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New fast least-squares algorithm for estimating the best-fitting parameters due to simple geometric-structures from gravity anomalies



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ABSTRACT

A new fast least-squares method is developed to estimate the shape factor (q -parameter) of a buried structure using normalized residual anomalies obtained from gravity data. The problem of shape factor estimation is transformed into a problem of finding a solution of a non-linear equation of the form $f(q) = 0$ by defining the anomaly value at the origin and at different points on the profile (N -value). Procedures are also formulated to estimate the depth (z -parameter) and the amplitude coefficient (A -parameter) of the buried structure. The method is simple and rapid for estimating parameters that produced gravity anomalies. This technique is used for a class of geometrically simple anomalous bodies, including the semi-infinite vertical cylinder, the infinitely long horizontal cylinder, and the sphere. The technique is tested and verified on theoretical models with and without random errors. It is also successfully applied to real data sets from Senegal and India, and the inverted-parameters are in good agreement with the known actual values.

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Introduction

The gravity method has many applications such as hydrocarbon exploration [1], mineral exploration [2], cavity detection [3], engineering applications [4,5], geothermal activity [6], archaeological sites investigations [7,8], weapons inspection [9] and hydrological investigations [10]. It is known that the

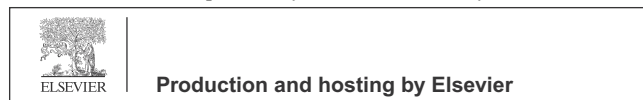
gravity data interpretation is non-unique where different subsurface causative targets may yield the same gravity anomaly; however, *a priori* information about the geometry of the causative target may lead to a unique solution [11]. Various quantitative interpretation methods of the gravity data over inhomogeneous structures have been developed. These methods can be classified into two categories:

Category I include two- (2D) and three-dimensional (3D) gravity tomography and inversion for arbitrary structures, which are found to be adequate in most cases [12–14]. The 3D gravity inverse problem solution based on rigorous and full forward modeling demands high computer resources, computational time and *a priori* information for the model parameters we invert for.

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Category II is based on describing the measured gravity anomaly due to isolated buried structures that can be approximated by some simple geometric-shaped bodies, such as a semi-infinite vertical cylinder, a horizontal cylinder or a sphere. In this case, fast quantitative interpretation methods based on geometrically simple anomalies can be utilized to estimate the shape and the other associated model parameters of the body that best fits the measured data.

The research we propose in this paper falls in category II. Several numerical methods have been developed to estimate the nature of the sources such as: Walsh transform technique [15], analytic signal [16], a simple formula approach [17], graphical method [18], least-squares minimization approach [19], use of moving average residuals [20,21], solving two quadratic equations [16], and use of horizontal gradient residuals [22].

Also, numerous numerical methods have been developed to estimate only the depth of the sources such as: using characteristic points and distances [23,24], ratio techniques [25,26], transformation techniques [27–29], least-squares approaches [30,31], Euler deconvolution technique [32].

A new fast least-squares inversion algorithm is developed which estimates the shape factor parameter (q -parameter) using a non-linear least-squares sense. The q -parameter estimation problem is transformed into the problem of finding also a solution of a non-linear function $f(q) = 0$. The solution is obtained by minimizing a function in the least-squares way. After knowing the shape factor, the depth (z -parameter) parameters and the amplitude coefficient (A -parameter) parameter is estimated using simple formulas. Using the entire measured data make the results produced more reliable and realistic, and helps minimize the uncertainties due to the non-uniqueness and ill-posedness of the inverse problem solution.

So, the proposed method has been tested on noise-free synthetic data sets. In order to analyze this method better, we examine the effect of noise in the data, the effect of the error response of the chosen function related to N -value and error in the choice of the origin point. Finally, the fast algorithm is applied to two real data sets from Senegal and India and the interpreted shape and depth parameters are in good agreement with the known actual values.

