Implications of μ-τ flavored CP symmetry of leptons

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ABSTRACT: We discuss gauge models incorporating μ-τ flavored CP symmetry (called $\mathsf{CP}^{\mu\tau}$ in the text) in combination with $L_\mu - L_\tau$ invariance to understand neutrino mixings and discuss their phenomenological implications. We show that viable leptogenesis in this setting requires that the lightest right-handed neutrino mass must be between 10^9–10^{12} GeV and for effective two hierarchical right-handed neutrinos, leptogenesis takes place only in a narrower range of 5×10^{10}–10^{12} GeV. A multi-Higgs realization of this idea implies that there must be a pseudoscalar Higgs boson with mass less than 300 GeV. Generically, the vev alignment problem can be naturally avoided in our setting.

Contents

1 Introduction

Phenomenal success of experimental research in neutrino physics in the last two decades have led not only to unequivocally establishing that neutrinos have mass but also to an almost complete determination of flavor mixings between the different lepton generations. The missing parts are: (i) the Dirac CP phase, (ii) neutrino mass hierarchy and (iii) a knowledge of whether neutrinos are Majorana or Dirac fermions. Assuming that there are no extra sterile neutrinos, the discovery of the CP phase for neutrinos would put flavor information on leptons on the same footing as quarks. If neutrinos are Majorana fermions, there would be two more phases present in the flavor space and for complete information, one will need information on them. The latest global fits [1, 2] of neutrino parameters point to a preference for a negative value for the Dirac CP phase, $-\pi < \delta_{\mathsf{CP}} < 0$. A key focus of experimental research in neutrino physics at the moment is therefore to determine the Dirac CP phase in addition to answering the question of whether neutrinos are Dirac or Majorana particles and their mass hierarchy. An additional motivation to determine

the Dirac CP phase comes from its possible connection to understanding the origin of matter and anti-matter asymmetry in the universe via leptogenesis [3, 4]. While it is well known that non-observation of a non-zero Dirac CP phase does not preclude leptogenesis, its observation can nonetheless provide important insight into the latter [5–8].

On the theory front, understanding of the lepton mixing angles θ_{ij} has been one of the two major driving forces of much of the research in this field, the other being to probe the scale of neutrino masses. In the former case, symmetries have been used as a main tool, motivated by the observation that mixing angles $\theta_{23} \sim \frac{\pi}{4}$ and $\sin\theta_{12} \sim \frac{1}{\sqrt{3}}$, suggesting their possible group theoretic origin [9, 10]. Among the very first symmetries studied for neutrinos is the μ-τ exchange symmetry [11–23], which not only predicted maximal θ_{23} but also that $\theta_{13} = 0$. Many other symmetries such as S_4, A_4, $\Delta(3n^2)$, etc., were considered later on. The so-called tri-bi-maximal (TBM) mixing pattern [24–26] which embodied all these three features, i.e., $\theta_{23} \sim \frac{\pi}{4}$, $\sin\theta_{12} \sim \frac{1}{\sqrt{3}}$ as well as $\theta_{13} = 0$, together with the symmetry techniques to obtain this pattern, gave a big boost to this approach. Discovery of a non-zero and large value for θ_{13} [27–31] was a turning point in this research since it ruled out the tri-bi-maximal mixing pattern. Since then, many attempts have been made to combine flavor symmetries with CP transformation to accommodate a non-zero θ_{13} while trying to predict the Dirac CP phase [32–51], sometimes without imposing CP explicitly [52–54].

In this paper, we pursue this line of research and consider a simple approach based on a generalized definition of CP transformation that mixes it with μ-τ exchange (called $\mathsf{CP}^{\mu\tau}$ from now on) [32–34]. This symmetry is known to accommodate a non-zero θ_{13} while at the same time predicting a Dirac CP phase $\delta \sim \pm 90^0$ [32–34, 52] if the charged lepton mass matrices are taken diagonal. There are also models where one has deviations from the exact $\mathsf{CP}^{\mu\tau}$ limit [55, 56]. A key challenge to building such models has been that in the $\mathsf{CP}^{\mu\tau}$ symmetry limit, the muon and tau lepton Yukawa couplings are degenerate, leading to same masses. In ref. [32–34], explicit soft breaking terms were introduced to generate the $\mu\tau$ mass splitting. Another uncomfortable feature of these models has been its apparent inability to explain the origin of matter via leptogenesis [32–34]. We address both these issues in this paper. Our goal is to present a model where starting with a high scale symmetry, we find a low energy effective theory where the neutrino sector maintains exact $\mathsf{CP}^{\mu\tau}$ symmetry whereas in the charged lepton sector, the symmetry is spontaneously broken so as to allow the muon and tau masses to be different. We give two examples: one with an extended Higgs sector and another with an extension involving heavy vector like fermions. The former has interesting implications for Higgs physics that we discuss below. We also show that there exists a limited range of seesaw scales where successful leptogenesis can take place, when lepton flavor effects are taken into account.

As a part of this investigation, we also identify the combination of family lepton numbers $\mathsf{L}_\mu - \mathsf{L}_\tau$ [57, 58] (which we denote as $\mathrm{U}(1)_{\mu-\tau}$) as the largest natural abelian symmetry that can be imposed in conjunction with $\mathsf{CP}^{\mu\tau}$, thus providing the simplest example of combining an abelian symmetry with CP, yet with predictive CP violation at low energies. We arrive then at a natural setting where $G_l = \mathrm{U}(1)_{\mu-\tau}$ can be the residual symmetry of the charged lepton sector (ensuring diagonal mass matrix) and $G_\nu = \mathbb{Z}_2^{\mathsf{CP}}$, generated by

$\mathsf{CP}^{\mu\tau}$, is the residual symmetry of the neutrino sector. Because of the properties of $\mathrm{U}(1)_{\mu-\tau}$ and $\mathsf{CP}^{\mu\tau}$, these residual symmetries can be maintained separately in each sector without perturbing interactions in the scalar potential, thus avoiding the vev alignment problem of flavor symmetry models with larger nonabelian groups.

New results of the paper are: (i) construction of a model with natural residual symmetries G_l and G_ν but without soft breaking of $\mathsf{CP}^{\mu\tau}$; (ii) discussion of how one can implement successful leptogenesis in these models and constraints imposed by it on the seesaw scale and (iii) implications for neutrino-less double beta decay and Higgs physics.

This paper is organized as follows: in section 2, we review the consequences of $\mathsf{CP}^{\mu\tau}$ on the neutrino mass matrix and PMNS. Sections 3 and 4 present general consequences of $\mathsf{CP}^{\mu\tau}$ symmetry on neutrino-less double beta decay and leptogenesis. In section 5, we introduce the generalized CP like symmetries and show how $\mathsf{CP}^{\mu\tau}$ symmetry emerges as the trivial automorphism of gauged $\mathrm{U}(1)_{\mu-\tau}$ symmetry. We then present a multi-Higgs implementation of the symmetry in section 6, together with some phenomenological implications. Our paper is summarized in section 7. The appendices contain the proof of the uniqueness of $\mathsf{CP}^{\mu\tau}$, the $\mathsf{CP}^{\mu\tau}$ symmetry in the real basis and another realization of the idea where $G_l \times G_\nu$ is exact at high energies, which uses heavy vector like fermions instead of extra weak scale Higgs doublets.

2 Maximal θ_{23} and Dirac CP phase from $\mathsf{CP}^{\mu\tau}$: a review

The latest global fits [1, 2] of neutrino parameters still allows maximal atmospheric angle $\theta_{23} = 45°$ within 2σ and also point to a preference for negative values for the Dirac CP phase, $-180° < \delta_{\mathsf{CP}} < 0$. It was pointed out in [32–34] that maximal θ_{23} and maximal δ_{CP}, i.e.,

$$\theta_{23} = \pi/4 \ \text{ and } \ \delta_{\mathsf{CP}} = \pm\pi/2\,, \tag{2.1}$$

follow from the neutrino mass matrix invariant under $\mathsf{CP}^{\mu\tau}$ symmetry. In the flavor basis (fixed by some G_l), it corresponds to the relation:

$$X^{\mathsf{T}} M_\nu X = M_\nu^*\,, \tag{2.2}$$

where

$$X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\,. \tag{2.3}$$

Clearly, this symmetry can be implemented in the neutrino sector as the composition of $\mu\tau$ interchange symmetry with CP conjugation. We will show a simple and natural setting where this symmetry survives in the neutrino sector but is broken in the charged lepton sector.

Let us review some aspects of $\mathsf{CP}^{\mu\tau}$. First, the symmetry (2.2) implies a neutrino mass matrix of the form [32–34, 52]

$$M_\nu = \begin{pmatrix} a & d & d^* \\ d & c & b \\ d^* & b & c^* \end{pmatrix}\,, \tag{2.4}$$

where a, b are real whereas c, d are complex a priori. It is necessary that both $c \neq 0$, $d \neq 0$, and $\text{Im}(d^2 c^*) \neq 0$, to ensure $\theta_{13} \neq 0$ [33] because a rephasing transformation can turn M_ν to a matrix invariant under the simpler (unitary) $\mu\tau$ interchange symmetry.

One can show that a matrix of the form (2.4) can be always diagonalized by a matrix of the form [32–34]

$$U_0 = \begin{pmatrix} u_1 & u_2 & u_3 \\ w_1 & w_2 & w_3 \\ w_1^* & w_2^* & w_3^* \end{pmatrix}, \tag{2.5}$$

where u_i are real and conventionally positive. Application of complex conjugation on M_ν and U_0 shows that the diagonalization of (2.4),

$$U_0^{\mathsf{T}} M_\nu U_0 = \text{diag}(\pm m_i), \tag{2.6}$$

already leads to real diagonal entries, so that the Majorana phases are trivial, i.e., either 1 or i. Therefore, we can write for the complete diagonalization matrix,

$$U_\nu = U_\nu^{(0)} K_\nu, \tag{2.7}$$

where $U_\nu^{(0)}$ has the form (2.5) and K_ν is diagonal and contains the Majorana phases $(K_\nu)_{ii} = 1$ or i. We denote the different possibilities by

$$\text{diagonal of } K_\nu^2 \sim (+++), (-++), (+-+) \text{ or } (++-), \tag{2.8}$$

which correspond to the CP parities of ν_{iL} assuming $\mathsf{CP}^{\mu\tau}$.

It is easy to see that U_0 obeys

$$|(U_0)_{\mu j}| = |(U_0)_{\tau j}|, \quad \text{for } j = 1, 2, 3. \tag{2.9}$$

The equality for $j = 3$ signals maximal θ_{23}. The equality for $j = 1, 2$, easily seen in the standard parametrization, leads to [32–34]

$$\sin \theta_{13} \sin \delta_{\mathsf{CP}} = 0. \tag{2.10}$$

This signals maximal δ_{CP} since $\theta_{13} \neq 0$.

