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Energy Procedia 14 (2012) 1041 – 1046
Energy Procedia

2011 2nd International Conference on Advances in Energy Engineering (ICAEE 2011)

The Cyclostationary Characteristic Analysis of the Time-Frequency Overlapped Signal in Single-Channel

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Abstract: the characteristic analysis of the time-frequency overlapped signal (TFOS) in single-channel is a powerful challenge in electronic countermeasures. So, in this paper, the time-frequency overlapped degrees (P_t, P_f) of signals are proposed to measure the level overlapped of in time domain and frequency domain. Afterwards, based on cyclostationary principle, the proposition about the cyclostationary characteristic of TFOS is proved. Moreover, the cyclic spectrum of TFOS is deduced. Theoretical analysis and experimental results show that the linear sum of multi-cyclostationary signals is still cyclostationary signal.
Keywords: time-frequency overlapped degree, cyclostationary, cyclic spectrum density, single-channel signal.

1. Introduction

In complex electromagnetism environment, the intercepted probability of time-frequency overlapped signals (TFOS, i.e. Multi-source signals are overlapped in time domain and frequency domain) increases rapidly in single-channel. But because of the complexity of TFOS, the overlap part of intercepted signals is usually ignored in signal processing. Therefore, the false alarm rate and lost alarm rate are increased in fact. Due to the important study significance in electronic countermeasures, analysis technology of TFOS is concerned by more and more researchers in recent years [1-2].

Cyclostationary analysis may offer some advantages to analyze TFOS for their unique characteristics in cyclic domain. So, based on cyclostationary principle, the cyclic spectral analysis of TFOS is an active area of research. Gardner [3] had been studying the cyclostationary property of non-stationary signals. Based on examples, G.B. Giannakis [4] had been analyzed the limit sum and accumulate of cyclostationary signals. The detection and modulation identification of TFOS is analyzed with known carrier frequencies and code rates [1-2]. But more attention in present literatures emphasizes on estimating carrier frequency and code rate of signals and analyzing relations of cyclostationary signals and stationary signals. How to measure the overlap level of signals in time domain and frequency domain

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doi:10.1016/j.egypro.2011.12.887

is not discussed, and cyclostationary characteristics of TFOS are not still proved the by reasonable deducing process.

In this paper, our study proposes the time-frequency overlap degree (TFOD) of multi-cyclostationary signals to descript the time-frequency overlap level of TFOS ,and gives a deduce work to prove the linear superposition of multi-cyclostationary signals is still the cyclostationary signal, and discusses the separatebility of TFOS in cyclic domain, based on cyclic spectrum density (CSD) analysis of TFOS. Both theoretical analysis and simulation verify that the inference in this paper is reasonable and efficient.

2. The time-frequency overlapped degree of TFOS

2.1 Signal model

Consider a single-channel system, TFOS signal is given as[1-2][5]

$$S(t) = \sum_{i=1}^{N} A_i S_i(t, T_i) + n(t) \tag{1}$$

where, $N \in Z^+$, $S_i(t,T_i)$ is cyclostationary signals, and $T_i(T_a \neq T_b, a, b \in i)$ represent the sampling period. A_i is the amplitude of $S_i(t,T_i)$. $n(t)$ is zero mean Gauss noise. When $N = 1$, $S(t)$ is the non-overlap signal. In this paper, $N \geq 2$ is discussed.

2.2 Time-frequency overlapped degree

In present, there is not still uniform definition for describing the overlap level of TFOS. Assume TFOS is comprised of two signals. Then, the time-frequency overlapped degree is obtained as

$$P_t = \begin{bmatrix} T_{xx}/T_x & \cancel{T_{xy}/T_x} \\ \cancel{T_{yx}/T_y} & T_{yy}/T_y \end{bmatrix} \times 100\%, \ P_f = \begin{bmatrix} B_{xx}/B_x & \cancel{B_{xy}/B_x} \\ \cancel{B_{yx}/B_y} & B_{yy}/B_y \end{bmatrix} \times 100\% \tag{2}$$

where, $P_t$and P_f represent the overlapped degree in time domain and frequency domain, respectively. $T_x$and $T_y$are respectively the time-width of the signal x and y. T_{xy} is the overlapped time-width of x and y. T_{xx} and T_{yy} are respectively themselves overlapped time-width of x and y ($T_{xy}=T_{yx}$). $0<P_t<1$, $P_t=1$indicate that one signal is completely covered by other signals in time domain. Similarly, in the P_f matrix, T is replaced by B. Where, B represent the frequency-bandwidth of signals and $0< P_f<1$, $P_f=1$indicate that one signal is completely covered by other signals in frequency domain.