Methodology

The general formula of a gravity anomaly generated by a semi-infinite vertical cylinder, an infinitely long horizontal cylinder, or a sphere (Fig. 1) at a point $P(x_i)$ along a profile [26] is given by:

$$g(x_i, z, q) = A \frac{z^m}{(x_i^2 + z^2)^q}, \quad i = 1, 2, 3, \dots, L \quad (1)$$

where

$$A = \begin{cases} \frac{4}{3} \pi G \sigma R^3 \\ 2\pi G \sigma R^2 \\ \pi G \sigma R^2 \end{cases}, \quad m = \begin{cases} 1 \\ 0 \end{cases}, \quad q = \begin{cases} \frac{3}{2} & \text{for a sphere} \\ 1 & \text{for a horizontal cylinder} \\ \frac{1}{2} & \text{for a vertical cylinder } R \ll z \end{cases}$$

In Eq. (1), z is the depth to the body (km), q is the shape factor (dimensionless), A ($\text{mGal} \times \text{km}^{2q-m}$) is the amplitude coefficient whose unit is shape factor dependent, x_i is the coordinate of the measurement station (km), σ is the density contrast (g/cc), G is the universal gravitational constant, and R is the radius (km). The shape factors of a sphere (3D), an infinitely long horizontal cylinder (2D), and a semi-infinite vertical cylinder (3D) are 1.5, 1.0, and 0.5, respectively.

At the origin ($x_i = 0$), the Eq. (1) gives the following relationship:

$$A = g(0)z^{2q-m}, \quad (2)$$

Using Eq. (2), we obtain the following normalized gravity anomaly form:

$$F(x_i, z, q) = \left(\frac{z^2}{x_i^2 + z^2} \right)^q, \quad (3)$$

where $F(x_i, z, q) = \frac{g(x_i, z, q)}{g(0)}$.

Again, for all shapes, Eq. (3) gives the following value at $x_i = \pm N$

$$T = \left(\frac{z^2}{N^2 + z^2} \right)^q, \quad N = \pm 1, 2, 3, \dots \quad (4)$$

From Eq. (4), we obtain the following equation for the depth (z):

$$z = N \sqrt{\frac{T^{1/q}}{1 - T^{1/q}}}. \quad (5)$$

Substituting Eq. (5) into Eq. (3), we obtain the following equation for the shape factor (q):

$$F(x_i, q) = \left(\frac{N^2 P(q)}{x_i^2 + P(q)(N^2 - x_i^2)} \right)^q, \quad (6)$$

where $P(q) = T^{1/q}$.

The unknown shape factor (q) in Eq. (6) can be obtained by minimizing:

$$\varphi(q) = \sum_{i=1}^M [L(x_i) - F(x_i, q)]^2, \quad (7)$$

where $L(x_i)$ denotes the normalized observed gravity anomaly at x_i .

Setting the derivative of $\varphi(q)$ to zero with respect to q leads to

$$f(q) = \sum_{i=1}^M [L(x_i) - W(x_i, q)] W^*(x_i, q) = 0, \quad (8)$$

where $W^*(x_i, q) = \frac{d}{dq} W(x_i, q)$

Eq. (8) can be solved for q using the standard methods for solving nonlinear equations [33], and its iteration form can be expressed as:

$$q_f = f(q_j), \quad (9)$$

where q_j is the initial shape factor and q_f is the revised shape factor; q_f will be used as the q_j for the next iteration. The iteration stops when $|q_f - q_j| \leq e$, where e is a small predetermined real number close to zero.

Once the q -parameter is known, the depth (z -parameter) can be estimated from Eq. (5) and the amplitude coefficient (A -parameter) can be determined from Eq. (2). Theoretically, one N -value is enough to determine the shape factor and the other model parameters. In real data, more than one N -value is desirable because of the presence of noise in data.

