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ORGINAL ARTICLE

Study on motion of rigid rod on a circular surface using MHPM

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Abstract In this paper motion of rigid rod on a circular surface is studied analytically. A new analytical method called modified homotopy perturbation method (MHPM) is applied for solving this problem in different initial conditions to show capability of this method. The governing equation for motion of a rigid rod on the circular surface without slipping have been solved using MHPM. The efficacy of MHPM for handling nonlinear oscillation systems with various small and large oscillation amplitudes are presented in comparison with numerical benchmarks. Outcomes reveal that MHPM has an excellent agreement with numerical solution. The results show that by decreasing the oscillation amplitude, the velocity of rigid rod decreases and for $A=\pi/3$ the velocity profile is maximum.

1. Introduction

The complexity of handling the nonlinear vibration problems due to computation burden of numerical solution in one hand and deficiency of traditional analytical techniques to develop a general method that include all types of nonlinear equation of motions persuades researchers to apply novel analytical methods for solving the critical problems and broaden frontier in non-linear engineering problems.

Perturbation method as a pioneer method for solving nonlinear equations has some drawbacks in physical nonlinear problems because of its necessity for predicting a small parameter [1–3]. Various more effective analytical methods have been investigated and utilized for dealing with nonlinear equations in practicing engineering field. Such equations allow one to create physical insights through the physics of the problem and also parametric investigations. Vibration of mechanical systems associated with nonlinear properties have been handled using various analytical methods.

In order to eliminate the limitations of the conventional perturbation methods and providing an analytical approximate solution; a combination of homotopy method and perturbation method named homotopy perturbation method

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(HPM) was developed by He [4]. The HPM was successfully applied to discontinuous nonlinear oscillators and obtain bifurcation of nonlinear wave equations [5–8]. Frequency and displacement for nonlinear vibration equation of cantilever beam excited by harmonic forces were found utilizing HPM and observed that the accuracy of results increase with higher order approximations [9]. Vibrations of a mass grounded by linear and nonlinear springs in series have been interpreted by applying HPM [10]. Ayazi et al. solved nonlinear differential equation of beam elastic deformation with two fixed end and under uniform distributed load by HPM which does not need small parameters in comparison with conventional perturbation [11]. Also, several modified homotopy perturbation method (MHPM) developed so that become more suitable for nonlinear oscillations problems [12–14].

Adomian decomposition method (ADM) as an effective approach for handling the linear and nonlinear ordinary and partial differential equations has been utilized in area of applied mathematics and engineering problems because of its immediate solution terms without any transformation, linearization, discretization or physically restrictive assumptions [15]. In [16], a comparison of HPM and ADM was made revealing that the former is more powerful than the latter. Hosseini and Jafari proposed a modified ADM suitable for high order system of nonlinear differential equations with greater rate of convergence [17]. Wu adopted ADM in companion with Riemann-Liouville derivative to deal with a fractional nonlinear differential of equation [18].

Variational iteration method (VIM) is other rapidly convergent successive approximations of the exact solution for nonlinear differential equations [19]. One priority of using VIM is its ability to solve linear and nonlinear differential equation without any need to the so called Adomian polynomials which causes computational burden [20]. The outperform performance of VIM rather perturbation method in some nonlinear equations were shown in [21,22]. Deflection and deformation of flexible beam and plate was studied using VIM and results were assessed by other analytical and exact solutions by Choobbasti et al. [23]. Ganji et al. found better performance of VIM rather homotopy perturbation method (HPM) for greater value of nonlinearity parameter while they showed that for small amount of nonlinear parameter both VIM and HPM provide highly accurate numerical solutions for nonlinear problems in comparison with other methods [24].

He's frequency-amplitude formulation approach was introduced to attain analytical approximate periodic solutions for various nonlinear oscillatory systems [25–27]. A Semi-numerical-analytical solution technique called differential transformation method (DTM) has been utilized to derive approximate explicit analytical solutions for nonlinear oscillator equations in comparison with other analytical approaches [28].

