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Propagation of cosmic rays in the foam-like Universe

A.A. Kirillov *, E.P. Savelova, P.S. Zolotarev

Branch of Uljanovsky State University in Dimitrovgrad, Dimitrova Street 4., Dimitrovgrad 433507, Russia

ABSTRACT

The model of a classical spacetime foam is considered, which consists of static wormholes embedded in Minkowski spacetime. We examine the propagation of particles in such a medium and demonstrate that a single thin ray undergoes a specific damping in the density of particles depending on the traversed path and the distribution of wormholes. The missing particles are scattered around the ray. Wormholes was shown to form DM halos around point-like sources and, therefore, the correlation predicted between the damping and the amount of DM may be used to verify the topological nature of dark matter.

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1. Introduction

The nature of dark matter (DM) represents one of the most important and yet unsolved problems of the modern astrophysics. Indeed, while the presence of DM has long been known [1] and represents a well established fact (e.g., see Refs. [2,3] and references therein), there is no common agreement about what DM is. In the simplest picture DM represents some non-baryonic particles (predicted numerously by particle physics) which should be sufficiently heavy to be cold at the moment of recombination and those give the basis to the standard (cold dark matter) CDM models. The latter turn out to be very successful in reproducing properties of the Universe at very large scales (where perturbations are still on the linear stage of the development) which led to a wide-spread optimistic believe that non-baryonic particles provide indeed an adequate content of DM.

However the success of CDM models at very large scales is accompanied with a failure at smaller (of the galaxies size) scales. Indeed, cold particles which interact only by gravity should necessary form cusps ($\rho_{DM} \sim 1/r$) in centers of galaxies[1] [4] (see also Ref. [5] where the problem of cusps in CDM is discussed in more detail), while observations definitely show the cored ($\rho_{DM} \sim$ const) [6] distribution. The only way to destroy the cusp and get the cored distribution is to introduce some self-interaction in DM or to consider warm DM. Both possibilities are rejected at large scales by observing $\Delta T/T$ spectrum (e.g., see Ref. [3] and references therein). By other words DM displays so non-trivial proper-

ties (it is warm or self-interacting in galaxies, however it was cold at the moment of recombination and it is still cold on larger (than galaxies) scales) that it is difficult to find particles capable of reconciling such observations.

These facts support the constant interest to different alternatives of the DM hypothesis which interpret the observed discrepancy between luminous and gravitational masses as a violation of the law of gravity. Such violations (or modifications of general relativity (GR)) have widely been discussed, e.g., see Refs. [7, 8]. However, it turns out to be rather difficult to get a modification of GR which is flexible enough to reconcile all the variety of the observed DM halos. Moreover, the weak lensing observations of a cluster merge in Ref. [9] seem to reject most of modifications of GR in which a non-standard gravity force scales with baryonic mass.

The more viable picture of DM phenomena was suggested in Ref. [10] (see also references therein) and developed recently in Refs. [11,12]. It is based on the fact that on the very early (quantum) stage the Universe should have a foam-like topological structure [13]. There are no convincing theoretical arguments of why such a foamed structure should decay upon the quantum stage— relics of the quantum stage foam might very well survive the cosmological expansion, thus creating a certain distribution of wormholes in the Friedman space. Moreover, the inflationary stage in the past [14] should enormously stretch characteristic scales of the relic foam. The foam-like structure, in turn, was shown to be flexible enough to account for the all the variety of DM phenomena [10,11]; for parameters of the foam may arbitrary vary in space to produce the observed variety of DM halos in galaxies (e.g., the universal rotation curve for spirals constructed in Ref. [15] for the foamed Universe perfectly fits observations). Moreover, the topological origin of DM phenomena means that the DM halos surrounding point-like sources appear due to the scattering on topological defects and if a source radiates, such a halo turns out to be luminous too [11] which seems to be the only way to explain

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[1] The presence of cusps formed by the development of adiabatic perturbations follows straightforwardly from the conservation of the circulation theorem in the hydrodynamics. By other words the fact that the distribution of DM should have cusps in galaxies is equivalent to the fact that DM should represent cold non-baryonic particles.

naturally the observed absence of DM fraction in intracluster gas clouds [9].

