*} D_{0}^{*}$ molecular type currents with $J^{P C}=1^{-+}$. Our findings indicate that the $X(4350)$ narrow structure observed by the Belle Collaboration in the process $\gamma \gamma \rightarrow$ $X(4350) \rightarrow J / \psi \phi$, cannot be described by using an exotic $1^{-+}$, $D_{s}^{*} D_{s 0}^{*}$ current.


Fig. 4. The $1^{-+}$meson mass, described with a $D^{*} D_{0}^{*}$ molecular current, as a function of the sum rule parameter for $\sqrt{s_{0}}=5.2 \mathrm{GeV}$ (solid line), $\sqrt{s_{0}}=5.3 \mathrm{GeV}$ (dotted line), $\sqrt{s_{0}}=5.4 \mathrm{GeV}$ (dot-dashed line), and $\sqrt{s_{0}}=5.5 \mathrm{GeV}$ (dashed line). The crosses indicate the upper and lower limits in the Borel region.

## Acknowledgements

This work has been partly supported by FAPESP and CNPqBrazil.

## References

[1] S.-K. Choi, et al., Belle Collaboration, Phys. Rev. Lett. 91 (2003) 262001.
[2] K. Abe, et al., Belle Collaboration, Phys. Rev. Lett. 100 (2008) 142001, arXiv:0708.1790.
[3] T. Aaltonen, et al., CDF Collaboration, Phys. Rev. Lett. 102 (2009) 242002, arXiv:0903.2229.
[4] M. Nielsen, F.S. Navarra, S.H. Lee, arXiv:0911.1958.
[5] X. Liu, S.-L. Zhu, Phys. Rev. D 80 (2009) 017502.
[6] N. Mahajan, Phys. Lett. B 679 (2009) 228, arXiv:0903.3107.
[7] T. Branz, T. Gutsche, V.E. Lyubovitskij, Phys. Rev. D 80 (2009) 054019.
[8] R.M. Albuquerque, M.E. Bracco, M. Nielsen, Phys. Lett. B 678 (2009) 186.
[9] G.-J. Ding, Eur. Phys. J. C 64 (2009) 297.
[10] J.R. Zhang, M.Q. Huang, arXiv:0905.4178.
[11] X. Liu, H.W. Ke, Phys. Rev. D 80 (2009) 034009.
[12] R. Molina, E. Oset, arXiv:0907.3043.
[13] C.P. Shen, et al., Belle Collaboration, arXiv:0912.2383.
[14] Fl. Stancu, arXiv:0906.2485.
[15] J.R. Zhang, M.Q. Huang, arXiv:0905.4672.
[16] R.M. Albuquerque, M. Nielsen, Nucl. Phys. A 815 (2009) 53, arXiv:0804.4817.
[17] X. Liu, G.Z. Luo, Z.F. Sun, arXiv:0911.3694.
[18] Z.-G. Wang, arXiv:0912.4626.
[19] M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B 147 (1979) 385.
[20] L.J. Reinders, H. Rubinstein, S. Yazaki, Phys. Rep. 127 (1985) 1.
[21] For a review and references to original works, see e.g. S. Narison, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. 17 (2002) 1, hep-th/0205006;
S. Narison, World Sci. Lecture Notes Phys. 26 (1989) 1;
S. Narison, Acta Phys. Polon. B 26 (1995) 687;
S. Narison, Riv. Nuovo Cimento 10 (2) (1987) 1;
S. Narison, Phys. Rep. 84 (1982) 263.
[22] B.L. Ioffe, Nucl. Phys. B 188 (1981) 317; B.L. Ioffe, Nucl. Phys. B 191 (1981) 591, Erratum.
[23] S. Narison, Phys. Lett. B 466 (1999) 345; S. Narison, Phys. Lett. B 361 (1995) 121; S. Narison, Phys. Lett. B 387 (1996) 162; S. Narison, Phys. Lett. B 624 (2005) 223.
[24] R.D. Matheus, et al., Phys. Rev. D 75 (2007) 014005.
[25] S.H. Lee, A. Mihara, F.S. Navarra, M. Nielsen, Phys. Lett. B 661 (2008) 28, arXiv:0710.1029.
[26] S.H. Lee, M. Nielsen, U. Wiedner, arXiv:0803.1168.
[27] M.E. Bracco, S.H. Lee, M. Nielsen, R. Rodrigues da Silva, Phys. Lett. B 671 (2009) 240, arXiv:0807.3275.
[28] S.H. Lee, K. Morita, M. Nielsen, Nucl. Phys. A 815 (2009) 29, arXiv:0808.0690.
[29] S.H. Lee, K. Morita, M. Nielsen, Phys. Rev. D 78 (2008) 076001, arXiv:0808.3168.
[30] C. Amsler, et al., Phys. Lett. B 667 (2008) 1.


[^0]:    * Corresponding author.

    E-mail addresses: rma@if.usp.br (R.M. Albuquerque), jdias@if.usp.br (J.M. Dias), mnielsen@if.usp.br (M. Nielsen).