Several new applications of analytical solutions for nonlinear differential equations is following:

Ghasemi et al. [29] applied optimal homotopy asymptotic method (OHAM) and homotopy perturbation method (HPM) to obtain the temperature distribution in a flat-plate airheating solar collector.

Application of least square method (LSM) for electrohydrodynamic flow (EHD flow) in a circular cylindrical conduit was studied by Ghasemi et al. [30].

Recently, Hatami and Ganji [31,32] and Hatami et al. [33,34] used analytical methods for solving some useful and applicable problems in the fields of nanofluids and nanoparticles.

A complicated and also practical problem of the motion of a rigid rod rocking back and forth on the circular surface without slipping was adopted from Gaylord [35] and is used in present paper to be solved by proposed version of MHPM. This method has been used as a reliable benchmark for evaluating the novel analytical approaches to nonlinear oscillation system [36–38]. In following Mathematical formulation and geometry of problem are presented in Section 2. Section 2 is specified to explain the basic idea of MHPM. Application of MHPM to motion of rigid rod on the circular surface is developed in Section 4. Results of applying MHPM to solve equation motion of rigid rod on the circular surface are discussed in Section 5. Paper concludes in Section 6.

2. Statement of the problem

The schematic view of rigid rod on the circular surface is illustrated in Figure 1 and the equation of motion of problem concerned is considered as follows [35]:

$$\ddot{u} + a(u^2\ddot{u} + u\dot{u}^2) + bu\cos(u) = 0, \quad (1)$$

with initial conditions:

$$u(0) = A, \quad \dot{u}(0) = 0 \quad (2)$$

where:

$$a = \frac{12r^2}{l^2}, \quad b = \frac{12gr}{l^2} \quad \text{and} \quad u = \theta(t) \quad (3)$$

3. Fundamentals of modified homotopy perturbation method (MHPM)

In this section, a new modification of the HPM is recapitulated. The generalized equation is introduced as

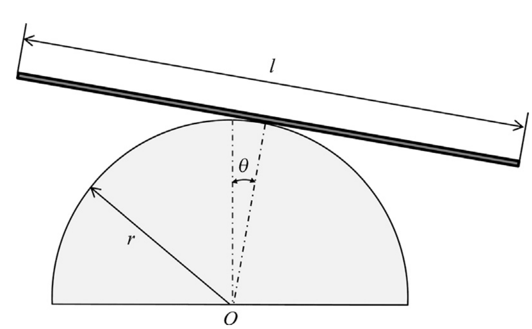

Figure 1 Geometry of problem.

follows:

$$\ddot{u}+N(u,\dot{u},\ddot{u},t)=0, \tag{4}$$

with boundary conditions:

$$u(0)=A, \quad \dot{u}(0)=0 \tag{5}$$

To explain the basic idea of this method, we consider the following nonlinear differential equation:

$$A(u)-f(r)=0, \quad r\in\Omega \tag{6}$$

subject to boundary condition:

$$B(u,\partial u/\partial n)=0, \quad r\in\Gamma \tag{7}$$

where A, B, $f(r)$ and Γ are general differential operator, boundary operator, known analytical function, and the boundary of domain Ω respectively.

Generally speaking the operator A can be divided into a linear part L and a nonlinear part $N(u)$. Eq. (6) can so, be rewritten as:

$$L(u)+N(u)-f(r)=0 \tag{8}$$

A homotopy of Eq. (6) $v(r,p):\Omega\times[0,1]\to R$ is constructed so that satisfied:

$$H(v,p)=L(v)-L(u_0)+pL(u_0)+p[N(v)-f(r)]=0 \tag{9}$$

where p is embedding parameter and u_0 is an initial guess approximation of Eq. (6) which satisfies the boundary condition. According to MHPM, the solution is expanded into series of p in the form:

$$u=\sum_{i=1}^{n}p^iu_i \tag{10}$$

Frequency is expanded in similar way as:

$$1=\omega^2-\sum_{i=1}^{n}p^i\alpha_i \tag{11}$$

Substituting Eqs. (11) and (10) in Eq. (9) and equating the terms with powers of p, we can obtain a series of linear equation. The approximate for the solution and frequency are:

$$u=\lim_{p\to 1}\sum_{i=0}^{n}u_i \tag{12}$$

$$\omega^2=1+\lim_{p\to 1}\sum_{i=0}^{n}\alpha_i \tag{13}$$

where α_i are arbitrary parameters that should be determined.