3 Neutrino-less double beta decay in theories with $\mathsf{CP}^{\mu\tau}$

For Majorana neutrinos, there is a nonzero probability of neutrino-less double beta decay to occur. The rate depends on the square of the modulus of

$$m_{ee} \equiv \sum_i m_i U_{ei}^2. \tag{3.1}$$

In general, this quantity depends on the Dirac CP phase (depending on the convention) and Majorana CP phases. For the theory invariant under $G_\nu = \mathbb{Z}_2^{\mathsf{CP}}$ and $G_l \subset \mathrm{U}(1)_{\mu-\tau}$, $\delta_{\mathsf{CP}} = \pm\pi/2$, only a discrete choice of possibilities for the Majorana phases remain. We obtain

$$m_{ee} = \sum_i m_i' U_{ei}^{(0)2}, \tag{3.2}$$

where $U_{ei}^{(0)}$ are real positive quantities fixed by θ_{12}, θ_{13}, cf. (2.7), and $m_i' = \pm m_i$ are the light neutrino masses with its CP parities.
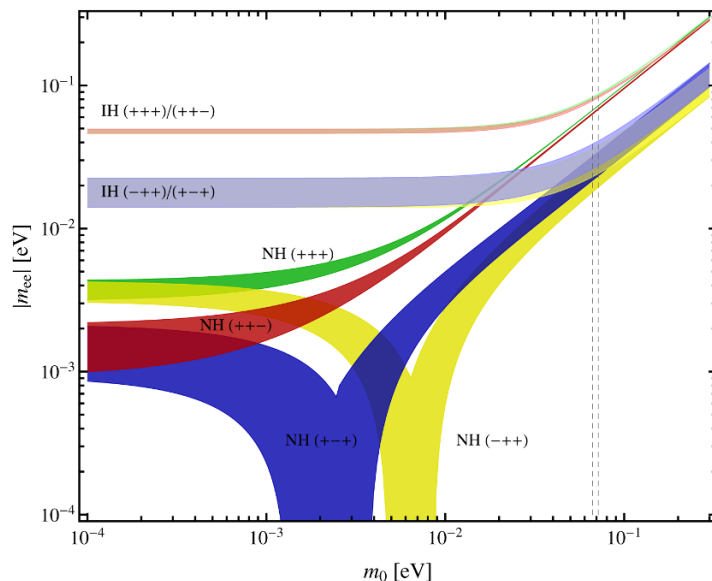
Figure 1. $|m_{ee}|$ as a function of the lightest mass m_0 (m_1 for NH and m_3 for IH) for CP parities $K^2_{\nu ii}$ of the light neutrinos ν_{iL}: $(+++)$ (green), $(-++)$ (yellow), $(+-+)$ (blue) and $(++-)$ (red). Darker colors denotes NH and lighter colors denotes IH. For the latter, light blue and yellow (light red and green) are largely overlapped. We use the 3-σ allowed ranges for $\Delta m^2_{21}, \Delta m^2_{31}, \theta_{12}, \theta_{13}$ of ref. [2]. The vertical dashed lines shows the current bound coming from the cosmological data on $\sum m_i$; cf. (3.3).

In figure 1 we show the discrete possibilities for $|m_{ee}|$ as a function of the lightest neutrino mass m_0 (m_1 for NH and m_3 for IH). We vary Δm^2_{21}, Δm^2_{31}, θ_{12}, θ_{13} within their 3-σ allowed values [2] ($\theta_{23} = \pi/4$ is fixed from symmetry). We can see that some CP parities can be distinguished if independent information on the mass hierarchy and sufficiently precise information of the absolute mass scale is known. Specially for IH, we can distinguish between $(+ + +)/(+ + -)$ CP parities for ν_L and $(- + +)/(+ - +)$. For NH, some cases can be distinguished for some ranges of the absolute mass scale. For example the $\tilde{S}_4$ ($A_4 \rtimes \mathbb{Z}^{\mathsf{CP}}_2$) model of ref. [37] lies in the lower (NH) yellow $(- + +)$ band. With enough precision, even in the quasi-degenerate spectrum we can distinguish between $(+++)/(++-)$ and $(-++)/(+-+)$ CP parities. Notice that some bands would completely overlap in the $m_0 \to 0$ limit. Regions similar to the ones we show here can be seen, in the general phenomenological analysis of ref. [59] (see its figure 2 with dashed curves denoted as $(\pm\pm)$), but without the underlying symmetry discussion. Note that this predictions for neutrinoless double beta decay is the same as for the strictly CP conserving case at low energies but in our case the Dirac CP phase is maximal instead of being 0 or π, a fact that can be distinguished in future oscillation experiments.

Also shown in figure 1 are the cosmological bounds for m_0,

$$\begin{aligned} \text{NH}: \quad & m_0 = m_1 < 0.0716\,\text{eV}\,, \\ \text{IH}: \quad & m_0 = m_3 < 0.0665\,\text{eV}\,. \end{aligned} \tag{3.3}$$

These values are obtained from the cosmological bound of $\sum m_i < 0.23$ at 95% C.L. reported by the Planck collaboration [60] when 3-σ range of Δm^2_{21} and Δm^2_{31} are considered.

4 Leptogenesis

Neutrino mass mechanisms are widely considered to have a connection to the origin of matter via leptogenesis [61–63]. In this section, we discuss this in the class of models we are discussing here. The first consideration of leptogenesis with $\mathsf{CP}^{\mu\tau}$ symmetry was made in [33]. The authors concluded that leptogenesis is not possible because $\mathsf{CP}^{\mu\tau}$ invariance of the neutrino sector ensured that all elements $(\lambda\lambda^\dagger)^2_{ij}$ were real leading to vanishing CP asymmetry, with λ being the N_R Yukawa coupling in the basis where the RHNs are mass eigenstates. Such a conclusion, however, is only valid for the case where the heavy neutrinos are hierarchical and charged lepton flavor effects are unimportant (the so-called one-flavor approximation), i.e., for $T \sim M_1 \gtrsim 10^{12}\text{GeV}$, where M_1 is the mass of the lightest right-handed neutrino. Below that temperature, the tau lepton enters into thermal equilibrium due to its Yukawa interaction with τ_R and flavor effects must be considered (the so called flavored leptogenesis [64, 65]). We will see that successful leptogenesis is possible even with $\mathsf{CP}^{\mu\tau}$ symmetry in the intermediate range $10^9\text{GeV} \lesssim M_1 \lesssim 10^{12}\text{GeV}$ if flavor effects are taken into account. Therefore, we do not need small $\mathsf{CP}^{\mu\tau}$ breaking for successful leptogenesis as in ref. [56]. Surprisingly, $\mathsf{CP}^{\mu\tau}$ symmetry seems to preclude successful leptogenesis for $M_1 \lesssim 10^9\text{GeV}$ for hierarchical heavy right-handed neutrinos because both τ and μ flavors are in thermal equilibrium; see section 4.1. This result holds even if the resonant enhancement of CP asymmetries due to quasi-degenerate heavy right-handed neutrinos are considered; see section 4.2.

To prove our assertion, let us first review the consequences of $\mathsf{CP}^{\mu\tau}$ on the quantities relevant for leptogenesis. It is clear from the form of U_ν in (2.7) that $\mathsf{CP}^{\mu\tau}$ implies the CP property

$$XU_\nu^* = U_\nu K_\nu^2 \quad \text{or} \quad U_\nu^* = X^\dagger U_\nu K_\nu^2 \,. \tag{4.1}$$

This can be also generically inferred from the relation (2.2). As can be checked explicitly in the CP-basis, K_ν^2 corresponds to the CP parities of ν_{iL} considering $\mathsf{CP}^{\mu\tau}$ is conserved in the neutrino sector. A similar relation is also valid for U_R, the matrix that diagonalizes M_R:

$$U_R^* = XU_R K_R^2 \quad \text{and} \quad U_R = U_R^{(0)} K_R \,. \tag{4.2}$$

Note that the previous relation assumes M_R is in the symmetry basis. We also assume the charged lepton mass matrix (squared) is diagonal (flavor basis) so that the PMNS matrix is $U = U_\nu$.

Let us write the type-I seesaw Lagrangian in the form

$$-\mathcal{L} = y_\alpha \bar{L}_\alpha H l_{\alpha R} + \bar{N}_{iR}\lambda_{i\alpha}\tilde{H}^\dagger L_\alpha + M_i \bar{N}_{iR} N_{iR}^c \,, \tag{4.3}$$

where the sum of repeated indices is implicit. In this basis, the CP asymmetries depend only on λ and the heavy masses M_i.

In the symmetry basis, λ_{sym} obeys

$$X^\dagger \lambda_{\text{sym}} X = \lambda_{\text{sym}}^* \,. \tag{4.4}$$

In the basis of (4.3), we have

$$\lambda = U_R^\dagger \lambda_{\text{sym}} \,, \tag{4.5}$$

and it obeys

$$\lambda^* = K_R^2 \lambda X \,. \tag{4.6}$$

4.1 Hierarchical heavy neutrinos

We can see the consequences of $\mathsf{CP}^{\mu\tau}$ on leptogenesis for the case where the right-handed neutrinos N_{iR} have hierarchical masses and only the decay of lightest state N_1 is relevant for leptogenesis. Our discussion, however, apply also to cases where the hierarchy is mild. In our notation, the flavored CP asymmetries for the decay $N_1 \to l_\alpha + \phi$, $\alpha = e, \mu, \tau$, read (see e.g. [61–63])

$$
\begin{aligned}
\epsilon_\alpha = \frac{1}{8\pi(\lambda\lambda^\dagger)_{11}} \sum_{j\neq 1} &\left\{ \mathrm{Im}\left[\left(\lambda\lambda^\dagger\right)_{j1} \lambda_{j\alpha}\lambda_{1\alpha}^*\right] g(x_j) \right. \\
&\left. + \mathrm{Im}\left[\left(\lambda\lambda^\dagger\right)_{1j} \lambda_{j\alpha}\lambda_{1\alpha}^*\right] \frac{1}{1-x_j} \right\},
\end{aligned}
\tag{4.7}
$$

where $x_j \equiv M_j^2/M_1^2$ and

$$g(x) \equiv \sqrt{x}\left[\frac{1}{1-x} + 1 - (1+x)\ln\left(\frac{1+x}{x}\right)\right] \equiv \frac{\sqrt{x}}{1-x} + f(x)\,, \tag{4.8}$$

where $f(x)$ is the vertex function. The part proportional to $f(x)$ corresponds to the one-loop vertex contribution while the rest corresponds to the self-energy contribution for N_R. We are assuming that N_{iR} masses are hierarchical, i.e., $M_3 - M_1 > M_2 - M_1 \gg \Gamma_1$. We comment on the possibility of resonant enhancement in section 4.2.

Now if we apply the symmetry properties (4.6) of λ in (4.7), we conclude that

$$\epsilon_e = 0\,, \quad \epsilon_\mu = -\epsilon_\tau\,. \tag{4.9}$$

For example, note that $\lambda_{j\mu}^* = K_{Rjj}^2 \lambda_{j\tau}$ and $K_{Rjj}^4 = 1$ for all j. The $\mathsf{CP}^{\mu\tau}$ symmetry also relates the μ and τ washout parameters as

$$\tilde{m}_\mu = \tilde{m}_\tau\,, \tag{4.10}$$

where

$$\tilde{m}_\alpha \equiv \frac{|\lambda_{1\alpha}|^2 v^2}{M_1}\,, \tag{4.11}$$

and $v = 174\text{GeV}$ in the SM; they quantify the strength of N_1 decay and also its inverse decays into L_α. Therefore, it is clear that the CP asymmetries for the N_1 decaying into all flavors,

$$\epsilon^{(1)} = \epsilon_e + \epsilon_\mu + \epsilon_\tau\,, \tag{4.12}$$

is vanishing and leptogenesis at the high scale $T \sim M_1 \gtrsim 10^{12}\text{GeV}$ can not proceed.

