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Study of the effects of the Reynolds number on circular porous slider via variational iteration algorithm-II

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ABSTRACT

In this paper the problem of the porous slider where the fluid is injected through the porous bottom is considered. The similarity transformation reduces the governing equations into coupled nonlinear ordinary differential equations. The resulting equations are solved by using He's variational iteration algorithm-II. The resulting series solution contains the well known Reynolds number. The influence of the Reynolds number on the velocity field has been discussed graphically.

1. Introduction

The flow between porous plates has been studied by many authors, notably Berman [1], Proudman [2], Terrill [3] and Elkouh [4]. It was Morgan and Cameron [5] who first gave an analytical survey of study of porous bearings with the aid of hydrodynamic conditions. There are various types of porous bearings but the most common is slider bearings [6]. Sliding friction [7] is greatly reduced if a fluid is forced between two solid surfaces moving relative to each other. Porous sliders are important in fluid cushioned moving pads. To gain insight into the real situation an attempt is made for the analysis of the three-dimensional problem involving the Reynolds number [8].

Wang [9] carried out a numerical analysis of the problem for moderately large Reynolds numbers. However, the numerical methods are comparatively tedious and difficult due to stability problem. For simple geometries the semi-analytical numerical method provides accurate results and has advantages over pure numerical methods and analytical methods such as finite difference, finite element [10], Adomian decomposition method [11] and Laplace decomposition method [12]. The variational iteration algorithm-II [13,15] for solving differential and integral equation, linear or nonlinear, has been the subject of extensive analytical and numerical studies. The method, well addressed in [13–15], has a significant advantage in that it provides the solution in a rapid convergent series with elegantly computable components.

The objectives of this paper are two-fold: first, to introduce the advantages of the VIM-II, which primarily lie in its ability to avoid the unnecessary calculations of other iteration methods, namely, the VIM and ADM, the VIM-II does not require the calculation of Adomian polynomials for the nonlinear terms that appear in differential equations, as a solution can be obtained without the incorporation of these polynomials; second, to illustrate through this new approach, effects of the Reynolds number on the circular porous slider.

2. Formulation of the problem

The present paper calculates the flow field due to a circular porous slider Fig. 1. For three-dimensional flow, Bujurke introduced the following transforms [7]:

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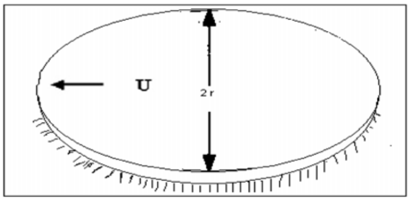

Fig. 1. Schematic diagram of the problem.

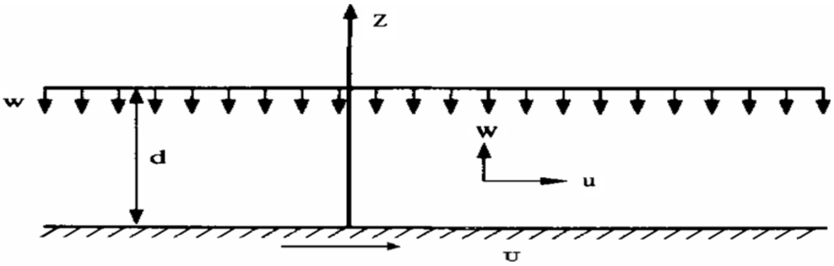

Fig. 2. System of coordinate axes.

$$u = Uf(\eta) + \frac{Wx}{d}h'(\eta), \qquad v = \frac{Wyh'(\eta)}{d}, \qquad w = -2Wh(\eta),$$
$$\frac{-p}{\rho} = \frac{W^2K_2}{2d}(x^2+y^2) + \frac{1}{2}W^2 - vW_z + A, \tag{2.1}$$

where $\eta = z/d$ and K, A are constants (Fig. 2). Substituting Eq. (2.1) into the constant density Navier–Stokes equations yields a system of nonlinear ordinary differential equations in the form

$$(h')^2 - 2hh'' = K + \frac{h'''}{R}, \tag{2.2}$$

or after differentiating Eq. (2.2)

$$-2hh''' = \frac{h''''}{R}, \tag{2.3}$$

and

$$fh' - 2hf' = \frac{1}{R}f''. \tag{2.4}$$

Corresponding boundary conditions take the form

$$h(0) = h'(0) = 0, \qquad h(1) = \frac{1}{2}, \qquad h'(1) = 0,$$
$$f(0) = 1, \qquad f(1) = 0 \tag{2.5}$$

where $R = Wd/\gamma$ is the Reynolds number.