Eventually substituting $\cos(u)=1-(u^2/2)+(u^4/24)+\cdots$ in Eq. (1) gives:

$$\ddot{u}+a(u^2\ddot{u}+u\dot{u}^2)+b\left(u-\frac{u^3}{2}+\frac{u^5}{24}\right)=0 \tag{14}$$

4. Solution with MHPM

In order to apply MHPM to Eq. (14), it should be rearranged as following form:

$$\ddot{u}+1\cdot u=-p\cdot\left[a(u^2\ddot{u}+u\dot{u}^2)+(b-1)u+b\left(-\frac{u^3}{2}+\frac{u^5}{24}\right)\right],\quad p\in[0,1] \tag{15}$$

where p is embedding parameter.

According to the MHPM, the solution u and 1 as coefficients of u expanded as following terms:

$$u=\sum_{i=1}^{n}p^iu_i \tag{16}$$

$$1=\omega^2-\sum_{i=1}^{n}p^i\alpha_i \tag{17}$$

Substituting Eqs. (16) and (17) into Eq. (15) yields:

$$p^0:\ddot{u}_0+\omega^2u_0=0 \tag{18}$$

$$p^1:\ddot{u}_1+\omega^2u_1-\alpha_1u_0+au_0^2\ddot{u}_0+\frac{b}{24}u_0^5-\frac{b}{2}u_0^3+au_0\dot{u}_0^2+(b-1)u_0=0 \tag{19}$$

$$p^2:\ddot{u}_2+\omega^2u_2-\alpha_1u_1-\alpha_2u_0+au_0^2\ddot{u}_1+2au_0u_1\ddot{u}_0+2au_0\dot{u}_0\dot{u}_1+au_1\dot{u}_0^2+\frac{5}{24}bu_0^4u_1+(b-1)u_1-\frac{3}{2}bu_0^2u_1=0 \tag{20}$$

By solving Eq. (18), it has obtained:

$$u_0(t)=A\cos(\omega t) \tag{21}$$

Substituting of Eq. (21) into the right side of Eq. (19) gives:

$$\ddot{u}_1+\omega^2u_1=\rho(\omega t),$$

which:

$$\rho(\omega t)=-\alpha_1A\cos(\omega t)-aA^3\cos(\omega t)^3\omega^2+\frac{b}{24}bA^5\cos(\omega t)^5-\frac{1}{2}bA^3\cos(\omega t)^3+aA^3\cos(\omega t)\sin(\omega t)^2\omega^2+(b-1)A\cos(\omega t) \tag{22}$$

By using Fourier series, we can achieve secular term:

$$\rho(\omega t)=\sum_{n=0}^{\infty}\delta_{2n+1}\cos[(2n+1)\omega t]\approx\delta_1\cos(\omega t),$$

$$\delta_1=\frac{4}{\pi}\int_0^{\frac{\pi}{2}}\rho(\phi)\cos(\phi)\,d\phi$$
$$=\frac{4}{\pi}\int_0^{\frac{\pi}{2}}\Big(-\alpha_1A\cos(\varphi)-aA^3\cos(\varphi)^3\omega^2+\frac{b}{24}bA^5\cos(\varphi)^5-\frac{1}{2}bA^3\cos(\varphi)^3+aA^3\cos(\varphi)\sin(\varphi)^2\omega^2+(b-1)A\cos(\varphi)\cos(\varphi)\Big)d\varphi=$$
$$-A\varphi_1+Ab-\frac{3}{8}bA^3-\frac{a}{2}A^3\omega^2+\frac{5}{192}bA^5-A \tag{23}$$