In the same way, we can define the $N \times N$ matrix to describe the time-frequency overlapped level of TFOS. Here, TFOS is made up of N cyclostationary signals.

3 The cyclostationary characteristic of TFOS

3.1 Cyclostationary principles

The zero-mean nonstationary random signal $s(t)$ is cyclostationary when its autocorrelation is a periodic function of time. The cyclic autocorrelation function of time series $s(t)$ can be defined by[3-4]

$$R_s^\alpha(\tau) = \frac{1}{T}\int_{-T/2}^{T/2} R_s(t,\tau)\exp(-j2\pi\alpha t)dt = \frac{1}{T}\int_{-T/2}^{T/2} E[s(t)s^*(t-\tau)]\exp(-j2\pi\alpha t)dt \tag{3}$$

where, α is cyclic frequency. The cyclic spectrum of $R_s^\alpha(\tau)$ can be obtained by Fourier transform using (3) as the input.

$$S_s^\alpha(f) = \int_{-\infty}^{+\infty} R_s^\alpha(\tau)\exp(-j2\pi f\tau)d\tau \tag{4}$$

3.2 Cyclostationary characteristics

Proposition [4]**:** Let $S_i(t,T_i)$ is cyclostationary signal and mutual independence. Then, $S(t)$ is still cyclostationary signal. Where, the period of $S_i(t,T_i)$ is T_i and $E[S_i(t,T_i)]=0$. $n(t)$ is Gauss Noise.

Prove: $S_i(t,T_i)$ is the cyclostationary signal, then, $R_{s_i}(t,u)=R_{s_i}(t+T_i,u+T_i)=R_{s_i}(t+T,u+T)$ is gained when the length of data is unlimited and the lowest common multiple of $\{T_i, 1<i\le N\}$ is T. The mean-value of the $S(t)$ and $S(t+T)$ can be given as

$$m_S(t)=E\left\{\sum_{i=1}^{N}A_iS_i(t)+n(t)\right\}=E\left\{\sum_{i=1}^{N}A_iS_i(t)\right\}+E\{n(t)\}=0，m_S(t+T)=E\left\{\sum_{i=1}^{N}A_iS_i(t+T)+n(t+T)\right\}=0$$

As a consequence, $m_S(t)=m_S(t+T)$. The autocorrelation function of $S(t)$ can be obtained as

$$\begin{aligned}
R_S(t,u)&=E\left\{S(t)S^*(u)\right\}=E\left\{[\sum_{i=1}^{N}A_iS_i(t)+n(t)]\times[\sum_{j=1}^{N}A_jS_j^*(u)+n(u)]\right\}\\
&=E\left\{\sum_{i=1}^{N}A_i^2S_i(t)S_i^*(u)+\sum_{i=1}^{N}\sum_{j=1\cap j\ne i}^{N}A_iA_jS_i(t)S_j^*(u)+\sum_{i=1}^{N}A_iS_i(t)*n(t)\right.\\
&\qquad\left.+\sum_{j=1}^{N}A_jS_j^*(u)*n(u)+n(t)n(u)\right\}\\
&=\sum_{i=1}^{N}A_i^2R_{s_i}(t,u)+\sum_{i=1}^{N}\sum_{j=1\cap j\ne i}^{N}A_iA_jm_{s_i}(t)m_{s_j}^*(u)\\
&=\sum_{i=1}^{N}A_i^2R_{s_i}(t+T,u+T)+\sum_{i=1}^{N}\sum_{j\ne i}^{N}A_iA_jm_{s_i}(t+T)m_{s_j}^*(u+T)\\
&=E\left\{[\sum_{i=1}^{N}A_iS_i(t+T)+n(t+T)]\times[\sum_{j=1}^{N}A_jS_j^*(u+T)+n(t+T)]\right\}\\
&=R_S(t+T,u+T)
\end{aligned}\tag{5}$$

The equation (5) shows that the single-channel TFOS $S(t)$ is still the cyclostationary signal.