While the foam-like structure of the Universe is capable of providing a quite good description of DM phenomena, it is necessary to look for some independent tests to verify the topological nature of DM. Effects of the spacetime foam attract the more increasing attention (e.g., see Refs. [16–20] and references therein). However most of effects considered [18–20] assume the foamy structure at extremely small scales (which correspond to energies higher than 200 GeV). DM phenomena, however, suggest that the characteristic scale of the spacetime foam L (and respectively of wormholes) should be of the galaxy scale, e.g., of the order of a few kpc. The fact that the fundamental length scale for the quantum dynamics of spacetime need not be equal to the Planck length was also discussed recently in Ref. [17].

In the present Letter we consider the propagation of cosmic rays in the foam-like Universe. To this end we consider the model of the spacetime foam [12], which consists of a static gas of wormholes embedded in the Minkowski space. However contrary to above mentioned papers (e.g., see Ref. [19] where effects of cosmic ray interactions in a small-scale foam have been considered) we assume that the characteristic scale of such a foam is of the order of a galaxy size. We demonstrate that the scattering on the topological structure is described by a specific term in the Boltzmann equation. We show that in a foamed space a single thin ray of particles emitted undergoes a specific damping in the density of particles depending on the traversed path and the distribution of wormholes, while the missing particles are scattered and form a halo around the ray. Such halo however has a very low density and is difficult to observe. It turns out that the damping traces rather rigidly the amount of wormholes which in the foam-like Universe form DM halos in galaxies. Thus, there should exist a rather strong correlation between the damping and the distribution of DM in a galaxy which presumably can be used to verify the topological nature of Dark matter.

2. Boltzmann equation

In the present section for the sake of simplicity we consider the flat Minkowski space, while the generalization to the case of Friedman models is straightforward. Basic elements of relativistic kinetic theory can be found in standard textbooks, e.g., Ref. [21]. Let $f(r, p, t)$ be the number of particles in the interval of the phase space $d\Gamma = d^3r\, d^3p$. This function obeys the equation

$$\frac{\partial f}{\partial t} + \dot{r}\frac{\partial f}{\partial r} + \dot{p}\frac{\partial f}{\partial p} = C[f] + \alpha(r, p, t)$$
$$- |v| \int \beta(\Gamma, \Gamma') f(\Gamma')\, d\Gamma', \qquad (1)$$

where $C[f]$ stands for collisions between particles, $\alpha(r, p, t)$ stands for the rate of emission of particles in the phase volume $d\Gamma$, and $\beta(\Gamma, \Gamma')$ describes the scattering on wormholes. For the sake of convenience we also distinguished the multiplier $|v| = p/m$. Our aim is to find an explicit expression for $\beta(\Gamma, \Gamma')$.

We consider first a single wormhole, which represents a couple of conjugated spheres $S_\pm$ of the radius a and with a distance $d = |\vec{R}_+ - \vec{R}_-|$ between centers of spheres. The interior of the spheres is removed and surfaces are glued together. The gluing procedure defines the two type of wormholes passable (traversable) and impassable. The impassable wormhole appears when before gluing we turn out one of surfaces $S_\pm$. The impassable wormhole works merely as a couple of independent spherical mirrors (absolute mirrors, since they reflect gravitons as well). The passable wormhole works like a couple of conjugated mirrors, so that while an incident particle falls on one mirror the reflected particle comes from the conjugated mirror.

Consider an arbitrary point $\vec{r}$ on the sphere S_-, i.e., $\vec{r} \in S_-$ and therefore $\xi_-^2 = (\vec{r} - \vec{R}_-)^2 = a^2$. The gluing procedure transforms this point into a conjugated point $\vec{r}' \in S_+$ which has the form $\vec{r}' = \vec{R}_+ + \vec{\xi}_+$ where $\vec{\xi}_+$ relates to $\vec{\xi}_-$ by some rotation $\xi_+^\alpha = U_\beta^\alpha \xi_-^\beta$. Then for the traversable wormhole we find

$$\int \beta(\Gamma, \Gamma') f(\Gamma')\, d\Gamma' = (f - f'_+)\delta(\xi_+ - a) + (f - f'_-)\delta(\xi_- - a),$$

where we used the notations $\vec{\xi}_\pm = \vec{r} - \vec{R}_\pm$, $f'_\pm = f(r_\pm, p_\pm, t)$,

