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A note on "Generalized fuzzy rough approximation operators based on fuzzy coverings"

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ABSTRACT

In this note, we show by examples that Theorem 5.3, partial proof of Theorem 5.3′, Lemma 5.4 and Remark 5.2 in [1] contain slight flaws and then provide the correct versions.

1. Theorem 5.3. on Page 850 is incorrect. The following example shows that this theorem does not hold in general.

Example 1. Let $U = \{a, b, c\}$, a fuzzy relation R on U is given as follows:

R	a	b	c
a	0.5	0.2	0.4
b	0.2	0.3	0.6
c	0.4	0.6	0.7

$A = \{(a, 0.4), (b, 0.8), (c, 0.3)\}$, $\mathcal{T} = \mathcal{T}_L$, and $\mathcal{I} = \mathcal{I}_L$.

It is easy to see that R is symmetric and $\mathcal{T}$-transitive. By the definitions of $\underline{C''_R}$, $\overline{C''_R}$, $\underline{R}$ and $\overline{R}$, we have $\underline{C''_R}(A)(a) = 1$, $\underline{R}(A)(a) = 0.9$, $\overline{C''_R}(A)(c) = 0.1$ and $\overline{R}(A)(c) = 0.4$. Thus, $\underline{C''_R} \neq \underline{R}$ and $\overline{C''_R} \neq \overline{R}$.

Theorem 1 (The correction of Theorem 5.3). *If $R \in F(U \times U)$ is symmetric and $\mathcal{T}$-transitive, then $\underline{C''_R} \supseteq \underline{R}$ and $\overline{C''_R} \subseteq \overline{R}^{\mathcal{T}}$.*

Proof. For any $A \in F(U)$, $x \in U$,

$$\underline{C''_R}(A)(x) = \wedge_{y\in U}\mathcal{I}(R(y,x), \wedge_{z\in U}\mathcal{I}(R(y,z), A(z))) = \wedge_{z\in U}\wedge_{y\in U}\mathcal{I}(R(y,x), \mathcal{I}(R(y,z), A(z))) = \wedge_{z\in U}\mathcal{I}(\vee_{y\in U}\mathcal{T}(R(y,x), R(y,z)), A(z))$$
$$= \wedge_{z\in U}\mathcal{I}(\vee_{y\in U}\mathcal{T}(R(x,y), R(y,z)), A(z)) \geqslant \wedge_{z\in U}\mathcal{I}(R(x,z), A(z)) = \underline{R}(A)(x).$$

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Thus $\underline{\mathcal{C}''_R}(A) \supseteq \underline{R}(A)$ for any $A \in F(U)$.

$$\overline{\mathcal{C}''_R}(A)(x) = \vee_{y\in U}\mathcal{T}(R(y,x), \vee_{z\in U}\mathcal{T}(R(y,z),A(z))) = \vee_{z\in U}\vee_{y\in U}\mathcal{T}(R(y,x),\mathcal{T}(R(y,z),A(z)))$$
$$= \vee_{z\in U}\mathcal{T}(\vee_{y\in U}\mathcal{T}(R(y,x),R(y,z)),A(z)) = \vee_{z\in U}\mathcal{T}(\vee_{y\in U}\mathcal{T}(R(x,y),R(y,z)),A(z)) \leqslant \vee_{z\in U}\mathcal{T}(R(x,z),A(z)) = \overline{R}(A)(x).$$

Thus $\overline{\mathcal{C}''_R}(A) \subseteq \overline{R}(A)$ for any $A \in F(U)$. □

2. Conditions in Lemma 5.4 on Page 851 are not equivalent. A counterexample is shown as follows:

Example 2. Let $U = \{a,b\}$. A fuzzy relation R on U is given as follows:

R	a	b
a	0.4	0.5
b	0.3	0.1

$\mathcal{T} = \mathcal{T}_M$ and $\mathcal{I} = \mathcal{I}_G$. By the definitions of $\underline{R}$ and $\overline{R}$, we obtain $\underline{R}(1_a)(a) = \underline{R}(1_b)(b) = 0$, however, R is not symmetric.

Theorem 2 (The correction of Lemma 5.4). *The following statements are equivalent:*

(1) *R is symmetric.*
(2) $\overline{R}(1_x)(y) = \overline{R}(1_y)(x), \quad \forall x,y \in U.$
(3) $\underline{R}(1_x \Rightarrow_{\mathcal{I}} \hat{\alpha})(y) = \underline{R}(1_y \Rightarrow_{\mathcal{I}} \hat{\alpha})(x), \quad \forall x,y \in U,\ \forall \alpha \in [0,1].$

Proof. It directly follows from Theorems 4.2 (3) and 4.7 of [2]. □

3. Proof of Theorem 5.3′ on Page 851 should be modified as follows.

Suppose that $\underline{\mathcal{C}''_R} = \underline{R}$. For any x, $y \in U$, from Theorem 4.6 (7) of [1], we have

$$\underline{R}(1_x \rightarrow_{\mathcal{I}} \hat{\alpha})(y) = \underline{\mathcal{C}''_R}(1_x \rightarrow_{\mathcal{I}} \hat{\alpha})(y) = \underline{\mathcal{C}''_R}(1_y \rightarrow_{\mathcal{I}} \hat{\alpha})(x) = \underline{R}(1_y \rightarrow_{\mathcal{I}} \hat{\alpha})(x).$$

By Theorem 4.7 of [2], we can conclude that R is symmetric. □

4. In Section 4.3 on Page 849, Li et al. gave the following assertion that $co_{\mathcal{N}}\left(\underline{\mathcal{C}'_{FR}}(A)\right) = \overline{\mathcal{C}'_{FR}}(co_{\mathcal{N}}(A))$, $co_{\mathcal{N}}\left(\overline{\mathcal{C}'_{FR}}(A)\right) = \underline{\mathcal{C}'_{FR}}(co_{\mathcal{N}}(A))$ and $co_{\mathcal{N}}(\underline{\mathcal{C}''_{FR}}(A)) = \overline{\mathcal{C}''_{FR}}(co_{\mathcal{N}}(A))$.