When $M_1 \lesssim 10^{12}\text{GeV}$, the tau Yukawa interactions enter in equilibrium (also the muon flavor below 10^9GeV) and distinct leptonic flavors may contribute differently to leptogenesis. In this case, the residual baryon asymmetry can be written as [61–65]

$$Y_{\Delta B} \simeq \frac{12}{37} Y_{N_1}^{\text{eq}} \sum_\alpha \epsilon_\alpha \eta_\alpha\,, \tag{4.13}$$

where the sum over α is performed only over the flavors that can be resolved by interactions at the period of leptogenesis (one, two or three flavors). The quantity $Y_{N_1}^{\text{eq}}$ is the thermal density of N_1 per total entropy density and is given by $Y_{N_1}^{\text{eq}} = \frac{135\zeta(3)}{4\pi^4 g_*} \approx 3.9 \times 10^{-3}$, where the last numerical value is for the SM degrees of freedom below the N_1 mass ($g_* = 106.75$). The factor $12/37$ corresponds to the reduction of asymmetry in $\Delta_\alpha = \text{B}/3 - \text{L}_\alpha$ to $\text{B} - \text{L}$ in the SM due to sphalerons.[1]

When $10^9 \lesssim M_1 \lesssim 10^{12}$GeV only the τ Yukawa interactions are in equilibrium and then only the τ flavor and its orthogonal combination are resolved by interactions. In this case, the asymmetry in (4.13) can be approximated by

$$Y_B \simeq \frac{12}{37} \times Y_{N_1}^{\text{eq}} \times \left[\epsilon_2 \eta\left(\frac{417}{589}\tilde{m}_2 \right) + \epsilon_\tau \eta\left(\frac{390}{589}\tilde{m}_\tau \right) \right], \qquad (4.14)$$

where $\epsilon_2 = \epsilon_e + \epsilon_\mu$, $\tilde{m}_2 = \tilde{m}_e + \tilde{m}_\mu$, and

$$\eta(\tilde{m}_\alpha) \simeq \left(\left(\frac{\tilde{m}_\alpha}{2.1 m_*} \right)^{-1} + \left(\frac{m_*/2}{\tilde{m}_\alpha} \right)^{-1.16} \right)^{-1}. \qquad (4.15)$$

The mass $m_* \equiv \frac{16\pi^2 v_u^2}{3 M_{\text{pl}}} \sqrt{\frac{g_* \pi}{5}} \approx 1\,\text{meV}$ quantifies the expansion rate of the Universe. The factors $417/589$ and $390/589$ correspond to the diagonal entries of the A matrix and quantifies the effects of flavor in the washout processes when changing from the asymmetry in lepton doublets to asymmetries in Δ_α, see e.g. [61–63]. We can see that the properties (4.9) of $\text{CP}^{\mu\tau}$ leads to a partial cancellation of the baryon asymmetry in (4.14) but it is nonzero because the τ flavor and its orthogonal combination are washed out differently. The question is then quantitative. We show some cases leading to successful leptogenesis in section 4.3.

For $M_1 \lesssim 10^9$GeV, the μ Yukawa interactions are also fast enough so that the three flavors can be resolved. For such a low scale, the CP asymmetries are usually too small to lead to a successful leptogenesis. In the $\text{CP}^{\mu\tau}$ symmetric case, the baryon asymmetry is in fact vanishing. With the three flavors resolved, the baryon asymmetry can be approximated by

$$Y_B \simeq \frac{12}{37} \times Y_{N_1}^{\text{eq}} \times \left[\epsilon_e \eta\left(\frac{151}{179}\tilde{m}_e \right) + \epsilon_\mu \eta\left(\frac{344}{537}\tilde{m}_\mu \right) + \epsilon_\tau \eta\left(\frac{344}{537}\tilde{m}_\tau \right) \right]. \qquad (4.16)$$

Due to the properties (4.9) and (4.10), the baryon asymmetry vanishes within this analytic approximation. Note that this is true even for mild hierarchies for M_i and the leptogenesis scale cannot be lowered by tuning the values of the masses.

Therefore, as long as $\text{CP}^{\mu\tau}$ symmetry is valid at the leptogenesis scale, the *only* temperature range for which leptogenesis might be viable for hierarchical N_{iR} is the intermediate scale $T \sim M_1$ where

$$10^9\text{GeV} \lesssim M_1 \lesssim 10^{12}\text{GeV}. \qquad (4.17)$$

It is worth emphasizing that CP violation in our case comes from maximal Dirac CP phase of the low-energy sector thereby giving a symmetry setting for some scenarios of

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[1]For the case of two Higgs doublets, this factor is 10/31 but numerically very close.

leptogenesis driven by low-scale CP violation [5–8]. All these properties follow from the G_l conservation in the charged lepton sector and $\mathsf{CP}^{\mu\tau}$ conservation of the neutrino sector; see section 5.

4.2 Resonant leptogenesis

For the usual type-I seesaw scenario, the CP asymmetry produced by N_1 decay usually decreases as we lower the mass of N_1 since the Yukawa couplings decrease and also the washout effects get stronger. For $M_1 \ll 10^9 \text{GeV}$, successful leptogenesis is not possible for hierarchical N_{iR}. However, when some of the masses, say M_1 and M_2, are quasi-degenerate, it is possible to resonantly enhance the CP asymmetry leading to the resonant leptogenesis scenario [66]. In fact, (4.7) is singular in that limit because perturbation theory breaks down. We can regulate such a behavior by resummation methods [66]. We will see in the following that $\mathsf{CP}^{\mu\tau}$ still leads to (4.9) and it largely suppresses the CP asymmetries if μ and τ flavors have equal washout strengths.

Suppose $M_3 \gg M_2 \approx M_1$ and also the resonant condition

$$M_2 - M_1 \sim \Gamma_{1,2} \ll M_{1,2} \,. \tag{4.18}$$

The resummed flavored CP asymmetry for $N_1 \to L_\alpha + \phi$, neglecting M_3 and vertex contributions, can be approximated by [66] (see also [67])

$$\epsilon_\alpha^{(1)} \approx f_{\text{reg}}^{12} \frac{\text{Im}\big[(\lambda\lambda^\dagger)_{21}\lambda_{1\alpha}^*\lambda_{2\alpha}\big] + \frac{M_1}{M_2}\text{Im}\big[(\lambda\lambda^\dagger)_{12}\lambda_{1\alpha}^*\lambda_{2\alpha}\big]}{(\lambda\lambda^\dagger)_{11}(\lambda\lambda^\dagger)_{11}} \,, \tag{4.19}$$

where

$$f_{\text{reg}}^{12} \equiv \frac{\big(M_1^2 - M_2^2\big)M_1\Gamma_2^{(0)}}{\big(M_1^2 - M_2^2\big)^2 + \big(M_1\Gamma_2^{(0)}\big)^2} \,. \tag{4.20}$$

One can see that (4.19) is a regulated version of (4.7), neglecting the contribution of $f(x)$ (vertex) and regulating the function $\sqrt{x_2}/(1-x_2)$ by f_{reg}^{12}. See [67] for a discussion about other regulator functions used in the literature. The N_2 decay is also resonantly enhanced as

$$\epsilon_\alpha^{(2)} \approx \epsilon_\alpha^{(1)} \,. \tag{4.21}$$

Thus with appropriate λ we can have an enhanced CP asymmetry of order one compared to $\epsilon \sim 10^{-6}$ required for successful leptogenesis in the conventional case.

Now, since the Yukawa structure in (4.19) is the same as in the hierarchical case (4.7), the consequences of $\mathsf{CP}^{\mu\tau}$ are the same: the flavored CP asymmetries $\epsilon_\alpha^{(1)}, \epsilon_\alpha^{(2)}$ obey (4.9). Therefore, if the effects of washout for μ and τ flavors are the same, the CP asymmetries for μ and τ will cancel each other precluding leptogenesis even when $M_1 \sim M_2 \lesssim 10^9 \text{GeV}$. This would be the case in the analytic approximation (4.16) arising from the classical Boltzmann equation solutions. However, to properly quantify the baryon asymmetry, including washout effects, a full flavored and quantum description is necessary and we will not address it here. Moreover, when the three right-handed neutrinos are quasi-degenerate, a more complicated expression holds for the CP asymmetries [67] and it is not clear if the properties (4.9) will still hold.

4.3 Quantitative analysis and N_3 decoupled case

To assess quantitatively if leptogenesis can be successful with $G_F = G_l \times G_\nu$ symmetry, we can use the Casas-Ibarra parametrization that uses a complex orthogonal matrix R:

$$R = \hat{M}_R^{-1/2}(\lambda v)U_\nu \hat{M}_\nu^{-1/2}\,, \tag{4.22}$$

where the hatted matrices correspond to the diagonalized matrices and λ is in the basis (4.3).

We can see that the $\mathsf{CP}^{\mu\tau}$ symmetry implies

$$R^* = K_R^2 R K_\nu^2\,. \tag{4.23}$$

This means that there is no CP violating effect coming from R when there is $\mathsf{CP}^{\mu\tau}$ symmetry. A similar result was found for usual CP symmetry in [5–8]. CP invariance in R is more apparent if we eliminate the potential purely imaginary i factors as in

$$R = K_R^* R^{(0)} K_\nu\,. \tag{4.24}$$

where $R^{(0)}$ is a real matrix, as can be seen from the properties of R. Therefore, $R^{(0)}$ obeys

$$R^{(0)\mathsf{T}} K_R^2 R^{(0)} = K_\nu^2\,, \quad R^{(0)} K_\nu^2 R^{(0)\mathsf{T}} = K_R^2\,. \tag{4.25}$$

This is just the defining relation for a *real* orthogonal matrix when $K_R^2 = K_\nu^2 = \mathbb{1}$ or a *real* hyperbolic[2] $R^{(0)}$ in O(2,1), when $K_R^2 = K_\nu^2 = \mathrm{diag}(-1,1,1)$ or any independently permuted diagonal entries for K_R^2 or K_ν^2. There is no other possibility and we conclude that the CP parities of ν_{iL} (N_{iR}) are either all equal or only one is different.

When M_i are hierarchical, the flavored CP asymmetries in (4.7) can be approximated to [5–8, 61–63]

$$\epsilon_\alpha = -\frac{3M_1}{16\pi v^2}\frac{\mathrm{Im}\left\{\sum_{ij}\sqrt{m_i m_j}\,m_j R_{1i}R_{1j}U^*_{\alpha i}U_{\alpha j}\right\}}{\sum_j m_j |R_{1j}|^2}\,, \tag{4.26}$$

where $M_1 \ll M_2, M_3$ is assumed. One can check (4.12) also in this form from the properties for R and $U_{\alpha j}$ in eqs. (4.1) and (4.24). Hence we only need ϵ_τ.

If we eliminate the CP parities K_ν, K_R, we obtain

$$\epsilon_\tau = -\frac{3M_1'}{16\pi v^2}\frac{\sum_{ij}\sqrt{m_i m_j}\,m_j' R_{1i}^{(0)}R_{1j}^{(0)}\mathrm{Im}\left\{U_{\tau i}^{(0)*}U_{\tau j}^{(0)}\right\}}{\sum_j m_j\left(R_{1j}^{(0)}\right)^2}\,, \tag{4.27}$$

where $M_1' = (K_R)_{11}^2 M_1 \equiv \pm M_1$ and $m_j' \equiv (K_\nu)_{jj}^2 m_j = \pm m_j$ are the masses including the CP parities. We can simplify further as

$$\epsilon_\tau = \frac{3M_1'}{16\pi v^2 \tilde{m}}\frac{J_{\mathsf{CP}}}{|U_{e1}U_{e2}U_{e3}|}\left\{B_{12}R_{11}^{(0)}R_{12}^{(0)} - B_{13}R_{11}^{(0)}R_{13}^{(0)} + B_{23}R_{12}^{(0)}R_{13}^{(0)}\right\}\,, \tag{4.28}$$

---

[2]Lorentz transformations in 2+1 dimensions.

where

$$
\begin{aligned}
B_{ij} &\equiv \sqrt{m_i m_j}\left(m_j' - m_i'\right)|U_{ek}|\,, \\
\tilde{m} &\equiv \sum_\alpha \tilde{m}_\alpha = \sum_j m_j |R_{1j}|^2 = \sum_\alpha \sum_{ij} \sqrt{m_i m_j}\, R_{1i}^{(0)} R_{1j}^{(0)} \operatorname{Re}\!\left(U_{\alpha i}^{(0)*} U_{\alpha j}^{(0)}\right),
\end{aligned}
\tag{4.29}
$$

with $(ijk) = (123)$ or permutations and J_{CP} is the Jarlskog invariant

$$
J_{\mathsf{CP}} \equiv \operatorname{Im}\!\left[U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^*\right].
\tag{4.30}
$$

To obtain (4.28), we have multiplied and divided by $U_{11}^{(0)} U_{12}^{(0)} U_{13}^{(0)} = |U_{11} U_{12} U_{13}|$ and included the appropriate factors inside the imaginary part. Notice that we are assuming $\mathsf{CP}^{\mu\tau}$ and (2.5). We also used the fact that the Jarlskog invariant can be written in terms of different entries of U.