$$\vec{r}_\pm = \vec{R}_\mp + U^{\mp 1}\vec{\xi}_\pm, \qquad \vec{p}_\pm = U^{\mp 1}(\vec{p} - 2(pn_\pm)\vec{n}_\pm), \qquad (2)$$

and $\vec{n}_\pm = \vec{\xi}_\pm/a$. This defines the scattering matrix $\beta(\Gamma, \Gamma')$ for a single wormhole in the form $\beta(\Gamma, \Gamma') = \beta_+(\Gamma, \Gamma') + \beta_-(\Gamma, \Gamma')$ where

$$\beta_\pm(\Gamma, \Gamma') = \delta(\xi_\pm - a)\big[\delta(r - r')\delta(p - p') - \delta(r_\pm - r')\delta(p_\pm - p')\big]. \qquad (3)$$

In the case of a spherical mirror (i.e., of the impassable wormhole) this expression reduces to the more simple form

$$\beta(\Gamma, \Gamma') = \delta(|r - R| - a)\delta(r - r')\big[\delta(p - p') - \delta(p_1 - p')\big], \qquad (4)$$

where $\vec{p}_1 = \vec{p} - 2(pn)\vec{n}$.

Let $F(R_\pm, a, U)$ be the density of wormholes with parameters R_-, R_+, U and a, i.e.,

$$F(R_\pm, a, U) = \sum_n \delta(\vec{R}_- - \vec{R}_-^n)\delta(\vec{R}_+ - \vec{R}_+^n)\delta(a - a_n)\delta(U - U_n). \qquad (5)$$

Then the total scattering matrix is described by

$$\beta_\pm^{\text{tot}}(\Gamma, \Gamma') = \int \beta_\pm(\Gamma, \Gamma')F(R_\pm, a, U)\, d^3R_+\, d^3R_-\, dU\, da. \qquad (6)$$

We note that the distribution of wormholes (5) has in general quite irregular and random behavior and in practical problems it requires some averaging out $\bar{F}(R_\pm, a, U)$, while for a specific astrophysical object (e.g., a galaxy) it may possess sufficiently strong fluctuations $\delta F \sim \bar{F}$.

3. Topological damping of cosmic rays

In the present section we consider the first terms in the topological scattering matrix (3) and (6). Those terms define the capture of particles by wormholes which leads to a specific damping of cosmic rays. Indeed, let us neglect collisions[2] and the topological scattering in (1) and consider trajectories of particles $x(t) = x(x_0, p_0, t)$, $p(t) = p(x_0, p_0, t)$. Then we can take variables (x_0, p_0, t) as new coordinates (instead of (x, p, t)) and Eq. (1) transforms to

$$\frac{df}{dt} = \alpha(r(t), p(t), t) - |v(t)|\beta_1(r(t))f$$
$$+ |v(t)| \int \beta_2(\Gamma, \Gamma')f(\Gamma')\, d\Gamma', \qquad (7)$$

where β_1 describes the capture of particles, while β_2 describes the remission of the same particles by wormholes. Now if we consider the case when the source $\alpha(t)$ produces a single thin ray and assume that wormholes have isotopic distribution around the source, then almost all particles captured by wormholes leave the ray and will radiate from another regions of space and will have different (from the ray) directions. Then in the first order we can neglect the last term in r.h.s. of (7) and find the solution in the form

$$f = e^{-\tau}\tilde{f}, \qquad (8)$$

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[2] For the topological damping the absence of collisions is not essential though, since they modify merely the function $\tilde{f}$ in (8).

where $\tilde{f}$ obey the standard kinetic equation with topological terms omitted (i.e., $d\tilde{f}/dt = \partial\tilde{f}/\partial t + \dot{r}\partial\tilde{f}/\partial r + \dot{p}\partial\tilde{f}/\partial p = \alpha(t)$), while the optical depth $\tau(t)$ describes the damping along the ray

$$\tau(t) = \int_{t_0}^{t} \beta_1\big(r(t')\big)\big|v(t')\big|\,dt' = \int_{0}^{\ell} \beta_1\big(r(s)\big)\,ds, \tag{9}$$

where ℓ is the coordinate along the ray.