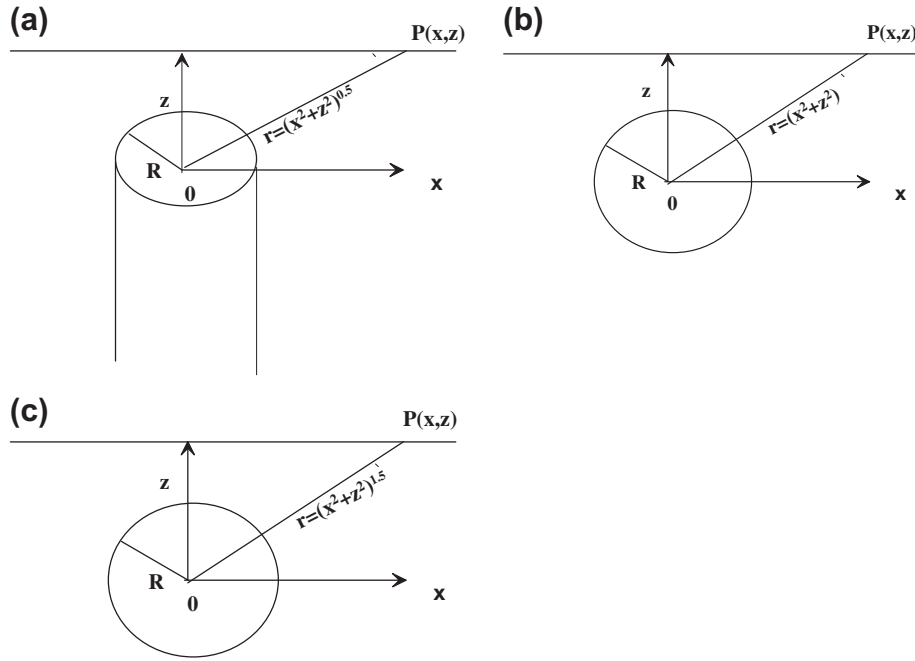


Fig. 1 Diagrams for various simple geometrical structures: (a) a semi-infinite vertical cylinder, (b) a horizontal cylinder, and (c) a sphere.

We then measure the goodness of fit between the observed and computed gravity data for each set of solutions. The standard error (μ) is used in this paper as a statistical preference criterion in order to compare the observed and calculated values. This μ is given by the following mathematical relationship [34]:

$$\mu = \sqrt{\frac{\sum_{i=1}^k [g(x_i) - g_c(x_i)]^2}{k}}, \quad (10)$$

where $g(x_i)$ is the observed gravity value and $g_c(x_i)$ is the calculated gravity value at the point x_i ($i = 1, 2, \dots, k$), respectively. k stands for the total number of data.

Results and discussions

This new fast least-squares inversion algorithm was tested on several synthetic datasets of a semi-infinite vertical cylinder (3D), an infinitely long horizontal cylinder (2D), and a sphere (3D) causative body. In order to assess and analyze this algorithm better, we will examine in the following two subsections the effect of the noise added to the data and the effect of error in the origin and T -value.

Effect of random noise

Synthetic examples of a semi-infinite vertical cylinder ($q = 0.5$, $A = 250 \text{ mGal} \times \text{unit}$, profile length = 20 units, sample interval = 1 unit, and $N = 3$ units), an infinitely long horizontal cylinder ($q = 1.0$, $A = 500 \text{ mGal} \times \text{unit}$, profile length = 20 units, sample interval = 1 unit, and $N = 4$ units), and a sphere ($q = 1.5$, $A = 1000 \text{ mGal} \times \text{unit}^2$, profile length = 20 units, sample interval = 1 unit, and $N = 6$ units) were defined. They were buried at different depths and interpreted using the introduced method (Eqs. (8), (5), and (2)) to estimate the shape factor (q -parameter), depth (z -parameter), and amplitude coefficient

(A -parameter), respectively. In all cases examined, the exact values of the q -, z -, and A -parameters were obtained. However, in studying the error response of the least-squares method, synthetic examples contaminated with 5% random errors were considered using the following formula:

$$\Delta g_{\text{rand}}(x_i) = g(x_i) + 5(\text{RND}(i) - 0.5), \quad (11)$$

where $\Delta g_{\text{rand}}(x_i)$ is the contaminated anomaly value at x_i , and $\text{RND}(i)$ is a pseudo-random number whose range is (0, 1). The interval of the pseudo-random number is an open interval, i.e., it does not include the extremes 0 and 1.

Following the proposed interpretation scheme, values of the most appropriate model parameters (q , z , and A) were computed and the percentages of error in model parameters were plotted against the model depth for comparison (Fig. 2).