3. Variational iteration algorithm-II

According to the classical variational iteration method [13,15], the correction functional can be written in the following form

$$h_{n+1}(\eta) = h_n(\eta) + \int_0^\eta \lambda_1\left(\frac{\partial^4 h_n}{\partial \xi^4} + 2Rh_n\frac{\partial^3 h_n}{\partial \xi^3}\right)\mathrm{d}\xi$$
$$f_{n+1}(\eta) = f_n(\eta) + \int_0^\eta \lambda_2\left(\frac{\partial^2 f_n}{\partial \xi^2} + 2Rh_n\frac{\partial f_n}{\partial \xi} - Rf_n\frac{\partial h_n}{\partial \xi}\right)\mathrm{d}\xi \tag{3.1}$$

with

$$\lambda_1 = \frac{(\xi-\eta)^3}{3!}, \qquad \lambda_2 = (\xi - \eta) \tag{3.2}$$

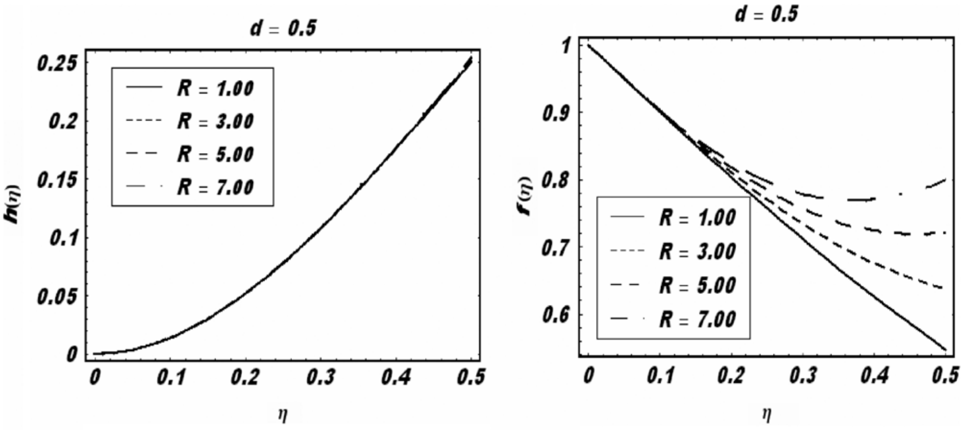

Fig. 3. Effect of the Reynolds number on h and f for $d = 0.5$.

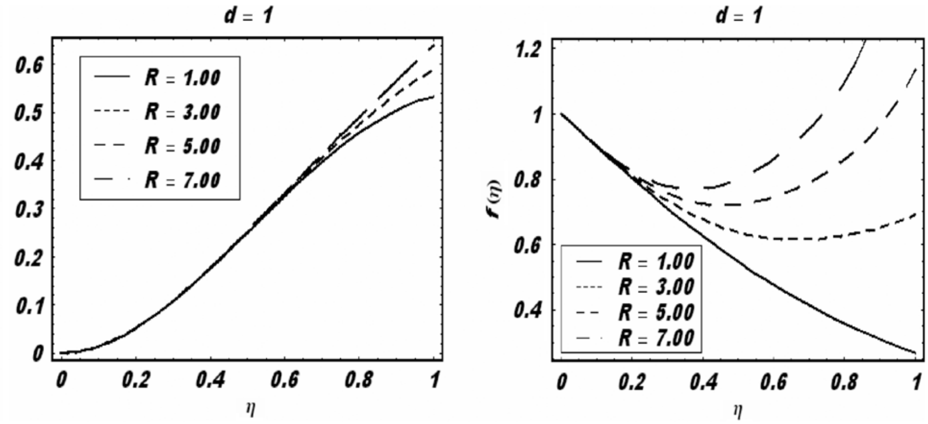

Fig. 4. Effect of the Reynolds number on h and f for $d = 1.0$.