For astrophysical implications (when the characteristic width of rays $L \gg a$) we can replace $\delta(\xi_\pm - a)$ in (3) with $\pi a^2 \delta(\vec{R}_\pm - \vec{r})$ (which means that the absorption of particles occurs at the positions $\vec{R}_\pm$, i.e., we neglect the throat size a). Then, from (6) we find

$$\beta_1(r) = \pi \sum_{n,s=\pm} a_n^2 \delta(\vec{R}_s^n - \vec{r}) = \pi \int a^2 n(r,a)\,da, \tag{10}$$

where $n = n_+ + n_-$, and

$$n_s(r,a) = \int \delta(\vec{R}_s - \vec{r}) F(R_\pm, a, U)\,d^3 R_+\,d^3 R_-\,dU.$$

For the sake of simplicity we consider the case when the distribution of wormholes reduces to $\bar{F}(R_\pm, a, U) = g(a)F(R_\pm, U)$. Then the value $\beta_1(r)$ can be expressed via the density of wormholes as

$$\beta_1(r) = \pi \bar{a}^2 n(r), \tag{11}$$

where $\bar{a}^2 = \int a^2 g(a)\,da$ and $n(r) = n_+(r) + n_-(r)$ is the total density of wormholes $n_\pm(r) = \sum_n \delta(\vec{R}_\pm^n - \vec{r})$.

4. Topological bias of a point source

Consider now the case of a stationary point-like source which radiates particles in an isotropic way, i.e., $\alpha(r,p,t) = \lambda(\varepsilon)\delta(\vec{r} - \vec{r}_0)$, where $\varepsilon = \sqrt{p^2 + m^2}$ and $\lambda(\varepsilon)$ is the distribution of the rate of emission of particles over the momenta. Then if we neglect the external force ($\dot{p} = 0$), collisions, and the scattering on the wormholes the stationary solution to (1) is

$$f_0(r,p) = \frac{m\lambda(\varepsilon)}{p|r - r_0|^2}\delta(\cos\theta - \cos\theta')\delta(\varphi - \varphi'), \tag{12}$$

where θ, φ define the direction of the vector $(\vec{r} - \vec{r}_0)$ and θ', φ' that of $\vec{p}$.

When the density of wormholes is low enough the topological term can be accounted for in the next order which defines the topological bias of the source $\alpha \to \alpha + \delta\alpha_{\text{halo}}$, where the halo density is given by

$$\delta\alpha_{\text{halo}}(\Gamma) = |v| \int \beta^{\text{tot}}(\Gamma, \Gamma') f_0(\Gamma')\,d\Gamma'.$$

Such a halo has the two terms $\delta\alpha_{\text{halo}}(\Gamma) = \delta\alpha_{1,\text{halo}} + \delta\alpha_{2,\text{halo}}$, where the first term describes the damping (11) and the second term defines the remission of particles. The exact form of the halo can be found by the image method as in Ref. [12]. Indeed, if we continue the solution to the whole space (we recall that the inner region of wormholes $|\vec{r} - \vec{R}_\pm^n| < a_n$ represents the non-physical region of space), the wormholes will produce secondary sources of particles. Thus, when we neglect the throat size ($a \ll R_\pm$) and assume the isotropic distribution over the matrix U, then upon averaging over U every wormhole will radiate in the isotropic way which defines the halo as

$$\delta\bar{\alpha}_{2,\text{halo}}(\vec{r}, \vec{p}) = \lambda(\varepsilon) B_2(\vec{r}),$$

where

$$B_2(\vec{r}) = \sum_{n,s=\pm} \frac{\pi a_n^2}{|\vec{R}_s^n - \vec{r}_0|^2}\delta(\vec{r} - \vec{R}_{-s}^n), \tag{13}$$

which defines an additional distribution of particles in the form $f(r,p) = f_0(r,p) + \delta\bar{f}(r,p)$

$$\delta\bar{f}(r,p) = \frac{m\lambda(\varepsilon)}{p}\sum_{n,s=\pm}\frac{\pi a_n^2}{|\vec{R}_s^n - \vec{r}_0|^2|\vec{r} - \vec{R}_{-s}^n|^2}$$
$$\times \delta\big(\cos\theta_{-s}^n - \cos\theta'\big)\delta\big(\varphi_{-s}^n - \varphi'\big).$$

The above expressions can be re-written via the distribution (5), e.g.,

$$B_2(\vec{r}) = \int \frac{\pi a^2}{|\vec{R} - \vec{r}_0|^2}N(r,R,a)\,d^3R\,da, \tag{14}$$

where $N(r,R,a) = N_+ + N_-$ and $N_s = \int \delta(\vec{R} - \vec{R}_{-s})\delta(\vec{r} - \vec{R}_s) \times F(R_\pm, a, U)d^3R_+\,d^3R_-\,dU$ (we point out to the obvious relation $n(r,a) = \int N(r,R,a)\,d^3R$ with the distribution $n(r,a)$ in (10)).