3.3 Spectral correlation function analysis

Suppose $E\{S_i(t)\}=0$, the spectral correlation function of the $S(t)$ can be computed as

$$
\begin{aligned}
R_S(t,\tau) &= E\left\{S(t)S^*(t-\tau)\right\} = E\left\{\sum_{i=1}^{N} S_i(t)\sum_{j=1}^{N} S_j^{\,*}(t-\tau)\right\} \\
&= E\left\{\left[\sum_{i=1}^{N} A_i S_i(t)+n(t)\right]\left[\sum_{j=1}^{N} A_j S_j^{\,*}(t-\tau)+n(t-\tau)\right]\right\} \\
&= E\left\{\sum_{i=1}^{N} A_i S_i(t)\sum_{j=1}^{N} A_j S_j^{\,*}(t-\tau)+\sum_{i=1}^{N} A_i S_i(t)*n(t-\tau)\right. \\
&\qquad \left.+\sum_{j=1}^{N} A_j S_j^{\,*}(t-\tau)*n(t)+n(t)n(t-\tau)\right\} \\
&= E\left\{\sum_{i=1}^{N} A_i A_i S_i(t) S_i^*(t-\tau)\right\}+E\left\{\sum_{i=1}^{N}\sum_{i\neq j}^{N} A_i A_j S_i(t) S_j^{\,*}(t-\tau)\right\}+E\left\{n(t)n(t-\tau)\right\} \\
&= \sum_{i=1}^{N} A_i^2 R_{S_i}(t,t-\tau)+P^2\delta(\tau)
\end{aligned}
\tag{6}
$$

The equation (3) can be transformed by using (6) as the input

$$
\begin{aligned}
R_S^{\alpha}(\tau) &= \lim_{\Delta T\to\infty}\frac{1}{\Delta T}\int_{-\Delta T/2}^{\Delta T/2} R_S(t,\tau)e^{-j2\pi\alpha t}dt = \lim_{\Delta T\to\infty}\frac{1}{\Delta T}\int_{-\Delta T/2}^{\Delta T/2}\left[\sum_{i=1}^{N} A_i^2 R_{S_i}(t,t-\tau)+P^2\delta(\tau)\right]e^{-j2\pi\alpha t}dt \\
&= \begin{cases} \displaystyle\sum_{i=1}^{N} A_i^2 R_{S_i}^{\alpha}(\tau), & \alpha\neq 0 \\ \displaystyle\sum_{i=1}^{N} A_i^2 R_{S_i}^{0}(\tau)+P^2\delta(\tau), & \alpha=0 \end{cases}
\end{aligned}
\tag{7}
$$

The cyclic spectrum $S_S^{\alpha}(f)$ of TFOS can be gained by using the equation (4) and (7)[5].

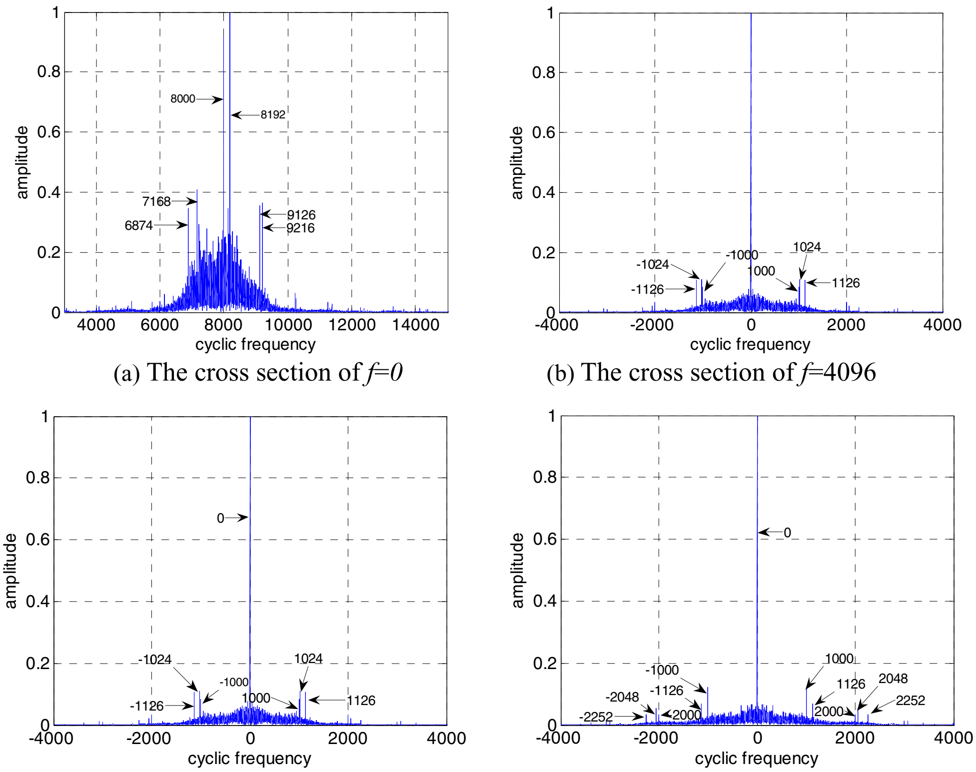
$$
S_S^{\alpha}(f)=\int_{-\infty}^{\infty} R_S^{\alpha}(\tau)e^{-j2\pi f\tau}d\tau=\begin{cases} \displaystyle\sum_{i=1}^{N} A_i^2 S_{S_i}^{\alpha}(f), & \alpha\neq 0 \\ \displaystyle\sum_{i=1}^{N} A_i^2 S_{S_i}^{0}(f)+S_n(f), & \alpha=0 \end{cases}
\tag{8}
$$