In the standard parametrization, the Jarlskog invariant is

$$
J_{\mathsf{CP}} = \left(s_{13} c_{13}^2\right)(s_{12} c_{12})(s_{23} c_{23}) \sin\delta_{\mathsf{CP}}\,.
\tag{4.31}
$$

Therefore, in the $\mathsf{CP}^{\mu\tau}$ symmetric case, we obtain

$$
\frac{J_{\mathsf{CP}}}{|U_{e1} U_{e2} U_{e3}|} = \pm\frac{1}{2}\,,
\tag{4.32}
$$

for $\delta_{\mathsf{CP}} = \pm\pi/2$, respectively [32]. We can see from (4.28) that ϵ_τ depends only on the low-energy CP violation coming from J_{CP}. Other than that, ϵ_τ only depends on the three $R_{1i}^{(0)}$, on the absolute neutrino scale and the discrete choice of ν_{iL} CP parities.

We can finally use Y_B in (4.14), ϵ_τ in (4.28) and $\tilde{m}_\alpha$ in (4.29) to calculate the baryon asymmetry produced by leptogenesis using the Casas-Ibarra parametrization. To simplify the numerical study even further, we employ the approximation where $M_3 \gg M_{1,2}$ and N_{3R} decouples. In that case, the R matrix can be written as [68]

$$
\begin{aligned}
\text{NH:} \quad R &= \begin{pmatrix} 0 & \star & \star \\ 0 & \star & \star \\ 1 & 0 & 0 \end{pmatrix}, \quad m_1 \to 0\,, \\[2mm]
\text{IH:} \quad R &= \begin{pmatrix} \star & \star & 0 \\ \star & \star & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad m_3 \to 0\,.
\end{aligned}
\tag{4.33}
$$

Then we can denote the different cases of CP parities for N_{iR} and ν_{iL} as in table 1. In the decoupling limit, when R is not real, we only have the cases [cf. (4.25)]

$$
\begin{aligned}
\text{NH:} &\quad (31), (12), (13), (21), (23)\,; \\
\text{IH:} &\quad (33), (11), (12), (21), (22)\,.
\end{aligned}
\tag{4.34}
$$

Note that, differently from the strength of double beta decay, leptogenesis also depends on the CP parities of the heavy right-handed neutrinos.

Case	K_R	K_ν	$R^{(0)}$
(00)	$\mathbb{1}_3$	$\mathbb{1}_3$	O(3)
(jk)	$(K_R)_{jj} = i$	$(K_\nu)_{kk} = i$	O(2,1)

Table 1. Possibilities for K_R, K_ν and $R^{(0)}$. In cases (jk), $j, k = 1, 2, 3$, K_R, K_ν have only one different diagonal entry as $\mathrm{diag}(i, 1, 1)$ or any permuted diagonal entries.

We show our results for leptogenesis induced by hierarchical N_{iR} and decoupled N_{3R} in figures 2 and 3. We use the maximum possible value for M_1 within flavored leptogenesis with τ-flavor in equilibrium: $M_1 = 10^{12}$GeV. Given the parametrization in (4.26) (M_1 only appears linearly in the prefactor), lowering M_1 leads to proportional lowering of ϵ_τ and also $|Y_B|$. Plots with smaller M_1 can be obtained by scaling down the lines proportionally. Note that $\theta_{23} = 45°$ (and $\delta_{\mathsf{CP}} = \pm\pi/2$) is fixed from symmetry and this makes the curves of $|Y_B|$ smoother, with less possibility of cancellations.

Let us begin with figure 2, left. We treat the case where all CP parities are equal for light and heavy neutrinos, i.e., cases (00)-NH and (00)-IH, and the figure shows the ratio of the baryon asymmetry of the model over its experimental value, $Y_B/Y_{B\exp}$, in terms of R_{12}. Since the third N_{3R} decouples, the same plots also applies to the case where the CP parity of N_{3R} is different from the rest, i.e., $K_R = \mathrm{diag}(1, 1, i)$. The property in (4.25) requires that we are only left with the cases (31)-NH [same as (00)-NH] and (33)-IH [same as (00)-IH]. Thus successful leptogenesis can happen for normal hierarchy [(00)-NH and (31)-NH] but not for the inverted hierarchy [(00)-IH and (33)-IH]. For normal hierarchy, we can read from the plot that the scale of M_1 can be lowered at most by a factor $|Y_B|_{\max}/Y_{B\exp} = 15.3$ and we need $0.65 \times 10^{11} \lesssim M_1 \lesssim 10^{12}$GeV. A similar situation of leptogenesis induced solely by δ_{CP} was also considered in ref. [5–7][For a further discussion on this issue, see 8]. Here we furnish a symmetry justification for that case.

In figure 2, right, the remaining cases for NH are considered, i.e., (12)/(23) and (13)/(22). We show the ratio $|Y_B|/Y_{B\exp}$ in terms of ξ, which parametrizes the nonzero R_{1i}. The cases (12) and (23) [(13) and (22)] are represented by the same blue (green) curve. We can see that the cases (13)-NH and (22)-NH do not lead to successful leptogenesis. For (12)-NH and (23)-NH, successful leptogenesis is also possible for $0.5 \times 10^{11} \lesssim M_1 \lesssim 10^{12}$GeV ($|Y_B|_{\max}/Y_{B\exp} = 20.2$).

Finally, figure 3 shows the remaining cases for IH: (11)/(12) and (12)/(22). We show again the ratio $|Y_B|/Y_{B\exp}$ in terms of ξ, which parametrizes the nonzero R_{1i}. In all cases leptogenesis is possible for slightly different ranges for M_1. For (11)/(21), we need $0.44 \times 10^{11} \lesssim M_1 \lesssim 10^{12}$GeV ($|Y_B|_{\max}/Y_{B\exp} = 22.8$). For (12)/(22), $2.3 \times 10^{11} \lesssim M_1 \lesssim 10^{12}$GeV ($|Y_B|_{\max}/Y_{B\exp} = 4.4$). If we assume negative δ_{CP}, preferred from global fits [1, 2], then the range for case (11)/(21) shrinks almost to the single value $M_1 \approx 10^{12}$GeV because the right portion of the curve leads to anti-matter dominance instead of matter dominance; see figure.

We conclude that successful leptogenesis is not possible for the cases (00)-IH, (33)-IH, (13)-NH and (22)-NH. Therefore, for IH, successful leptogenesis requires that the CP parity
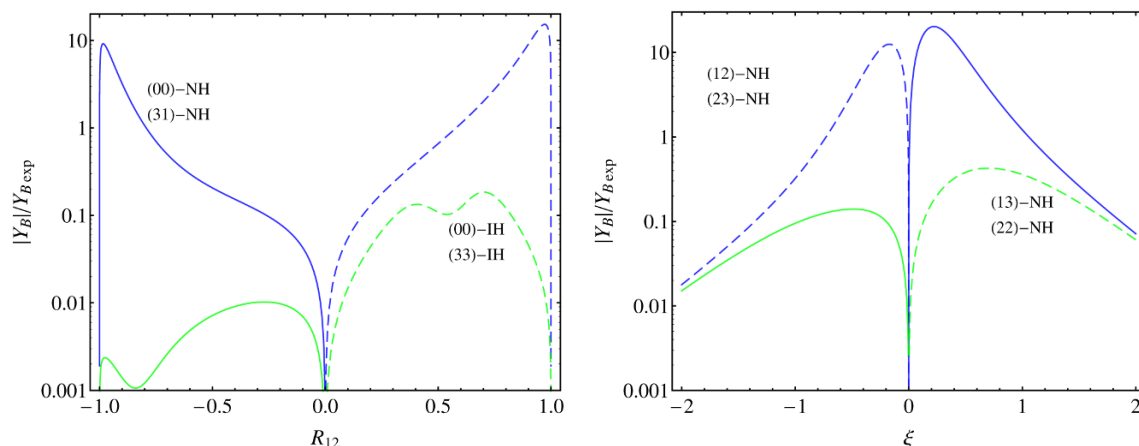
Figure 2. *Left*: ratio of $|Y_B|$ over $Y_{B\exp} = 8.75 \times 10^{-11}$ as a function of R_{12} for $M_1 = 10^{12}$GeV in the N_{3R} decoupling limit; the blue curve corresponds to both (00)-NH and (31)-NH, with $R_{11} = 0$, $|R_{12}|^2 + |R_{13}|^2 = 1$ and $R_{13} > 0$, while the green curve corresponds to both (00)-IH and (33)-IH, with $R_{13} = 0$, $|R_{11}|^2 + |R_{12}|^2 = 1$ and $R_{11} > 0$. *Right*: ratio of $|Y_B|$ over $Y_{B\exp}$, for $M_1 = 10^{12}$GeV and in the N_{3R} decoupled limit, as a function of ξ in $R_{1i} = (0, \cosh\xi, -i\sinh\xi)$ for (12)-NH (blue) and $R_{1i} = (0, -i\sinh\xi, \cosh\xi)$ for (13)-NH (green); the blue (green) curve also describes the case (23)-NH [(22)-NH], with R_{12}, R_{13} exchanged and $\xi \to -\xi$. We use the best-fit values of ref. [2] for θ_{12}, θ_{13} and the squared mass differences. The solid curves correspond to $Y_B > 0$ for $\delta_{\mathsf{CP}} = -90°$ (preferred, cf. [1, 2]) while the dashed curves correpond to $Y_B > 0$ for $\delta_{\mathsf{CP}} = 90°$.

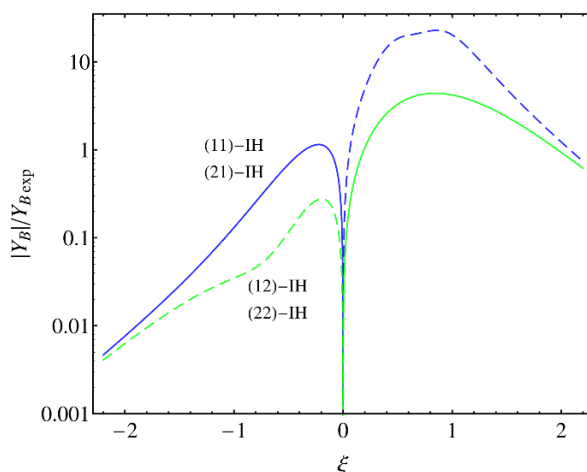**Figure 3**. Ratio of $|Y_B|$ over $Y_{B\exp} = 8.75 \times 10^{-11}$ as a function of ξ for $M_1 = 10^{12}$GeV in the N_{3R} decoupling limit; ξ is defined by $R_{1i} = (\cosh\xi, -i\sinh\xi, 0)$ for (11)-IH (blue) and $R_{1i} = (-i\sinh\xi, \cosh\xi, 0)$ for (12)-IH (green); the blue (green) also describes the case (21)-IH [(22)-IH], with R_{12}, R_{13} exchanged and $\xi \to -\xi$. The solid curves correspond to $Y_B > 0$ for $\delta_{\mathsf{CP}} = -90°$ (preferred, cf. [1, 2]) while the dashed curves correpond to $Y_B > 0$ for $\delta_{\mathsf{CP}} = 90°$.

of ν_{1L} or ν_{2L} be different of the rest. On the other hand, the cases (00)-IH and (33)-IH correspond to the largest value of $|m_{ee}|$ in figure 1. If this value of $|m_{ee}|$ were measured in future experiments, then $\mathsf{CP}^{\mu\tau}$ symmetric leptogenesis with hierarchical right-handed neutrinos and decoupled N_{3R} is excluded as the origin of the present baryon asymmetry of the Universe.