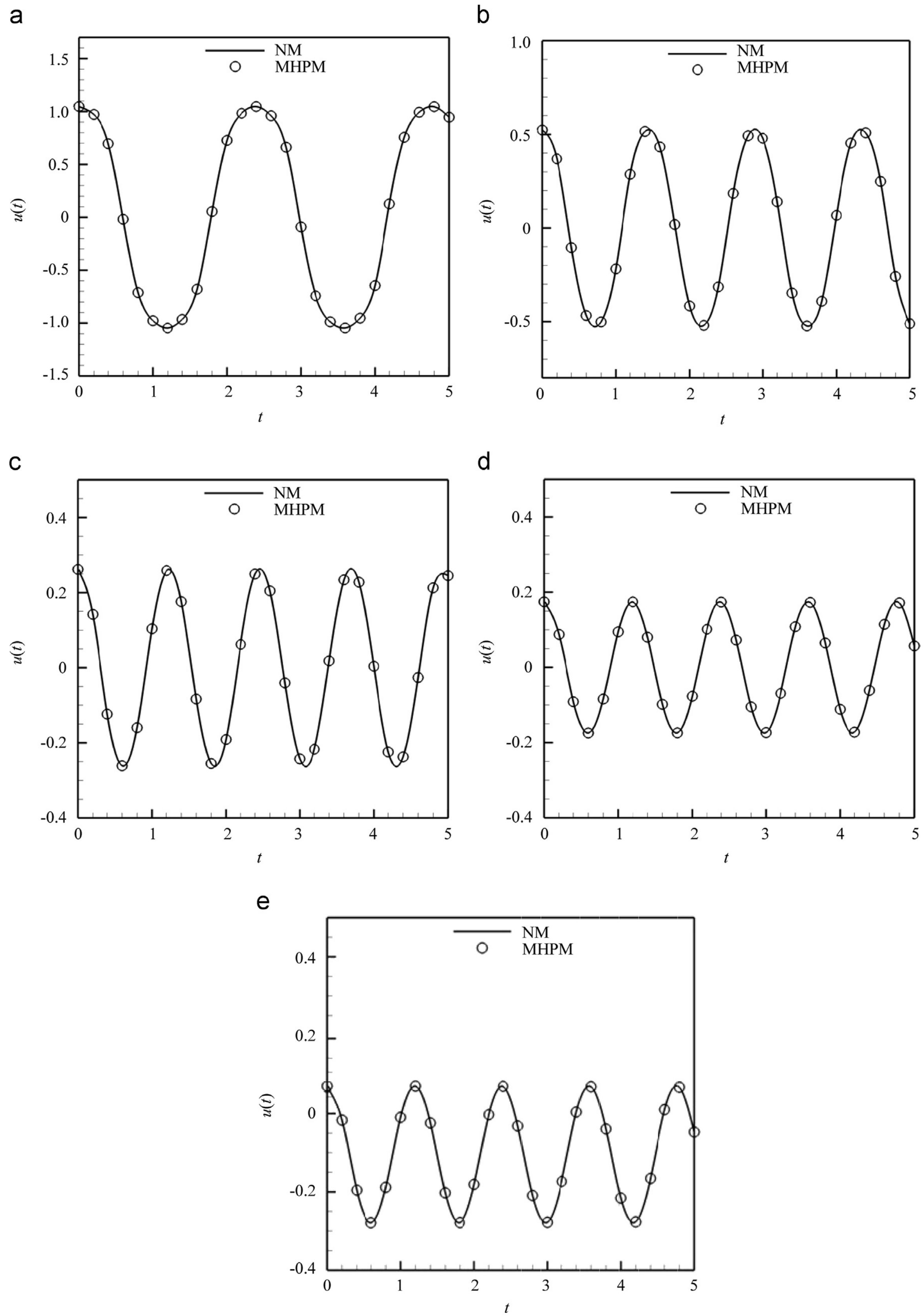

Figure 2 Comparison between numerical method and analytical solutions with $a = 3$, $b = 29.43$ when: (a) $A = \pi/3$, (b) $A = \pi/6$, (c) $A = \pi/12$, (d) $A = \pi/18$, and (e) $A = \pi/24$.