We verified numerically that the shape factor obtained is within 5% for the semi-infinite vertical cylinder, 2% for the horizontal cylinder and 1.7% for the sphere models. The depth obtained is within 7.6% for the semi-infinite vertical cylinder, 3.3% for the horizontal cylinder and 3.4% for the sphere models, whereas the amplitude coefficient is within 8.6% for the semi-infinite vertical cylinder, 9.1% for the horizontal cylinder and 9.5% for the sphere models (Fig. 2).

A noisy synthetic examples of a sphere model ($q = 1.5$, $A = 1000 \text{ mGal} \times \text{unit}^2$, profile length = 30 units, sample interval = 1 unit, and $N = 3$ units) was buried at different depths. It interpreted using the present method and three least-squares method [35] to estimate the shape factor, depth, and amplitude coefficient, respectively. The numerical result for the percentage errors in model parameters are summarized in Table 1. Table 1 shows that the results are modified by using the present algorithm are better than the other methods because our technique is robust in the presence of noise.

Good results are obtained by using the present algorithm—especially for shape and depth estimation, which is a primary concern in gravity prospecting and other geophysical work.

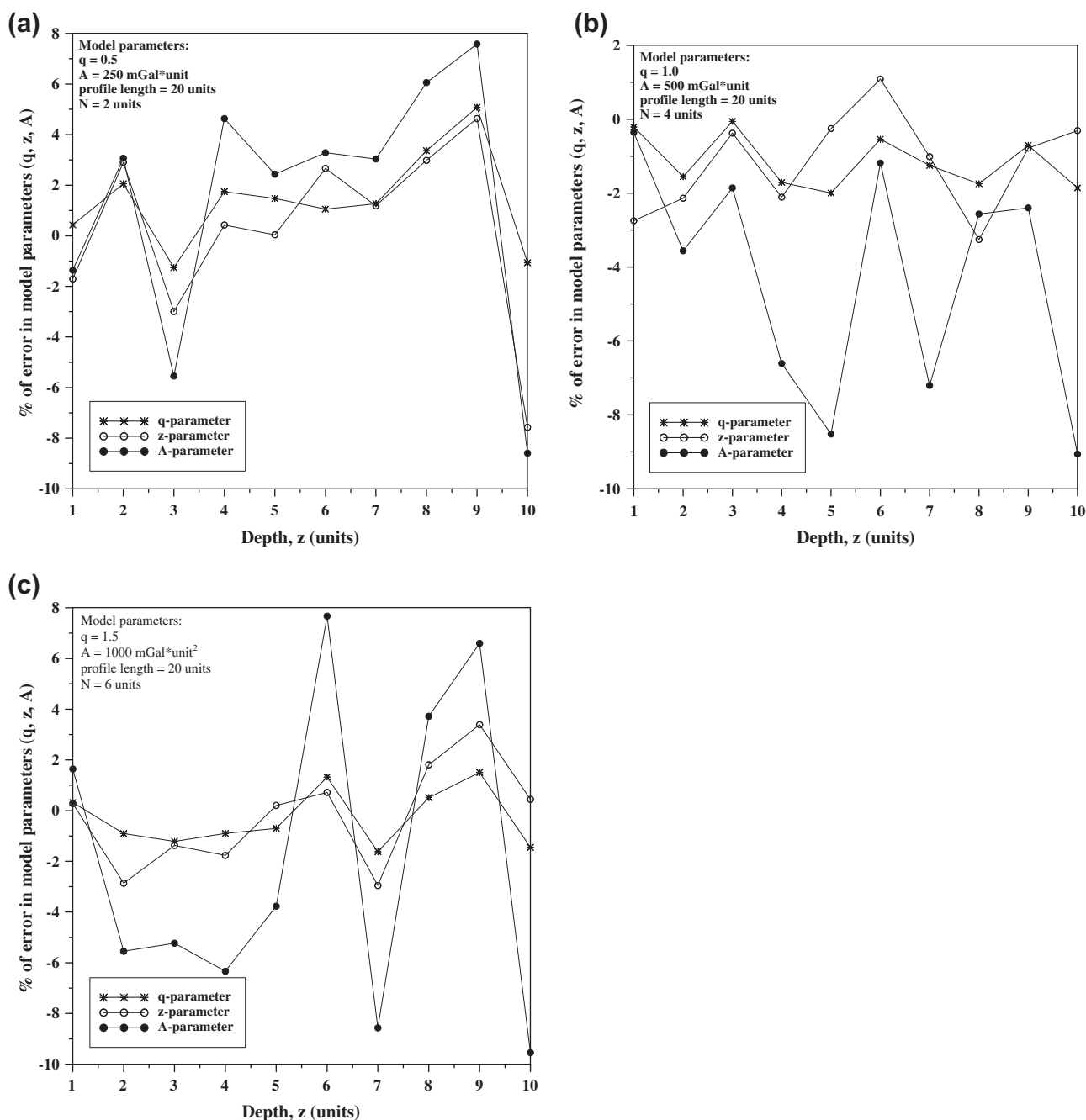


Fig. 2 Error response in model parameters (q , z , and A) estimates for (a) a semi-infinite vertical cylinder model, (b) a horizontal cylinder model, and (c) a sphere model. Abscissa: model depth. Ordinate: percent error in model parameters.

Table 1 Comparison results for the percentage errors in model parameters of a sphere model ($q = 1.5$, $A = 1000 \text{ mGal} \times \text{unit}^2$, profile length = 30 units, and sample interval = 1 unit).

	% Of error in The present method	Three least-squares method [35]
q -Parameter	1.6	3
z -Parameter	2.2	2.5
A -parameter	8.0	9

For synthetic data, we also verified that only a few points around $g(0)$ are needed to obtain the exact values of q -, z -, and A -parameters. However, the data with random errors require more points around $g(0)$.

Effect of errors in $g(0)$ and T

In studying the error response of the least-squares method, synthetic example of an infinitely long horizontal cylinder model ($q = 1.0$, $z = 5$ units, $A = 800 \text{ mGal} \times \text{unit}$, and profile