5 Symmetry choice and properties

We now turn to a theoretical discussion of $\mathsf{CP}^{\mu\tau}$ symmetry and follow it up in the subsequent section with a model realization. As already noted, a much pursued idea in the neutrino literature is that flavor symmetries may be behind the structure of masses and mixing angles of the leptons [9, 10]. A very predictive setting consists of assuming that the charged lepton sector and neutrino sectors are invariant under different groups G_l and G_ν, respectively. These groups are then part of a larger group G_F that may be entirely or partially valid at higher energies (the latter if some factor appears accidentally). A less ambitious variations of the above idea is (i) to allow more free parameters by requiring less symmetry for G_ν or G_l or (ii) including generalized CP (GCP) symmetries as part of the flavor group. Here we pursue a direction where we identify a minimal setting with G_l being abelian and G_ν being a GCP transformation. We find that we are largely restricted to $\mathsf{CP}^{\mu\tau}$ for G_ν.

To discuss our strategy, we assume Majorana neutrinos, with the leptonic Lagrangian below EWSB in the flavor basis to be

$$- \mathcal{L} = m_\alpha \bar{l}_{\alpha L} l_{\alpha R} + \overline{\nu_{\alpha L}^c}(M_\nu)_{\alpha\beta}\nu_{\beta L} + \text{h.c.}, \tag{5.1}$$

where the implicit sum over $\alpha = e, \mu, \tau$ is understood. Note that in the flavor basis, the interaction with W gauge bosons is diagonal, $W_\mu \bar{l}_{\alpha L}\gamma^\mu \nu_{\alpha L}$, and the PMNS matrix comes from the diagonalization of M_ν.

It is clear that the charged lepton part of (5.1) is invariant under three separate family lepton numbers $\mathsf{L}_e, \mathsf{L}_\mu, \mathsf{L}_\tau$, that should be broken in the neutrino part. Although these symmetries are automatically present whenever we diagonalize the charged lepton mass matrix [69], we assume some subgroup of it, G_l, is a symmetry of the theory at higher scales for the charged lepton sector (we allow for the fact that it may be accidental). Since charged leptons and left-handed neutrinos come from the same leptonic doublet L_α above the EW scale, the group G_ν should also act on the same space. Let us look for the minimal G_l and G_ν where the former is abelian and the latter is a GCP.

We assume G_l has a generic element acting on $L_\alpha = (l_{\alpha L}, \nu_{\alpha L})$ of the form (more generic forms are considered in appendix A)

$$G_l: \quad T = \begin{pmatrix} 1 & & \\ & e^{i\theta} & \\ & & e^{-i\theta} \end{pmatrix}. \tag{5.2}$$

For the moment, G_l can be a continuous U(1) group (which can therefore be the group U(1)$_{\mu-\tau}$ of $\mathsf{L}_\mu - \mathsf{L}_\tau$) or a discrete abelian group $\mathbb{Z}_n$, with $n \geq 3$ to avoid degenerate T.

We are in the basis where $T_L = T_{l_R} = T$ act all in the same way on left-handed doublets and right-handed singlets but they can be in different irreducible representations (irreps) if T is embedded in a larger group. In this case, G_l will refer to the group acting on the left-handed doublets L_α.

Next, we assume the symmetry of the neutrino sector of (5.1), G_ν, is composed of a generalized CP (GCP) symmetry [70, 71] of the form

$$G_\nu : \quad L(x) \to X L^{\mathsf{CP}}(\hat{x}), \tag{5.3}$$

where $L^{\mathsf{CP}} = -iCL^*$ is the usual CP transformation and X is a generic 3×3 unitary and *symmetric* matrix acting in the space of three families; $\hat{x}$ is the space inversion of x. Symmetric X guarantee that the application of (5.3) two times, leads to the identity. Note that a global rephasing is unimportant for X.

Now we *demand* that G_l and G_ν *close* as a group acting on L_α. If $G_{l,\nu}$ were unitary and we demanded that the product of its generators be finite, we would obtain von Dyck groups that were extensively studied in this context [72, 73].[3] Instead, (5.3) is a GCP symmetry and we should demand that the following composition of G_ν and G_l induce an automorphism [39]:

$$X T^* X^\dagger = T' \in G_l. \tag{5.4}$$

where T, T' are elements of the same group. This equation can be rewritten as

$$X = T' X T^{\mathsf{T}} \in G_l. \tag{5.5}$$

This equation and the previous one are not restricted to diagonal T but are valid for any unitary T in any basis.

If $G_l = \mathrm{U}(1)$, irrespective of the form in eq.(5.2), there are only two possible automorphisms:

$$(i) \ \ T' = T^{-1} \quad \text{or} \quad (ii) \ \ T' = T. \tag{5.6}$$

These are also automorphisms for all subgroups $\mathbb{Z}_n$ and, in particular, for $n = 3, 4$, they are the only ones. For general $\mathbb{Z}_n$, with $n \neq 3, 4$, additional automorphisms $T' = T^k$ are possible but not with the form (5.2). For these automorphisms, (5.5) and the form of T in (5.2) leads to

$$(i) \ \ X = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \quad \text{or} \quad (ii) \ \ X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \tag{5.7}$$

after rephasing some fields appropriately. The first case is just usual CP transformation and we can see that the charged lepton part of (5.1) is automatically invariant under such a transformation, thus leading to CP invariance in the whole theory. This symmetry prevents CP violation in the leptonic sector and hence we consider it no further. Instead we focus on the second case which we will denote as $\mathsf{CP}^{\mu\tau}$ and it is a well-known GCP symmetry in the literature called $\mu\tau$-reflection symmetry [32–34]. CP breaking arises in this setting because

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[3]For a different approach based on $\mathbb{Z}_2 \times \mathbb{Z}_2$, see [74, 75].

of the clash between the neutrino part and the charged lepton part in (5.1): the former is invariant under $\mathsf{CP}^{\mu\tau}$ while the latter is invariant under the usual CP (after rephasing). What distinguishes our work from the previous ones is that in previous works on $\mathsf{CP}^{\mu\tau}$, neither the symmetry G_l was identified nor its relation with G_ν was stressed as we do here. Also, in later approaches using GCP symmetry with finite flavor symmetries, much more complicated automorphism structures (compared to ours) needed to be studied for some groups [33, 37–51].

In fact, this settings is much more general: the two forms for X in (5.7) are *unique* for any diagonal T and the form for T in (5.2) is also *unique* for $G_l = \mathrm{U}(1)$ or $G_l = \mathbb{Z}_n$ with prime n or $n = 4, 6$. The uniqueness is up to simultaneous permutations of rows and columns that leaves T diagonal. This result is proved in appendix A, where we also show the first different form for T — it occurs for $\mathbb{Z}_8$.

Permutations of the above structure can be discarded for phenomenological reasons as follows. If we adopt T with nontrivial entries in (11)-(22) [or (11)-(33)], the structure of X would also be interchanged and we obtain the relations $|U_{e3}| = |U_{\mu3}|$ (or $|U_{e3}| = |U_{\tau3}|$), which leads (respectively) to

$$
\begin{aligned}
\mathsf{CP}^{e\mu}: & \quad \tan\theta_{13} = \sin\theta_{23}\,, \\
\mathsf{CP}^{e\tau}: & \quad \tan\theta_{13} = \cos\theta_{23}\,.
\end{aligned}
\tag{5.8}
$$

These relations are completely excluded because of small θ_{13}.

At last, we point out a remarkable property of the symmetries G_l generated by T and $G_\nu = \mathbb{Z}_2^{\mathsf{CP}}$ generated by $\mathsf{CP}^{\mu\tau}$: the two groups commute.[4] Therefore, our minimal flavor group, including GCP, can be just $G_F = G_l \times G_\nu$.[5] Generically, when G_F is a subgroup of U(3), $G_l \sim \mathbb{Z}_n$ and $G_\nu \sim \mathbb{Z}_2 \times \mathbb{Z}_2$ (or subgroup), their commutation is impossible because all mixing angles are nonzero. For that reason, the whole group containing G_l and G_ν tends to be a large nonabelian group. For example, the minimal group that leads to TBM is S_4 [76, 77] of order 24. To fix at least the nonzero θ_{13}, it must be much larger of order 150 or more [78–82].

The commutation of G_l and G_ν seems to have another remarkable feature, i.e., the *vev alignment problem*[6] often encountered in flavor symmetry model building — can be naturally avoided in the scalar sector (without supersymmetry) as our examples below show. The solution is simply that G_l (G_ν) can be broken in the neutrino sector (charged lepton sector) preserving G_ν (G_l) by using G_ν-invariant (G_l-invariant) fields with G_l (G_ν) charge. Hence, only complete invariants of both G_l and G_ν interact in the potential. Thus to avoid the contamination of G_l-breaking effects in the neutrino sector, we just need to avoid the coupling of G_l breaking scalars to neutrino fields (be it by additional symmetries). The same is valid for the charged lepton sector.[7]

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[4]This property is more transparent in the basis where G_l, in the continous case, is represented by SO(2) rather than U(1) and $\mathsf{CP}^{\mu\tau}$ is represented by usual CP which commutes; see appendix B.

[5]Obviously $\mathsf{CP}^{\mu\tau}$ may not commute with other symmetries such as the SM gauge group.

[6]This name is not entirely appropriate in our context (we use one-dimensional irreps, see also appendix B) and we specifically refer here to the possibility of different symmetry breaking scalars interacting through the potential.

[7]To see the advantage of our discussion relative to other flavor groups, we can compare our setting with those based on $A_4 = (\mathbb{Z}_2 \times \mathbb{Z}_2) \rtimes \mathbb{Z}_3$ group. We can take $G_l \simeq \mathbb{Z}_3$ and $G_\nu \simeq \mathbb{Z}_2$ and note that they do not

6 Model

The main challenge in model building with $\mathsf{CP}^{\mu\tau}$, is to keep it unbroken in the neutrino sector while breaking it sufficiently in the charged lepton sector (keeping G_l) to generate μ-τ mass splitting. We have found several ways to meet this challenge. Although our general setting can be implemented in many different ways, some distinction is possible on how G_l appears and how G_ν (GCP) is broken in the charged lepton sector. The different possibilities depend on how G_l appears, i.e., either

- G_l comes from a symmetry of the whole theory G_F at high scales; or

- G_l appears accidentally.

In section 5, we saw that the largest group for abelian G_l is $G_l = \mathrm{U}(1)_{\mu-\tau}$, which is the continuous symmetry of the combination $\mathsf{L}_\mu - \mathsf{L}_\tau$. Variations on this respect involve gauging $\mathrm{U}(1)_{\mu-\tau}$ or considering only a $\mathbb{Z}_n$ subgroup of it. The latter would allow embedding our $G_l \times G_\nu$ into a larger nonabelian discrete group. Either way, we use the nomenclature of $\mathrm{U}(1)_{\mu-\tau}$ to describe our models and only make some comments on variants.

Furthermore, our setting requires that only G_l be broken in the neutrino sector and only G_ν be broken in the charged lepton sector — the conservation of G_l and G_ν in the complementary sectors is what leads to predictions. That is achieved through the vacuum expectation value of scalars that we call as l-flavons and ν-flavons. They have the following properties:

- l-flavons: all conserve G_l but some need to break G_ν. Best candidate is a G_l invariant $\mathsf{CP}^{\mu\tau}$ *odd* scalar (we denote it as σ).

- ν-flavons: all conserve G_ν but some need to break G_l. Best candidates are scalars carrying G_l charge but $\mathsf{CP}^{\mu\tau}$ *even* (G_ν-invariant); we denote them as η's.

Since the alignment problem in the scalar potential can be avoided, we just need to prevent l-flavons (ν-flavons) to couple to the neutrino sector (charged lepton sector). Often that can be achieved by additional symmetries.