and the initial approximations are

$$h_0 = -\eta^3 + \frac{3\eta^2}{2}, \qquad f_0 = 1 - \eta. \tag{3.3}$$

However, according to the VIM-II [14], the iteration formula for Eqs. (2.3) and (2.4) is

$$\begin{aligned} h_{n+1}(\eta) &= h_0(\eta) + \int_0^\eta \frac{(\xi-\eta)^3}{3!}\left(2Rh_n\frac{\partial^3 h_n}{\partial \xi^3}\right)\mathrm{d}\xi, \\ f_{n+1}(\eta) &= f_0(\eta) + \int_0^\eta (\xi-\eta)\left(2Rh_n\frac{\partial f_n}{\partial \xi} - Rf_n\frac{\partial h_n}{\partial \xi}\right)\mathrm{d}\xi. \end{aligned} \tag{3.4}$$

Starting the initial approximation $h_0 = -\eta^3 + \frac{3\eta^2}{2}$ and $f_0 = 1 - \eta$, we have

$$\begin{aligned} h_1 &= \frac{3\eta^2}{2} - \eta^3 + \frac{R\eta^6}{20} - \frac{R\eta^7}{70}, \qquad f_1 = 1 - \eta + \frac{R\eta^3}{2} - \frac{R\eta^4}{4} + \frac{R\eta^5}{20}, \\ h_2 &= \frac{3\eta^2}{2} - \eta^3 + \frac{R\eta^6}{20} - \frac{R\eta^7}{70} - \frac{R^2\eta^9}{168} + \frac{3R^2\eta^{10}}{700} - \frac{3R^2\eta^{11}}{3850} - \frac{R^3\eta^{13}}{28600} + \frac{R^3\eta^{14}}{50960} - \frac{R^3\eta^{15}}{382200}, \\ f_2 &= 1 - \eta + \frac{R\eta^3}{2} - \frac{R\eta^4}{4} + \frac{R\eta^5}{20} - \frac{R^2\eta^6}{10} + \frac{27R^2\eta^7}{280} - \frac{43R^2\eta^8}{1120} + \frac{59R^2\eta^9}{10080} + \frac{R^3\eta^{11}}{6160} - \frac{19R^3\eta^{12}}{184800} + \frac{R^3\eta^{13}}{72800}, \end{aligned} \tag{3.5}$$

and so on. In a similar manner the rest of the components can be obtained by using (3.4).

4. Results and discussion

Figs. 3 and 4 have been plotted for the effects of R (Reynolds number) and d (heights of lubrication film) on the velocity components h and f. The effect of the Reynolds number on h and f for $d = 0.5$ are shown in Fig. 3. It is seen from the figures that with increase of the Reynolds number there are slight changes in h but change in f is more prominent as compared to h for the lubrication film thickness $d = 0.5$. The most interesting case is presented in Fig. 4 for $d = 1.0$. In this case both h and f show changes but once again change in f is more prominent. It is seen from the figures that h and f increase with increasing values of the Reynolds number. From here we can conclude that influence of the Reynolds number highly depends upon thickness of film.

5. Conclusion

The present case studies a three-dimensional problem using variational iteration algorithm-II (VIM-II). The influence of the Reynolds number has been discussed through graphs. The following observations have been made.

- With increasing R there are some effects of R on f but there is negligible change in h when the lubrication film thickness is $d = 0.5$.
- f increases dramatically for film thickness 1.0, whenever R shows a slight change in h.

In conclusion, the graphical behavior of the Reynolds number shows that effects of R highly depend upon the height of the film.

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