In this manner we see that both functions the damping of cosmic rays (10) and the distribution of secondary sources (the halo density) (14) are determined via the same function $N(r,R,a)$, i.e., the distribution of wormholes which has an irregular (random) behavior. Together with $N(r,R,a)$ functions $\beta_1(r)$ and $B_2(r)$ acquire the random character. However, due to the functional dependence on the only random function $N(r,R,a)$ such quantities should exhibit a rather strong correlation.

We point out that the interpretating of the cosmic rays damping possesses an ambiguity. For instance a suppression of the cosmic ray flux could be also due to other effects, like multiple scattering in the source itself (e.g,. see Ref. [22] and references therein). Such effects produce analogous correlation between the damping and the halo of the secondary sources. Moreover, the halo of the secondary sources (14) is rather difficult to observe; for the brightness of such a halo is very low (the intensities of the secondary sources are strongly suppressed by the factor a^2/R^2, where a is the effective section of the scatterer and R is the distance to the scatterer). However, the key point which allows to disentangle this specific topological damping from other effects is that the same distribution of wormholes defines the distribution of dark matter which we discuss in the next section.

5. Dark matter halos

As it was demonstrated in Ref. [12] (see also discussions in Refs. [10,11]) the distribution of wormholes (5) defines the density of dark matter halos in galaxies as well which is much more easier to observe. Indeed, in the presence of the gas of wormholes the modification of the Newton's potential was shown to be accounted for by the topological bias of sources, i.e., $\delta(r - r_0) \to \delta(r - r_0) + b(r, r_0)$, where the halo density $b(r, r_0)$ is determined via the same distribution of wormholes (5) by expressions analogous to (13), e.g., see for details Ref. [12]. The form of the bias function $b(r, r_0)$ however admits the direct measurement by observing rotation curves of galaxies (e.g., see Ref. [6]) and for the exact form of the bias see Refs. [10,15]). Indeed, in galaxies the topological bias relates the densities of dark and luminous matter as

$$\rho_{\text{DM}}(r) = \int \bar{b}(r - r')\rho_{\text{LM}}(r')\,d^3r', \tag{15}$$

which for the Fourier transforms takes the form $\rho_{\text{DM}}(k) = \bar{b}(k) \times \rho_{\text{LM}}(k)$. And for a point mass it defines the scale-dependent renormalization of the dynamic (or the total) mass within the radius R as

$$M_{\text{tot}}(R)/M = 1 + 4\pi \int_{0}^{R} b(r)r^2\,dr. \tag{16}$$

In observations the amount of DM is defined by the mass-to-luminosity ratio M/L. It is assumed that the luminosity traces the distribution of baryons ρ_{LM} which is measured by the observing the surface brightness. E.g., spirals can be modeled by an infinitely thin disk with surface mass density distribution (surface brightness) $\rho_{LM} = \sigma e^{-r/R_D}\delta(z)$, where R_D is the disc radius (the optical radius is $R_{opt} = 3.2 R_D$). The total dynamic mass is then defined by the rotation curve analysis (or by the dispersion of velocities in ellipticals) [6].

Observations show that the mass-to-luminosity ratio $M_{tot}(r)/L(r)$ for the sphere of the radius r increases with the distance r from the center of the galaxy in all galaxies. However if in HSB (high surface brightness) galaxies this ratio exceeds slightly the unity within the optical disk $M(R_{opt})/L \gtrsim 1$ which means that there is a small amount of DM, in LSB (low surface brightness) galaxies such a ratio may reach $M(R_{opt})/L \sim 10^3$. Such a correlation between the surface brightness and the amount of DM in galaxies could give an indirect evidence for the topological nature of DM; for in accordance to (11) the amount of wormholes defines the damping of cosmic rays and analogously the amount of wormholes defines the amount of dark matter in galaxies [12]. However, the basic mechanism which forms such a feature is different (e.g., see Ref. [23] and references therein). Indeed in smaller galaxies supernovae are more efficient in removing the gas from the central (stellar forming) region of a galaxy than in bigger galaxies and this creates the fact that in smaller objects the disc has a smaller baryonic density (a lower surface brightness).