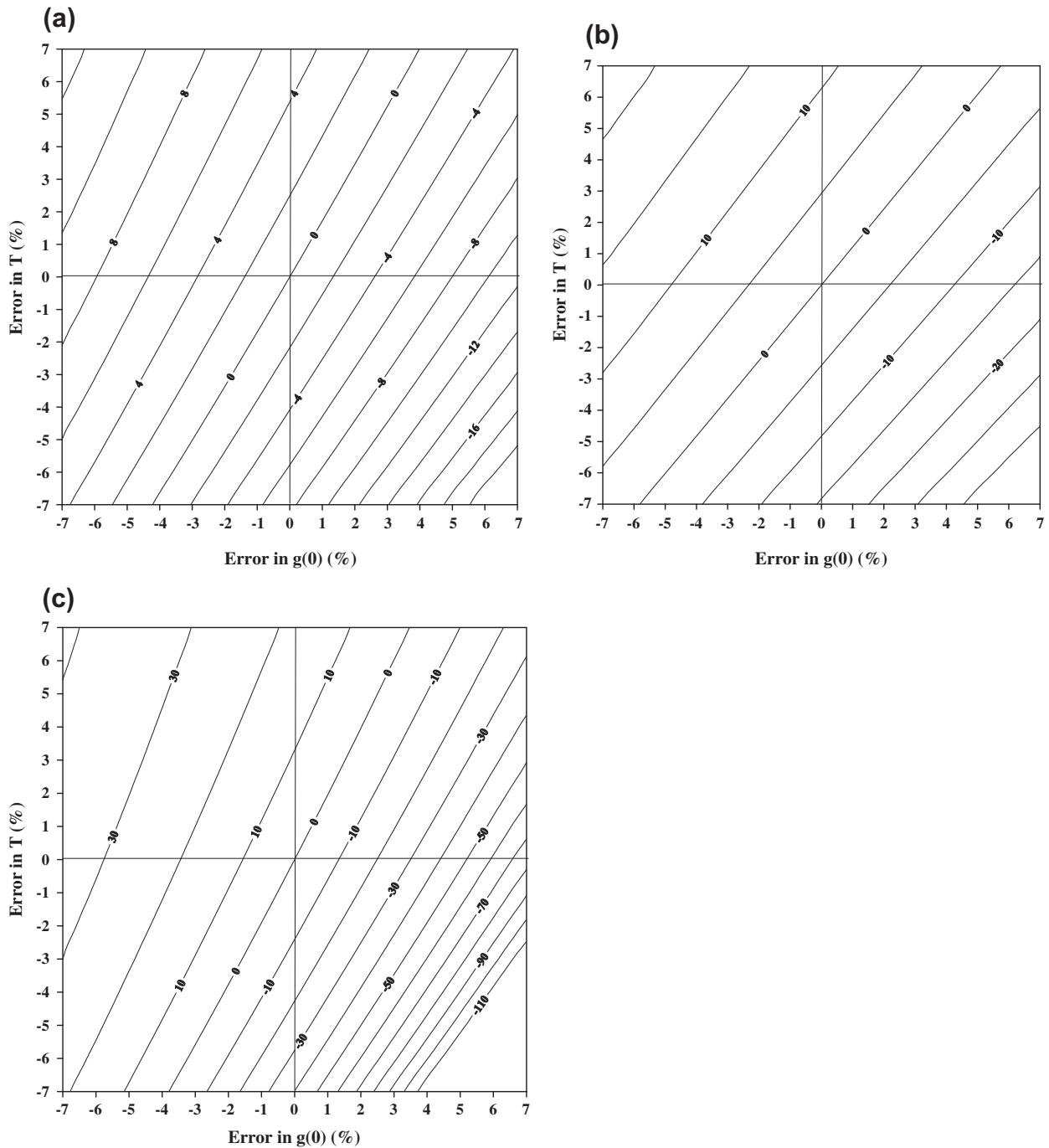


Fig. 3 A map showing error response (a) in shape factor (q -parameter), (b) in depth (z -parameter), and (c) in amplitude coefficient (A -parameter) estimates for a horizontal cylinder model ($q = 1.0$, $z = 5$ units, and $A = 800$ mGal \times unit, profile length = 40 units). Abscissa: percent error in $g(0)$. Ordinate: percent error in T .

length = 40 units) was considered in which errors of $\pm 1\%$, $\pm 2\%$, $\pm 3\%$, \dots , $\pm 7\%$ were assumed in both $g(0)$ and T . Following the same interpretation method, values of the three model parameters (q , z , and A) were computed and the percentage of errors in the model parameters were mapped, first using synthetic data without random noise (Fig. 3) and then using synthetic data with 20% random noise (Fig. 4). Figs. 3 and 4a show that the maximum error in the q -parameter is about 20% when both $g(0)$ and T have errors of 7% and -7% . Also,

Figs. 3 and 4b show the maximum error in the z -parameter is about 30% when $g(0)$ and T have 7% and -7% errors. On the other hand, Figs. 3 and 4c illustrate that the maximum error in the A -parameter is about 125% when $g(0)$ and T have 7% and -7% errors. Finally, when $g(0)$ and T are kept undisturbed, the percentage of error in model parameters is slightly smaller or greater than the imposed error. This demonstrates that the proposed method will give reliable model parameters solution even when both $g(0)$ and T are not correct and noisy.

Field examples

The Louga anomaly

The observed gravity anomaly profile is 32 km length, lying over the Louga area, west coast of Senegal, West Africa [36]. The anomaly profile was digitized at an interval of 0.5 km (Fig. 5a). The proposed inverse technique has been applied to the observed data to estimate the q -parameter, z -parameter

and A -parameter using the normalized field of the observed gravity data (Fig. 5b). Then we computed the standard error (μ) between the observed values and the values computed from estimated parameters q , z and A for each N -value. The results are shown in Table 2 for the cases of N -value. Also we computed the set of mean values and the optimum set ($\mu = 2.48$ mGal) is given at $N = 2$ km. The best-fit-model parameters are $q = 0.53$, $z = 4.94$ km and $A = 545.68$ mGal \times km (Fig. 5a). This suggests that the shape of the buried structure resembles a 3-D