One remark with respect to additional symmetries of flavons is in order. For the above setting, it is simpler if flavons do not carry other additive quantum numbers other than those of G_l or G_ν. For example, let us consider a ν-flavon η_2 carrying $\mathsf{L}_\mu - \mathsf{L}_\tau = 2$ (G_l) so that it couples with N_3 as $N_{3R}^2 \eta_2$. If η_2 carries no other quantum number, we can define its $\mathsf{CP}^{\mu\tau}$ transformation as[8]

$$\mathsf{CP}^{\mu\tau}: \quad \eta_2(x) \to \eta_2(\hat{x}), \tag{6.1}$$

_____

commute. In this case, G_ν invariant fields with G_l charge exist: take the $\mathbf{1}'$ or $\mathbf{1}''$ singlets (in actual models, additional flavons are necessary to partly break $\mathbb{Z}_2 \times \mathbb{Z}_2$ of A_4). However, there is no irrep with G_ν charge but without G_l charge in A_4. In actual A_4 models, usually triplets $\mathbf{3}$ with specifically aligned vevs are used to achieve the breaking $G_F \to G_l$ in the charged lepton sector (and also in the neutrino sector, hence the alignment problem).

[8]This possibility is raised for general discrete nonabelian symmetries in [39] but no model application was discussed.

i.e., η_2 is composed of two CP *even* real scalars. However, if η_2 also carries $\mathtt{B}-\mathtt{L}=-2$, and N_{iR} carries $\mathtt{B}-\mathtt{L}=-1$, then its N_3 coupling transform as

$$\mathsf{CP}^{\mu\tau}: \quad N_{3R}N_{3R}\eta_2 \to N_{2R}^{cp}N_{2R}^{cp}\eta_2 \,, \tag{6.2}$$

which maps a $\mathtt{B}-\mathtt{L}$ invariant term to a $\mathtt{B}-\mathtt{L}$ violating term. In this case, consistency with $\mathsf{CP}^{\mu\tau}$ requires the existence of another field η_{-2} with charges $\mathtt{L}_\mu-\mathtt{L}_\tau=-2$ and $\mathtt{B}-\mathtt{L}=-2$. The transformation property now would be

$$\mathsf{CP}^{\mu\tau}: \quad \eta_2(x) \to \eta_{-2}^*(\hat{x}) \,. \tag{6.3}$$

This corrects the transformation properties for (6.2) but allow $\mathsf{CP}^{\mu\tau}$ breaking if $|\langle\eta_2\rangle| \neq |\langle\eta_{-2}\rangle|$. Therefore, in our setting, we require that ν-flavons carry no other additive quantum number and hence a continuous $\mathtt{B}-\mathtt{L}$ symmetry cannot be implemented.

The exception to the above feature is when ν-flavons carry only a $\mathbb{Z}_2$ quantum number. In this case, since the representation is real, (6.1) can be maintained. This means that a $\mathbb{Z}_4$ subgroup of $\mathrm{U}(1)_{\mathtt{B}-\mathtt{L}}$, acting as

$$\mathbb{Z}_4^L: \quad \text{leptons} \sim i\,, \quad \nu\text{-flavons} \sim -1\,, \tag{6.4}$$

can still be implemented as a symmetry.

At last, we assume leptogenesis is successful in our setting and we will be seeking high scale ($M_R \gtrsim 10^{11}\mathrm{GeV}$) type-I seesaw implementations.

6.1 Multi-Higgs implementation

The model below illustrates the general aspects of our setting. In this case, G_l will be accidental and G_ν will be broken at a high scale and transmitted to the charged lepton sector to generate the $\mu\tau$ mass splitting. The symmetries at the high scale will be $\mathrm{U}(1)_{\mu-\tau} \times \mathbb{Z}_2^{\mathsf{CP}}$, a gauged $\mathrm{U}(1)_{\mu-\tau}$ (which is not exactly G_l at the low scale) and global $\mathsf{CP}^{\mu\tau}$. Another implementation where $G_F = G_l \times G_\nu$ is a symmetry of the high scale theory is given in appendix C.

All lepton fields transform alike under $\mathrm{U}(1)_{\mu-\tau}$, with $\mathtt{L}_\mu-\mathtt{L}_\tau$ charges

$$L_i \sim l_i \sim N_i \sim (0,1,-1)\,, \tag{6.5}$$

where $L_i, l_i \equiv l_{iR}, N_i \equiv N_{iR}$ (here we use L_i, l_i instead of L_α, l_α) are the three families of lepton doublets, lepton singlets and right-handed neutrino singlets, respectively. The $\mathsf{CP}^{\mu\tau}$ symmetry also acts similarly for all of the three type of fields, as (5.3) with the second X in (5.7), and should swap the second with the third family fields. Note that this GCP symmetry commutes with $\mathrm{U}(1)_{\mu-\tau}$ and it does not reverse its charges. The SM group charges, however, are reversed by this GCP symmetry.

We add two more Higgs doublets $\phi_{\pm 2}$ with $\mathrm{U}(1)_{\mu-\tau}$ charge ± 2 in addition to the SM doublet ϕ_0. The Lagrangian for the charged lepton sector is

$$-\mathcal{L}^l = y_0\bar{L}_1\phi_0 l_1 + y_2\bar{L}_2\phi_2 l_3 + y_{-2}\bar{L}_3\phi_{-2}l_2\,. \tag{6.6}$$

We prevent ϕ_0 from coupling to $\bar{L}_2 l_2$ and $\bar{L}_3 l_3$ by assigning

$$\mathbb{Z}_2: \quad l_2, l_3, \phi_{\pm 2} \text{ are odd.} \tag{6.7}$$

Such a symmetry also leads to the accidental symmetry

$$G_l: \quad L_2 \sim l_3 \sim e^{i\theta}, \quad L_3 \sim l_2 \sim e^{-i\theta}. \tag{6.8}$$

The Higgs doublets are invariant under this symmetry and so it leaves the symmetry invariant even after EWSB. It is this symmetry that will correspond to G_l at low energies and will differ from our original $\mathrm{U}(1)_{\mu-\tau}$ only for l_{iR}. The $\mathsf{CP}^{\mu\tau}$ acts in the same form for G_l as it does for $\mathrm{U}(1)_{\mu-\tau}$.

The $\mathsf{CP}^{\mu\tau}$ symmetry acts on the doublets as

$$\phi_0 \to \phi_0^*, \quad \phi_2 \to \phi_{-2}^*. \tag{6.9}$$

This implies y_0 is real and $y_{-2}^* = y_2$.

If we write

$$\langle \phi_{-2}^{(0)} \rangle = v_{-2} \quad \text{and} \quad \langle \phi_2^{(0)} \rangle = v_2, \tag{6.10}$$

the $\mathsf{CP}^{\mu\tau}$ breaking will come from

$$|v_{-2}| \gg |v_2|, \tag{6.11}$$

which induces the $\mu\tau$ mass splitting

$$m_\mu = |y_2 v_2| \ll m_\tau = |y_{-2} v_{-2}|. \tag{6.12}$$

Note that prior to EWSB $\mathsf{CP}^{\mu\tau}$ renders $\mu\tau$ flavors indistinguishable and the $|v_{-2}| \ll |v_2|$ leads physically to the same situation. The $\mathsf{CP}^{\mu\tau}$ breaking in (6.11) will be induced by a large vev of a CP odd scalar σ in the potential [83].

The Higgs potential is

$$\begin{aligned}
V_2 &= \mu_2\big(|\phi_2|^2 + |\phi_{-2}|^2\big) + \mu_0|\phi_0|^2, \\
V_4 &= \frac{1}{2}\lambda_0|\phi_0|^4 + \frac{1}{2}\lambda_1\big(|\phi_2|^2 + |\phi_{-2}|^2\big)^2 + \lambda_2|\phi_2|^2|\phi_{-2}|^2 \\
&\quad + \lambda_{22}\big(\phi_0^\dagger\phi_2\phi_0^\dagger\phi_{-2} + \text{h.c.}\big) + \lambda_{02}|\phi_0|^2\big(|\phi_2|^2 + |\phi_{-2}|^2\big) \\
&\quad + \lambda_{02}'\big(|\phi_0^\dagger\phi_2|^2 + |\phi_0^\dagger\phi_{-2}|^2\big), \\
\delta V &= \mu_\sigma\sigma\big(|\phi_2|^2 - |\phi_{-2}|^2\big) + \Big(\lambda_{-4}\phi_2^\dagger\phi_{-2}\eta_2^2 + \text{h.c.}\Big)
\end{aligned} \tag{6.13}$$

where σ is a CP odd scalar and η_2 is a CP-even scalar with $\mathrm{U}(1)_{\mu-\tau}$ charge 2 and will couple to N_2^2, N_3^2. We have omitted a term similar to the λ_2-term because only neutral vevs are sought and they are not relevant to the discussion below. We could also replace $\mathrm{U}(1)_{\mu-\tau}$ by $\mathbb{Z}_8$ by adding the terms $(\phi_2^\dagger\phi_{-2})^2$.

After σ and η_2 acquire vevs at the high scale, we get from δV and V_2 an effective quadratic term for $\phi_{\pm 2}$,

$$V_{2\mathrm{eff}} = M_2^2|\phi_2|^2 + M_{-2}^2|\phi_{-2}|^2 + M_{22}^2\phi_2^\dagger\phi_{-2} + \text{h.c.}, \tag{6.14}$$

where

$$M_2^2 = \mu_2 + \mu_\sigma \langle \sigma \rangle, \quad M_{-2}^2 = \mu_2 - \mu_\sigma \langle \sigma \rangle, \quad M_{22}^2 = \lambda_{-4} \langle \eta_2 \rangle^2. \tag{6.15}$$

Irrespective of the phases of $\lambda_{-4}, \langle \eta_2 \rangle$, we apply rephasing transformations so that M_{22}^2 is real and negative.

Now we adjust $\langle \sigma \rangle$ so that $|M_2^2| \simeq \varepsilon^{-1}|M_{22}^2| \simeq \varepsilon^{-2}|M_{-2}^2| \sim v_{\text{ew}}$. The phases of the vevs are trivial in the minimum when $\lambda_{22} < 0$. This leads to a high scale mass matrix for (ϕ_{-2}, ϕ_2) of the form:

$$M_\phi^2 = M_2^2 \begin{pmatrix} \varepsilon^2 & \sim \varepsilon \\ \sim \varepsilon & 1 \end{pmatrix} \tag{6.16}$$

The two approximate eigenvectors of this matrix are: $H' \approx \phi_2 + \varepsilon \phi_{-2}$ and $h_0 \approx \phi_{-2} - \varepsilon \phi_2$. By fine tuning we keep $\varepsilon \sim \frac{m_\mu}{m_\tau}$ and H' as superheavy whereas h_0 mass is negative and weak scale. Then below the scale of $\langle \eta \rangle$ and $\langle \sigma \rangle$, the effective charged lepton Yukawa couplings in (6.6) look like:

$$-\mathcal{L}_{\text{eff}}^l \simeq y_0 \bar{L}_1 \phi_0 l_1 + y_2 \varepsilon \bar{L}_2 h_0 l_3 + y_2^* \bar{L}_3 h_0 l_2. \tag{6.17}$$

After a 90° rotation of the right-handed charged leptons, this gives $m_\tau = |y_2^* \langle h_0^{(0)} \rangle|$ and $m_\mu = |y_2 \varepsilon \langle h_0^{(0)} \rangle|$ as desired for a realistic theory.

For the neutrino sector we add three singlet scalars η_k, $k = 0, 1, 2$, with $U(1)_{\mu-\tau}$ charge k; η_0 is a real scalar. When they acquire vevs (for $k \neq 0$), they break $U(1)_{\mu-\tau}$ without breaking $CP^{\mu\tau}$, as discussed previously, and they transform trivially under $CP^{\mu\tau}$:

$$CP^{\mu\tau}: \quad \eta_k(x) \to \eta_k(\hat{x}), \tag{6.18}$$

where $\hat{x} = (x_0, -\mathbf{x})$ for $x = (x_0, \mathbf{x})$. We also assume the symmetry $\mathbb{Z}_4^L$ in (6.4) where $\eta_k \sim -1$.