In the general case the relationship between the distribution of dark matter and that of wormholes is rather complicated, e.g., see Ref. [12]. Nevertheless, the renormalization of the intensity a point-like source (16) allows us to find a rather simple relation between the bias and the density of wormholes on scales $R \gg \bar{d}$ (where $d = |\vec{R}_+ - \vec{R}_-|$). We stress that the consideration below has a rather illustrative (or qualitative) character, while for actual measurements one has to use the exact relations in Ref. [12].

Indeed, the basic effect of a non-trivial topology is that it cuts some portion of the volume of the coordinate space. Therefore, the volume of the physically admissible region becomes smaller, while the density of particles emitted becomes higher. From the standard flat space standpoint this effectively looks as if the amplitude of a source renormalizes (16). Consider a ball of the radius r around a point-like source. E.g., for an isotropic source the number of particles emitted in the unit time in the solid angle $d\Omega = r^2 d\phi \, d\cos\theta$ remains constant $dN \sim f_0 \, d\Omega = $ const, which gives the standard distribution (12), i.e., $f_0 \sim I/4\pi r^2$.

Let us assume that wormholes have an isotropic distribution around the source and for the sake of illustration we shall assume that the distribution has also the structure $\bar{F}(R_\pm, a, U) = g(a)F(R_\pm, U)$. Then in the presence of wormholes the physical volume is

$$V_{ph}(r) = \frac{4}{3}\pi \left(r^3 - \Omega(r)\right),$$

where $\Omega(r) = 4\pi \int a^3 \int_0^r n(\tilde{r}, a)\tilde{r}^2 \, d\tilde{r}\, da$ defines the portion of the coordinate volume occupied by wormholes within the radius r and the density of wormholes $n(r, a)$ is defined in (10). Therefore, the actual value of the surface which restricts the ball is $S_{ph}(r) = \frac{d}{dr}V_{ph}(r)$ and we find for the density of particles $f \sim I/S_{ph}(r)$ which defines the renormalization of the source (13) $I(r)/I = 4\pi r^2/S_{ph}(r)$. Absolutely analogously we can use the Gauss divergency theorem to estimate the renormalization of the gravity source. Indeed, the Gauss theorem states that

$$\int_{S(R)} n\nabla G \, dS = 4\pi \int_{r<R} M\delta(r)\, dV = 4\pi M,$$

where G is the true Green function (or the actual Newton's potential). Then for isotropic distribution of wormholes it defines the normal projection of the force as $F_n(R) = n\nabla G = 4\pi M/S_{ph}(R)$. This can be rewritten as in the ordinary flat space (in terms of the standard Green function $G_0 = -1/r$ (i.e., the standard Newton's law) and the coordinate surface $S_{coor} = 4\pi R^2$) $F_n(R) = M'(R)/R^2$, where $M'(R)/M = 4\pi R^2/S_{ph}(R)$ which defines the bias function in the form (16) or

$$b(r) = \frac{1}{r^2}\frac{d}{dr}\frac{r^2}{\frac{d}{dr}V_{ph}(r)}. \tag{17}$$

We stress again that this function admits the direct measurement in galaxies [6,15]. Now by make use of the above expression for $V_{ph}(r)$ we find the behavior of the dynamic mass for a point source as

$$\frac{M_{tot}(r)}{M} = 1 + \frac{\gamma(r)}{(1 - \gamma(r))}, \tag{18}$$

where $\gamma(r) = \frac{4}{3}\pi \int a^3 n(r, a)\, da$ which can be estimated as $\gamma(r) \sim \frac{4}{3}\frac{\bar{a}^3}{\bar{a}^2}\beta_1(r)$. Thus, we see that both quantities the damping (i.e., the optical depth τ) and the amount of DM (the bias b) are expressed via the same function $n(r, a)$.