Avoiding secular term requires $\delta_1 = 0$.
From Eq. (11) and by setting $p = 1$:

$$\omega^2 = 1 + \alpha_1 \tag{24}$$

Therefore frequency is obtained as:

$$\omega_{\text{MHPM}} = \frac{\sqrt{6}}{24}\frac{\sqrt{(aA^2+2)b(-72A^2+5A^4+192)}}{aA^2+2} \tag{25}$$

5. Results and discussion

To illustrate the efficacy of MHPM for handling non-linear oscillation system, Eq. (1) is solved with various small and large oscillation amplitudes and outcomes are shown in Figure 2. It can be obviously seen that the results of numerical and analytical solutions are coincidence almost for all oscillation amplitudes. Also, for more investigations the results of numerical and analytical solutions at six steps starting from 0 to 5$^{\text{th}}$ seconds are inserted in Tables 1–5 for different values of A. It is observed that though there is no a constant trend in error extents as time goes on, the range of errors through the simulation are reasonably acceptable.

Table 1 Comparison between MHPM and numerical for $A = \pi/3$.

t	u-Numerical	u-MHPM	Error
0	1.05135	1.05174	0.000390
1	−1.0027	−1.0022	0.000500
2	0.77027	0.77052	0.000250
3	−0.105405	−0.105102	0.000303
4	−0.656757	−0.656510	0.000247
5	0.954054	0.954334	0.000280

Table 2 Comparison between MHPM and numerical for $A = \pi/6$.

t	u-Numerical	u-MHPM	Error
0	0.539583	0.539791	0.000208
1	−0.158631	−0.158408	0.000223
2	−0.340774	−0.340539	0.000235
3	0.504167	0.504425	0.000258
4	0.109524	0.109806	0.000282
5	−0.431845	−0.431437	0.000408

Table 3 Comparison between MHPM and numerical for $A = \pi/12$.

t	u-Numerical	u-MHPM	Error
0	0.262857	0.262949	0.000092
1	0.108571	0.108685	0.000114
2	−0.191429	−0.191382	0.000047
3	−0.245714	−0.245584	0.000130
4	0.0085714	0.0087424	0.000171
5	0.248571	0.248709	0.000138

Table 4 Comparison between MHPM and numerical for $A = \pi/18$.

t	u-Numerical	u-MHPM	Error
0	0.17622	0.17654	0.000320
1	0.10104	0.10123	0.000190
2	−0.075664	−0.075437	0.000227
3	−0.172795	−0.172580	0.000215
4	−0.113556	−0.113369	0.000187
5	0.058231	0.058506	0.000257

Table 5 Comparison between MHPM and numerical for $A = \pi/24$.

t	u-Numerical	u-MHPM	Error
0	0.0665644	0.0667825	0.000213
1	−0.0079754	−0.0077534	0.000222
2	−0.179141	−0.176270	0.002871
3	10.278528	−0.278237	0.000291
4	10.21227	−0.21209	0.000180
5	−0.0411043	−0.0410019	0.000102

6. Conclusions

The close solution of analytical research results and numerical ones proved the applicability of the proposed modified version to this kind of strongly nonlinear oscillator. It was obviously seen that the results of numerical and analytical solutions are coincidence almost for all oscillation amplitudes. MHPM profoundly succeeds to compensate the drawbacks of conventional HPM as well as numerical shortfalls in major applications. Less computation burden using less terms in expanded series which consequently leads less procedure time of solution in comparison with numerical integration and iteration procedure are main advantages of MHPM for solving complex vibration systems.

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