The Lagrangian for N,

$$\begin{aligned} -\mathcal{L} \supset & \frac{1}{2} k_1 \bar{N}_1 N_1^c \eta_0 + k_{23} \bar{N}_2 N_3^c \eta_0 \\ & + \frac{1}{2} k_2 \bar{N}_2 N_2^c \eta_2 + \frac{1}{2} k_3 \bar{N}_3 N_3^c \eta_2^* \\ & + k_{12} \bar{N}_1 N_2^c \eta_1 + k_{13} \bar{N}_1 N_3^c \eta_1^*, \end{aligned} \tag{6.19}$$

gives rise to M_R in the $CP^{\mu\tau}$ symmetric form (2.4) after η_k acquire generic vevs. GCP symmetry imposes real k_1, real k_{23}, $k_3 = k_2^*$, $k_{13} = k_{12}^*$. Given the necessary structure (2.4) and the requirement for $\theta_{13} \neq 0$, we indeed need both fields $\eta_{1,2}$. Note that $\mathbb{Z}_4^L$ prevents σ from coupling to N_{iR}.

It can be seen that $CP^{\mu\tau}$ symmetric M_R also leads to a $CP^{\mu\tau}$ symmetric M_R^{-1}. Such a structure is maintained from the neutrino Dirac mass matrix M_D coming from

$$-\mathcal{L} \supset f_0 \bar{N}_1 \tilde{\phi}_0^\dagger L_1 + f_2 \bar{N}_2 \tilde{\phi}_0^\dagger L_2 + f_3 \bar{N}_2 \tilde{\phi}_0^\dagger L_3, \tag{6.20}$$

where ϕ_0 is the same Higgs doublet that couples to electrons and quarks. The reality of f_0 and $f_3 = f_2^*$ follow from $CP^{\mu\tau}$ and and we obtain

$$M_D = \begin{pmatrix} x_\nu & & \\ & z_\nu & \\ & & z_\nu^* \end{pmatrix}. \tag{6.21}$$

The neutrino mass matrix given by the seesaw formula [84–88]

$$M_\nu = -M_D^\mathsf{T} M_R^{-1} M_D \,, \tag{6.22}$$

is $\mathsf{CP}^{\mu\tau}$ invariant and has the form (2.4) as advertised.

The leptogenesis aspects studied in section 4 has to be adapted in this case because $v = 174\,\text{GeV}$ has to be replaced by $v_u = v \sin\beta$. The plots shown in figures 2 and 3 apply now for $M_1/\sin^2\beta = 10^{12}\,\text{GeV}$ and limits for the M_1 window changes accordingly.

6.2 Higgs spectrum

At low energies, the scalar sector of this model acts like a lepton-specific (also called type-X) two Higgs doublet model [89] with the Higgs doublets being h_0 and ϕ_0, except for the Higgs couplings to electrons; cf. (6.17). Both of the doublets acquire vevs such that $\sqrt{\langle\phi_0\rangle^2 + \langle h_0\rangle^2} = v = 174\,\text{GeV}$. The ratio of vevs is given by $\langle h_0\rangle/\langle\phi_0\rangle = \tan\beta$ and the mixing between the real neutral Higgs fields is denoted by $\tan\alpha$. The effective Higgs potential in terms of ϕ_0 and h_0 is given by:

$$\begin{aligned}
V(\phi_0, h_0) = &-\mu_\phi^2|\phi_0|^2 - \mu_h^2|h_0|^2 + \frac{1}{2}\lambda_0|\phi_0^2|^2 + \frac{1}{2}\lambda_1|h_0^2|^2 \\
&+ \lambda_{02}|\phi_0|^2|h_0|^2 + \lambda_{02}'|\phi_0^\dagger h_0|^2 + \lambda_{22}\varepsilon\left(\phi_0^\dagger h_0 \phi_0^\dagger h_0 + \text{h.c.}\right).
\end{aligned} \tag{6.23}$$

The spectrum of Higgs states is given by [90, 91]

$$m_A^2 = -4\lambda_{22}\varepsilon v^2\,, \quad m_{H^+}^2 = -\left(\lambda_{02}' + 2\varepsilon\lambda_{22}\right)v^2\,, \tag{6.24}$$

where $v = 174\,\text{GeV}$ (we use a different normalization compared to [90, 91]), while the mass matrix for the CP even states, in the basis $\sqrt{2}(\text{Re}\,h_0 - v_{-2}, \text{Re}\,\phi_0 - v_0)$, is

$$M_{h,H}^2 = 2\begin{pmatrix} \lambda_1 v_{-2}^2 & \lambda_{345} v_0 v_{-2} \\ \lambda_{345} v_0 v_{-2} & \lambda_0 v_0^2 \end{pmatrix}, \tag{6.25}$$

where $\lambda_{345} = \lambda_{02} + \lambda_{02}' + 2\varepsilon\lambda_{22}$. We are using $\langle h_0\rangle \approx \langle\phi_{-2}\rangle \approx v_{-2}$.

Since our parameter λ_{22} comes from the high energy theory (decoupled ϕ_2), it can not be arbitrarily large. If we impose it to be perturbative, $|\lambda_{22}| < 4\pi$ we obtain an upper bound for the pseudoscalar A as

$$m_A = 2v\sqrt{\varepsilon|\lambda_{22}|} \lesssim 2v\sqrt{4\pi\frac{m_\mu}{m_\tau}} = 300\,\text{GeV}\,, \tag{6.26}$$

hence non-decoupling. This is smaller than $2m_t$ and $t\bar{t}$ cannot be produced. Neutral scalars in the 2HDMs are less constrained than the charged higgsses (e.g. from flavor observables [92, 93]) and the strongest limits are available for the MSSM (or type-II) [94]. Usually they appear as lower bounds on the heavy masses because the decoupling limit is usually a good description. Very light pseudoscalars of mass below $\mathcal{O}(10\text{GeV})$ can also have its couplings constrained [94–97]. Current LHC limits for the different types of 2HDM constrain the various 2HDMs to be close to the alignment limit [For a review and earlier references, see 98]. Even in this limit, a portion of the parameter space is already excluded. For example, only $\tan\beta \gtrsim 3$ is allowed by data (above 200GeV). Also, being an effective 2HDM, the triple Higgs coupling for the interaction h^3 is different from the SM and can be probed in the future [99].

7 Summary

We have presented a minimal setting where G_l is conserved in the charged lepton sector and G_ν is conserved in the neutrino sector. The largest G_l can be identified with the combination L_μ-L_τ symmetry and G_ν is generated by a generalized CP symmetry, $\mathsf{CP}^{\mu\tau}$, that combines CP with $\mu\tau$ exchange. When G_l is conserved in the charged lepton sector and G_ν is conserved in the neutrino sector, we obtain the usual prediction of maximal θ_{23} and δ_{CP} with nonzero θ_{13}. Additionally, Majorana phases are fixed up to discrete choices and they lead to very specific predictions for neutrino-less double beta decay and leptogenesis.

In our setting, the two symmetries G_l and G_ν commute and this feature allows us to naturally avoid the alignment problem in the scalar sector. Additional symmetries can be used to keep the G_l- and G_ν-breaking effects restricted to the neutrino sector and charged lepton sector, respectively. Additionally, continuous $\mathsf{B} - \mathsf{L}$ cannot be imposed (hence not gauged) in our setting and only a $\mathbb{Z}_4$ subgroup may be imposed to keep $\mathsf{CP}^{\mu\tau}$ naturally unbroken in the neutrino sector. Our construction also illustrates that generalized CP symmetries based on the *trivial* automorphism of flavor groups — much less considered in the literature — may still lead to interesting model constructions.

For the neutrino-less double beta decay, the discrete choice of Majorana phases (or CP parities) leads to specific strips that can be clearly distinguished in some cases; see figure 1. For example, for inverted hierarchy, the case of all equal CP parities or only ν_{3L} with different CP parity can be distinguished from the rest and can be potentially measured or falsified in the near future. We emphasize that, key predictions of these models are: (i) $\theta_{23} = 45^0$ and $\delta_{CP} = \pi/2$ simultaneously i.e. if experimentally measured values for either of these observables deviate from the above predictions, $\mathsf{CP}^{\mu\tau}$ violating terms will be necessary to keep these ideas viable.

The consequences of $\mathsf{CP}^{\mu\tau}$ for leptogenesis leads to the natural implementation of the purely flavored leptogenesis scenario where the total CP asymmetry due to N_1 decay is vanishing. Successful leptogenesis is possible only when flavored leptogenesis is considered and that must take place at the intermediate temperature range of 10^9–10^{12}GeV. Flavored leptogenesis below 10^9GeV seems to be precluded even if the CP asymmetry is resonantly enhanced by quasi-degenerate N_{1R} and N_{2R} if the μ- and τ-flavors are washed out equally. For effective two heavy and hierarchical right-handed neutrinos the window for successful leptogenesis is even narrower: 5×10^{10}–10^{12}GeV.

A Uniqueness of $\mathbf{CP}^{\mu\tau}$

We show here that the GCP defined by X in (5.7) for the abelian symmetry generated by T in (5.2) are the only possibilities for any $G_l = \mathrm{U}(1)$ or $G_l = \mathbb{Z}_n$, with prime n or $n = 4, 6$. A different possibility arises only for T (the possibilities for X are the same) beginning with $n = 8$. The case of $G_l = \mathrm{U}(1)$ was considered in the text. We only need to consider $G_l = \mathbb{Z}_n$.

To show the assertion, we generalize the form of T from (5.2) to

$$T = \begin{pmatrix} z_1 & & \\ & z_2 & \\ & & z_3 \end{pmatrix}, \tag{A.1}$$

where z_i are complex number of modulus unity. We also keep $\det T = z_1 z_2 z_3 = 1$ because its nontrivial contribution can be factored out to usual lepton number. Let us also consider more general automorphisms for $\mathbb{Z}_n$ in (5.5): $T' = T^k$ where k cannot divide n.

Then the consistency condition (5.5) can be recast in the following form:

$$z_i^k z_j = 1 \quad \text{if} \quad X_{ij} \neq 0. \tag{A.2}$$

Let us take the first row of X. Because X is nonsingular, at least one element of the first row has to be nonzero. Suppose two elements are nonzero. If $X_{11} \neq 0$ and $X_{12} \neq 0$, then condition (A.2) implies

$$z_1^{k+1} = z_1^k z_2 = 1, \tag{A.3}$$

and then $z_1 = z_2$ which is impossible because T is nondegenerate. The same conclusion is reached if any two of the elements of the first row is nonzero. The argument is independent of the row and hence only one element in each row (or column) can be nonzero. Listing all possibilities and selecting only the symmetric matrices, the nonzero entries of X coincides with the positions of the unity in (5.7), after eliminating similar forms that are related by the simultaneous permutations of rows and columns that keep T diagonal. Rephasing of fields leads to (5.7). Thus X is restricted to (5.7) except for permutations.

Now, for the first case of $X = \mathbb{1}$, we reach the conclusion that

$$z_1^{k+1} = z_2^{k+1} = z_3^{k+1} = 1. \tag{A.4}$$

This means $T^{k+1} = \mathbb{1}$ and if T is a faithful representation, $k + 1 = 0 \mod n$. Therefore, $k = -1$ is the only possibility.

For the second case of X being (23)-transposition, we have the conditions

$$z_1^{k+1} = z_2^k z_3 = z_3^k z_2 = 1. \tag{A.5}$$

This imposes conditions on z_1 and also $z_2^{k-1} = z_3^{k-1}$. For prime n the last relation is only possible if $k = 1$: this leads to (5.2) (we exclude $z_2 = z_3$). The cases $n = 4, 6$ do not lead to different forms because only $k = 1$ or $k = -1$ correspond to automorphisms. The cases so far proves the assertion.