6. Conclusions

For a homogeneous density of wormholes $n(r, a) = \bar{n}(a)$ and $\beta_1(r) = \bar{\beta}_1 = $ const, the damping is determined merely as $\tau(\ell) = \bar{\beta}_1 \ell$ where ℓ is the coordinate along the ray. Thus, the damping defines the characteristic scale[3] $L = 1/\bar{\beta}_1$ which has the order $L \sim \bar{a}/\gamma$ (where $\gamma = (\bar{a}/\lambda)^3$, $\lambda^3 \sim 1/\bar{n}$ is the volume per one wormhole, and $\bar{a}$ is a characteristic size of throats). The parameter γ can be extracted from observations of DM in galaxies, while the scale $\bar{a}$ represents here a free parameter which should be fixed from some additional and independent considerations. E.g., for the homogeneous distribution of wormholes the value of $\bar{a}$ defines the amount of dark energy. Indeed, consider one wormhole in the Minkowski space. Then the metric can be taken in the form (e.g., see Ref. [12])

$$ds^2 = dt^2 - f^2(r)\left(dr^2 + r^2 \sin^2\vartheta \, d\phi^2 + r^2 \, d\vartheta^2\right),$$

where $f(r) = 1 + \theta(a - r)(\frac{a^2}{r^2} - 1)$ and $\theta(x)$ is the step function.[4] Both regions $r > a$ and $r < a$ represent portions of the ordinary flat Minkowski space and therefore the curvature is $R^k_l \equiv 0$. However on the boundary $r = a$ it has the singularity which defines the scalar curvature as $R = -T = \frac{1}{a}\delta(r - a)$ where T stands for the trace of the stress energy tensor which one has to add to the Einstein equations to support such a wormhole. It is clear that such a source violates the averaged null energy condition, i.e., $T = \varepsilon + 3p < 0$ (e.g., for the Friedmann space this results in an acceleration of the scale factor $\sim t^\alpha$ with $\alpha = \frac{2\varepsilon}{3(\varepsilon+p)} > 1$), i.e., represents a form of dark energy. Every wormhole gives contribution $\int T r^2 \, dr \sim a$ to the dark energy, while the DE density is $\epsilon = \int an(a, r)\, da \sim \gamma/\bar{a}^2$. Thus, the parameter $\bar{a}$ can in principle be extracted from DE density observations. We note however that one has to be careful in using such a parameter in galaxies, since in the general case the value $\bar{a}$ is scale dependent (e.g., for the fractal distribution of wormholes the mean value is unstable).

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[3] We point out that such scale has only statistical meaning, since the actual distribution of wormholes cannot be utterly homogeneous, otherwise rays could not reach a sufficiently remote observer. In particular, there is evidence for the fractal structure of space (e.g., see discussions in Refs. [10,15]) which means that there always exist geodesics along which light propagates almost without the scattering.

[4] One can replace $f(r)$ with a smooth function, this however will not change the subsequent estimates.

At the optical radius r_{opt} a galaxy can be considered already as a point-like source of gravity and, therefore, for estimates we can use (18) instead of (15) and (17). In HSB galaxies the amount of dark matter within the optical radius r_{opt} is rather small $M/L \gtrsim 1$ which gives $M_{dyn}(r_{opt})/M \sim 1 + \gamma(r_{opt})$ with $\gamma(r_{opt}) \ll 1$ (i.e., $\lambda^3/\bar{a}^3 \gg 1$) and we can expect the topological damping to be negligible ($\tau \ll 1$). In LSB galaxies the mas-to-luminosity ratio may reach $M/L \sim 10^3$ which gives $\gamma(r_{opt}) \sim 1$ and we can expect a considerable damping $\tau \sim 1$. From the qualitative standpoint this feature agrees with the observed correlation between the surface brightness and the amount of dark matter in galaxies which can be considered as an indirect evidence for the topological nature of dark matter. The interpretation of such a feature is ambiguous though (e.g., see Ref. [23] and for other effects which lead to suppression of the cosmic ray flux see Ref. [22]). However we point out that both quantities γ and τ are functions of the same random distribution of wormhole $n(a, r)$ and therefore they should exhibit a rather strong correlation which may allow to verify the topological nature of DM.

Presumably, astrophysical objects which may also be used to test the topological nature of DM are large scale extragalactic relativistic jets in quasars, e.g., see Ref. [24] and references therein. The smaller jets which are widely observed in active galactic nuclei can also be used in LSB galaxies, where the amount of DM is considerable. However the crucial step here is the exact knowledge of the launching mechanism which may allow to find the discrepancy between the predicted profile of a jet and the actually observed one.

Acknowledgements

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