The equation (8) shows that the cyclic spectrum density (CSD) of TFOS is the linear sum of the CSD of multi-cyclostationary signals. The properties of the cyclic frequency of the cyclostationary signal are not interfered by the phenomenon of the overlap in time domain and frequency domain. This is consistent with the experimental analysis in literature [4]. Therefore, it is helpful to separate the TFOS, based on cyclostationary principle in cyclic domain, as long as the cyclic frequency of every overlapped original signal is deferent.

4. Experiment and analysis

Suppose the single-channel TFOS is comprised of two BPSK (*s1,s2*) and one QPSK (*s3*). Signal parameters are normalized and exampling frequency of the signal is 16384Hz. The carrier frequency (*f*) and code rate(*r*) of signals are f_{s1}=4096*Hz*, r_{s1}=1024*Baud*, f_{s2}=4000*Hz*, r_{s1}=1126*Baud*, f_{s1}=3800*Hz*, r_{s1}=1000*Baud*, respectively. The initial time and phase of signals are 0. The power ratio during signals is 1. The level of the Gauss white noise is 20dB. The simulations are based on the FFT Accumulation Method (FAM). Let signals arrived at receiver in the same time (P_t=1), the overlapped degrees of TFOS is $P_{f(s1,s2,s3)}$=[1,0.921,0.864; 1,1,0.963; 0.844,0.855,1]. Based on CSD of BPSK[3],[6], the cyclic frequency set (CFS) is $\left\{\alpha_{s1,2};\alpha_{s1,2}=\pm 2f_c \pm k/T, \alpha = k/T; k \in Z\right\}$. Where f_c stand for the carrier frequency. Moreover, the cyclic frequency set of QPSK is $\left\{\alpha_{s3};\alpha_{s3}=k/T;k\in Z\right\}$. So, the main CFS $\left\{\alpha_{s1};\alpha_{s1}=0,\pm 1024,\pm 2048,\pm 8192,\pm 9216,\pm 7168\right\}$ can be gained. Similarly, the CFSs of *s2*and *s3* are $\left\{\alpha_{s2};\alpha_{s2}=0,\pm 1126,\pm 2252,\pm 8000,\pm 9126,\pm 6874\right\}$ and $\left\{\alpha_{s3};\alpha_{s3}=0,\pm 1000,\pm 2000\right\}$, respectively. The CFS of TFOS is $\left\{\alpha;\alpha=\alpha_{s1}\cup\alpha_{s2}\cup\alpha_{s3}\right\}$.

The CSD of TFOS (BPSK and QPSK) is shown in Figure1. Because of inexistence the cyclic frequency ($2f_c$) in the cyclic frequency set of QPSK, there is not peak at $2f_c$ in Figure1 (a). Peaks are directly related to the code rate of BPSK and QPSK in Figure1 (b), (c) and (d). Locations of peaks are corresponding to the cyclic frequency set $\alpha=\left\{0,\pm 1000,\pm 2000,\pm 1024,\pm 2048,\pm 1126,\pm 2252\right\}$.



(a) The cross section of *f=0* (b) The cross section of *f*=4096

(c) The cross section of *f*=4000 (d) The cross section of *f*=3800

Figure2. The CSD analysis of the time-frequency overlapped signals (BPSK and QPSK)

5. Conclusions

Because multi-cyclostationary signals are overlapped in time domain and frequency domain, it is difficult to analyze the multi-cyclostationary signals overlapped. In this paper, based on the signal model of TFOS, the time-frequency overlapped level of signals is measured by the time-frequency degree. And then, the cyclostationary characteristics of TFOS are educed. Theoretical analysis and the experiment show that the linear sum of independence cyclostationary signals is still the cyclostationary signal. After multi-cyclostationary signals are mixed, cyclostationary characteristics of the original signal are not distributed by other signals.

Acknowledgments

This paper was supported by a grant from the National Natural Science Foundation of China (No. 60971103) and the Fundamental Research Funds for the Central Universities (NO.SWJTU11BR026).

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