In fact, the above three equalities do not hold in general. A counterexample is as follows:

Example 3. Let $U = \{a,b,c\}$, $K_1 = \{(a,0.4),(b,1),(c,0.3)\}$, $K_2 = \{(a,1),(b,0.4),(c,0.8)\}$,

$$K_3 = \{(a,0.1),(b,0.7),(c,1)\}, \quad \text{and} \quad A = \{(a,0.8),(b,0),(c,0.5)\}.$$

(1) In the definitions of $\underline{\mathcal{C}'_{FR}}$, $\overline{\mathcal{C}'_{FR}}$, $\underline{\mathcal{C}''_{FR}}$ and $\overline{\mathcal{C}''_{FR}}$, $\mathcal{T} = \mathcal{T}_M$, $\mathcal{I} = \mathcal{I}_G$, by calculating, we have $co_{\mathcal{N}}\left(\underline{\mathcal{C}'_{FR}}(A)\right)(c) = 1$, $\overline{\mathcal{C}'_{FR}}(co_{\mathcal{N}}(A))(c) = 0.4$, $co_{\mathcal{N}}\left(\underline{\mathcal{C}''_{FR}}(A)\right)(c) = 1$ and $\overline{\mathcal{C}''_{FR}}(co_{\mathcal{N}}(A))(c) = 0.7$.
(2) In the definitions of $\underline{\mathcal{C}'_{FR}}$ and $\overline{\mathcal{C}'_{FR}}$, $\mathcal{T} = \mathcal{T}_L$, $\mathcal{I} = \mathcal{I}_L$, by calculating, we have $co_{\mathcal{N}}\left(\overline{\mathcal{C}'_{FR}}(A)\right)(c) = 0.5$ and $\underline{\mathcal{C}'_{FR}}(co_{\mathcal{N}}(A))(c) = 0$.

The correct relationships are given in the next theorem.

Theorem 3. *Let $(U,\mathcal{C})$ be a generalized fuzzy approximation space. If $\mathcal{I}$ is an R-implicator based on a continuous t-norm $\mathcal{T}$ and $\mathcal{N}$ is a negator induced by $\mathcal{I}$, then*

$$\forall A \in IF(U), \quad co_{\mathcal{N}}\left(\underline{\mathcal{C}'_{FR}}(A)\right) \supseteq \overline{\mathcal{C}'_{FR}}(co_{\mathcal{N}}(A)), \quad co_{\mathcal{N}}\left(\overline{\mathcal{C}'_{FR}}(A)\right) \supseteq \underline{\mathcal{C}'_{FR}}(co_{\mathcal{N}}(A)), \quad co_{\mathcal{N}}\left(\underline{\mathcal{C}''_{FR}}(A)\right) \supseteq \overline{\mathcal{C}''_{FR}}(co_{\mathcal{N}}(A)).$$

Proof. For any $A \in IF(U)$, $x \in U$,

$$\overline{\mathcal{C}'_{FR}}(co_{\mathcal{N}}(A))(x) = \wedge_{C\in\mathcal{C}}\mathcal{I}(C(x), \vee_{y\in U}\mathcal{T}(C(y),(co_{\mathcal{N}}(A))(y))) = \wedge_{C\in\mathcal{C}}\mathcal{I}(C(x), \vee_{y\in U}\mathcal{T}(C(y),\mathcal{I}(A(y),0_L)))$$
$$\leqslant \wedge_{C\in\mathcal{C}}\mathcal{I}(C(x), \vee_{y\in U}\mathcal{I}(\mathcal{I}(C(y),A(y)),0_L)) \leqslant \wedge_{C\in\mathcal{C}}\mathcal{I}(C(x), \mathcal{I}(\wedge_{y\in U}\mathcal{I}(C(y),A(y)),0_L))$$
$$= \wedge_{C\in\mathcal{C}}\mathcal{I}(\mathcal{T}(C(x), \wedge_{y\in U}\mathcal{I}(C(y),A(y))),0_L) = \mathcal{I}(\vee_{C\in\mathcal{C}}\mathcal{T}(C(x), \wedge_{y\in U}\mathcal{I}(C(y),A(y))),0_L)$$
$$= \mathcal{N}(\vee_{C\in\mathcal{C}}\mathcal{T}(C(x), \wedge_{y\in U}\mathcal{I}(C(y),A(y)))) = (co_{\mathcal{N}}(\underline{\mathcal{C}'_{FR}}(A)))(x).$$

Thus we can conclude that $co_{\mathcal{N}}(\underline{\mathcal{C}'_{FR}}(A)) \supseteq \overline{\mathcal{C}'_{FR}}(co_{\mathcal{N}}(A))$. In the same way, the other relations can be proved. □

5. Based on the correct theorems, Remark 5.2 on Page 852 must be changed accordingly.

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