The first different form appears for $\mathbb{Z}_8$ and one example is

$$T = \begin{pmatrix} -1 & & \\ & \omega_8 & \\ & & \omega_8^3 \end{pmatrix}, \tag{A.6}$$

which obeys $XT^*X^{-1} = T^5$; ω_8 denotes $e^{i2\pi/8}$. If we allow X to be nonsymmetric, other possibilities appear such as for $\mathbb{Z}_7$ where X is the cyclic permutation and $T = \mathrm{diag}(\omega_7, \omega_7^2, \omega_7^4)$ (the same that appears for the T_7 group).

B $G_l \times G_\nu$ in the real basis

We show here how the $\mathsf{CP}^{\mu\tau}$ symmetry of (5.3) and the $\mathrm{U}(1)_{\mu-\tau}$ symmetry of (5.2) are rewritten in a real basis where $\mathsf{CP}^{\mu\tau}$ is just the usual CP transformation. In this basis, the commutation of $\mathsf{CP}^{\mu\tau}$ and $\mathrm{U}(1)_{\mu-\tau}$ is transparent and it also shows how the combination $\mathrm{U}(1)_{\mu-\tau} \times \mathbb{Z}_2^{\mathsf{CP}}$ leads effectively to a two-dimensional representation when the fields are complex, i.e., carrying quantum numbers other than $\mathrm{U}(1)_{\mu-\tau} \times \mathbb{Z}_2^{\mathsf{CP}}$.

It is clear that the charged lepton part of (5.1) breaks the $\mathsf{CP}^{\mu\tau}$ symmetry strongly as $m_\tau/m_\mu \sim y_\tau/y_\mu \sim 17$ (if l_α transform similarly to L_α and H transforms as usual). This breaking can be analyzed in a different basis. Since the matrix X in $\mathsf{CP}^{\mu\tau}$ is symmetric, there is a change of basis where X can be completely removed. We can concentrate in the $\mu\tau$ space where such a basis change is

$$\begin{pmatrix} L_\mu \\ L_\tau \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} \begin{pmatrix} L'_\mu \\ L'_\tau \end{pmatrix}. \tag{B.1}$$

For the right-handed singlets l_i we apply the same transformations. The CP transformation in the new basis will be just the usual

$$L'_i \to (-iC)L_i'^* \,, \tag{B.2}$$

and similarly for l_i.

The Yukawa coefficients in $\bar{L}_i Y_{ij} H l_j$ in the new basis will be just

$$Y = \begin{pmatrix} y_e & & \\ & y_\mu & \\ & & y_\tau \end{pmatrix} \to Y' = \begin{pmatrix} y_e & & \\ & \bar{y} & -i\Delta y/2 \\ & i\Delta y/2 & \bar{y} \end{pmatrix}, \tag{B.3}$$

where $\bar{y} \equiv (y_\tau + y_\mu)/2$ and $\Delta y \equiv y_\tau - y_\mu$. One can see that if $y_\tau \neq y_\mu^*$, CP is violated because the phases of $\bar{y}$ and $i\Delta y$ can not be simultaneously removed [keeping (B.4)] while in this basis M_ν should be a real matrix. For example, if $y_{\mu,\tau}$ are real CP is violated by $i\Delta y$. The latter term is however still invariant by the following SO(2) without being proportional to the identity:

$$\begin{pmatrix} L'_\mu \\ L'_\tau \end{pmatrix} \to \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} L'_\mu \\ L'_\tau \end{pmatrix}, \tag{B.4}$$

irrep	real basis	$\mathrm{U}(1)_{\mu-\tau}$ diagonal
$(0,\pm)$	1-dim real	1-dim real
$(q,\pm)$	2-dim real	1-dim complex
$(0,*)$	1-dim complex	1-dim complex
$(q,*)$	2-dim complex	2-dim complex

Table 2. Irreducible representations for $\mathrm{U}(1)_{\mu-\tau} \times \mathbb{Z}_2^{\mathsf{CP}}$.

In this basis it is clear that the CP transformation (B.2) commutes with the SO(2) transformation in (B.4).

In this basis it is also clear $\mathrm{U}(1)_{\mu-\tau} \times \mathbb{Z}_2^{\mathsf{CP}}$ have the irreducible representations shown in table 2, where $(q, \pm)$ denotes charge q for $\mathrm{U}(1)_{\mu-\tau}$ and CP parities $\pm$ while $*$ denotes a complex field transforming as $\phi \to \phi^*$ in the real basis or $\phi_q \to \phi_{-q}^*$ in the $\mathrm{U}(1)_{\mu-\tau}$ diagonal basis.

C Single Higgs implementation

In this implementation, the symmetry at the high scale is $G_F = G_l \times G_\nu$ where $G_l = \mathrm{U}(1)_{\mu-\tau}$ (gauged) and $G_\nu = \mathbb{Z}_2^{\mathsf{CP}}$. At low energy, right above the electroweak scale, we effectively have the SM with one Higgs doublet.

The neutrino sector is the same as in the multi-Higgs model of section 6.1, with additional simplification by eliminating η_0 and the symmetry $\mathbb{Z}_4^L$. If we replace $\mathrm{U}(1)_{\mu-\tau}$ by $\mathbb{Z}_3$, we can simplify further by identifying $\eta_1 = \eta_2^*$, and we are left with only one ν-flavon.

The charged lepton sector needs to be modified. We still assume $\mathsf{CP}^{\mu\tau}$ is spontaneously broken by a vev of a CP odd scalar, which now we rename as σ_-. We also need a CP even scalar σ_+. To confine the CP breaking to the charged lepton sector, we introduce a $\mathbb{Z}_2$ symmetry for which

$$\mathbb{Z}_2 : \quad \sigma_\pm, l_{iR} \text{ are odd,} \tag{C.1}$$

and the rest are even. Both $\sigma_\pm$ are invariant under $\mathrm{U}(1)_{\mu-\tau}$. We can write an effective Lagrangian as

$$-\mathcal{L}_{\text{eff}}^l = \frac{\sigma_e}{\Lambda_{\mathsf{CP}}} \bar{L}_e H l_e + \frac{\sigma_\mu}{\Lambda_{\mathsf{CP}}} \bar{L}_\mu H l_\mu + \frac{\sigma_\tau}{\Lambda_{\mathsf{CP}}} \bar{L}_\tau H l_\tau + \text{h.c.} \,. \tag{C.2}$$

where σ_α, $\alpha = e, \mu, \tau$ are some complex linear combinations of $\sigma_\pm$. GCP invariance requires

$$\begin{aligned} \sigma_e &= a_e \sigma_+ + i b_e \sigma_- \,, \\ \sigma_\mu &= a_\mu \sigma_+ + i b_\mu \sigma_- \,, \\ \sigma_\tau &= a_\tau \sigma_+ + i b_\tau \sigma_- \,, \end{aligned} \tag{C.3}$$

where a_e, b_e are real coefficients and $a_\tau = a_\mu^*$, $b_\tau = b_\mu^*$ are generally complex. The $\mu\tau$ mass splitting is generated from

$$\frac{m_\tau^2 - m_\mu^2}{v^2} = \frac{1}{\Lambda_{\mathsf{CP}}^2} \Big[|a_\mu^* u_+ + i b_\mu^* u_-|^2 - |a_\mu u_+ + i b_\mu u_-|^2 \Big] = \frac{u_+ u_-}{\Lambda_{\mathsf{CP}}^2} 4 \,\mathrm{Im}\big(a_\mu^* b_\mu\big) \,, \tag{C.4}$$

where $\langle \sigma_+ \rangle = u_+$ and $\langle \sigma_- \rangle = u_-$. We can see that CP breaking, and hence $\mu\tau$ mass splitting, requires both $u_\pm$ to be nonzero.

One example for a UV completion of (C.2) can be achieved by introducing three heavy vector-like charged lepton fields E_{iL} and E_{iR}, the latter with the same SM quantum number of l_{iR}. They are charged under $U(1)_{\mu-\tau}$ just like the rest of the leptons as (6.5) but they are even under the additional $\mathbb{Z}_2$ symmetry of (C.1). The Lagrangian is then

$$
\begin{aligned}
-\mathcal{L}^l = {} & y_1' \bar{L}_1 H E_{1R} + y_2' \bar{L}_2 H E_{2R} + y_3' \bar{L}_3 H E_{3R} \\
& + M_{E_i} \bar{E}_{iL} E_{iR} + \sigma_i \bar{E}_{iL} l_i \,,
\end{aligned}
\tag{C.5}
$$

where $y_3' = y_2'^*$ and σ_i are some linear combinations of $\sigma_\pm$ just like (C.3); M_{E_1} is real from GCP and $M_{E_3} = M_{E_2}$ can be taken real by convention. We obtain (C.2) for the charged leptons after integrating out the heavy leptons E_i, with the identification

$$
\begin{aligned}
\frac{\sigma_e}{\Lambda_{\mathsf{CP}}} &= -\frac{y_1'}{M_{E_1}} \sigma_1 \,, \\
\frac{\sigma_\mu}{\Lambda_{\mathsf{CP}}} &= -\frac{y_2'}{M_{E_2}} \sigma_2 \,, \\
\frac{\sigma_\tau}{\Lambda_{\mathsf{CP}}} &= -\frac{y_3'}{M_{E_3}} \sigma_3 \,.
\end{aligned}
\tag{C.6}
$$

In particular, the electron Yukawa is naturally small for $M_{E_1} \gg M_{E_2}$.

We should mention that $U(1)_{\mu-\tau}$ breaking would be induced in the charged lepton sector by the additional couplings between E_i and η_k as

$$
\begin{aligned}
-\mathcal{L}^l \subset {} & \mu_{12}' \bar{E}_{1L} E_{2R} \eta_1^* + \mu_{13}' \bar{E}_{1L} E_{3R} \eta_1 \\
& + \mu_{21}' \bar{E}_{2L} E_{1R} \eta_1 + \mu_{31}' \bar{E}_{1L} E_{3R} \eta_1^* \\
& + \mu_{23}' \bar{E}_{2L} E_{3R} \eta_2 + \mu_{32}' \bar{E}_{3L} E_{2R} \eta_2^* + \text{h.c.},
\end{aligned}
\tag{C.7}
$$

where $\mu_{32}' = \mu_{23}'^*$, $\mu_{13}' = \mu_{12}'^*$, $\mu_{31}' = \mu_{21}'^*$. However, we can assume that $U(1)_{\mu-\tau}$ breaking scale is much smaller than the bare mass terms for E_i as

$$
|\langle \eta_{1,2} \rangle| \ll M_{E_2} \ll M_{E_1} \,.
\tag{C.8}
$$

In this case, the $U(1)_{\mu-\tau}$ breaking effects can be neglected and (C.2) is effectively obtained after E_i are integrated out. Since $\langle \eta_k \rangle$ are related to N_R masses, more specifically to the generation of θ_{12}, θ_{13} and N_2, N_3 mass splitting, (C.8) means that N_R mass scale is much smaller than the E_i scale. An alternative way of avoiding $U(1)_{\mu-\tau}$ breaking in the charged lepton sector would be to use $\mathbb{Z}_4^L$.

As for the scale of $\langle \sigma_\pm \rangle$, we should have $\langle \sigma_\pm \rangle / M_{E_2} \gtrsim 10^{-2}$ for an order one y_3' coupling in (C.6), and it can lie below or above the $U(1)_{\mu-\tau}$ breaking scale. Anyhow, $\sigma_\pm$ does not couple to N_R at renormalizable level due to the $\mathbb{Z}_2$ symmetry and CP breaking is not induced at leading order to the neutrino sector since η_k only couple to CP even combinations σ_+^2 and σ_-^2. We assume, however, that all $\langle \eta_k \rangle$, $\langle \sigma_\pm \rangle$, are greater than the scale where leptogenesis takes place, typically 10^{11}GeV in our case, so that CP breaking in the charged lepton sector can be manifest.

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