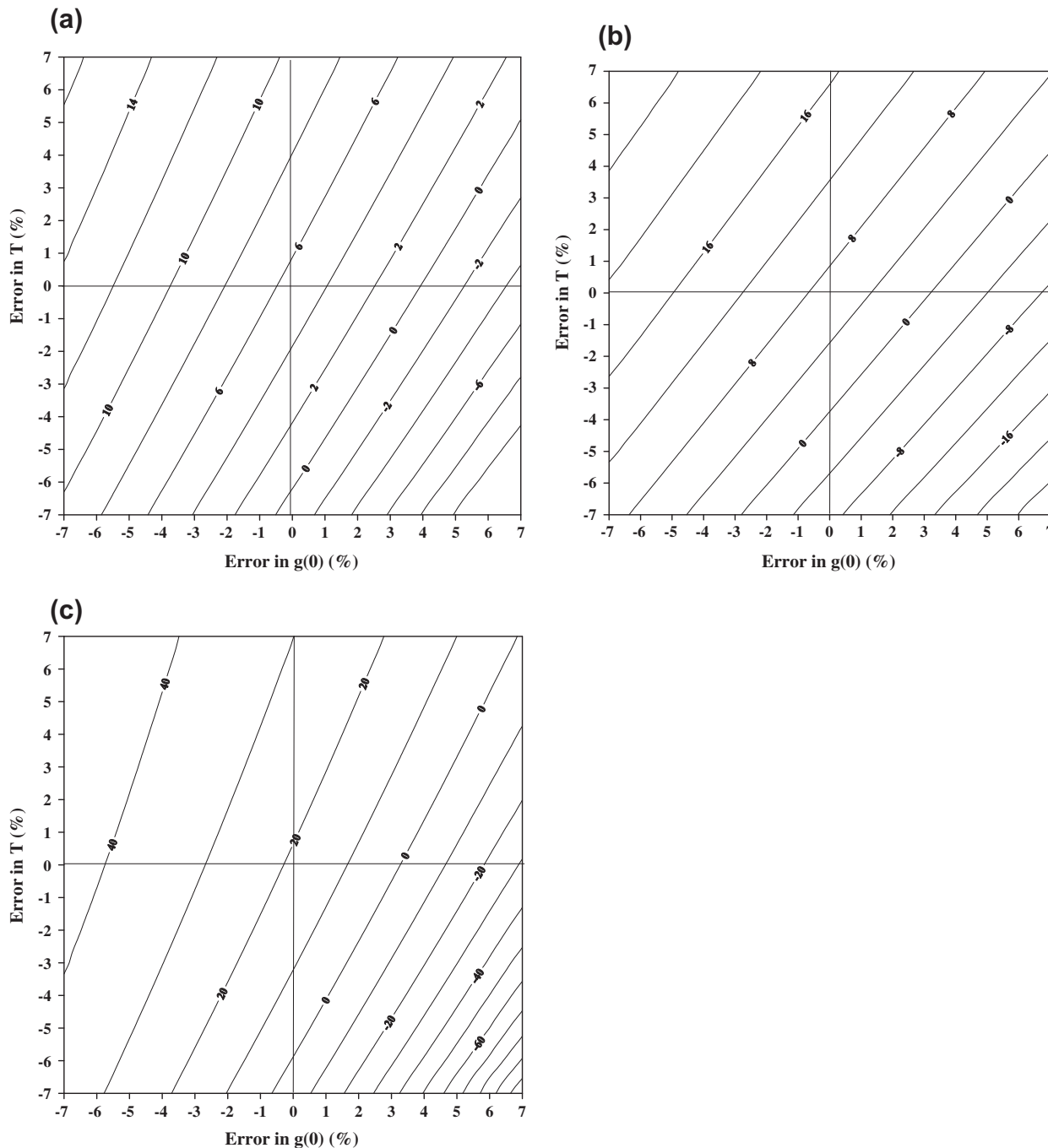


Fig. 4 A map showing error response (a) in shape factor (q -parameter), (b) in depth (z -parameter), and (c) in amplitude coefficient (A -parameter) estimates for a horizontal cylinder model ($q = 1.0$, $z = 5$ units, and $A = 800$ mGal \times unit, profile length = 40 units) after adding 20% random noise. Abscissa: percent error in $g(0)$. Ordinate: percent error in T .

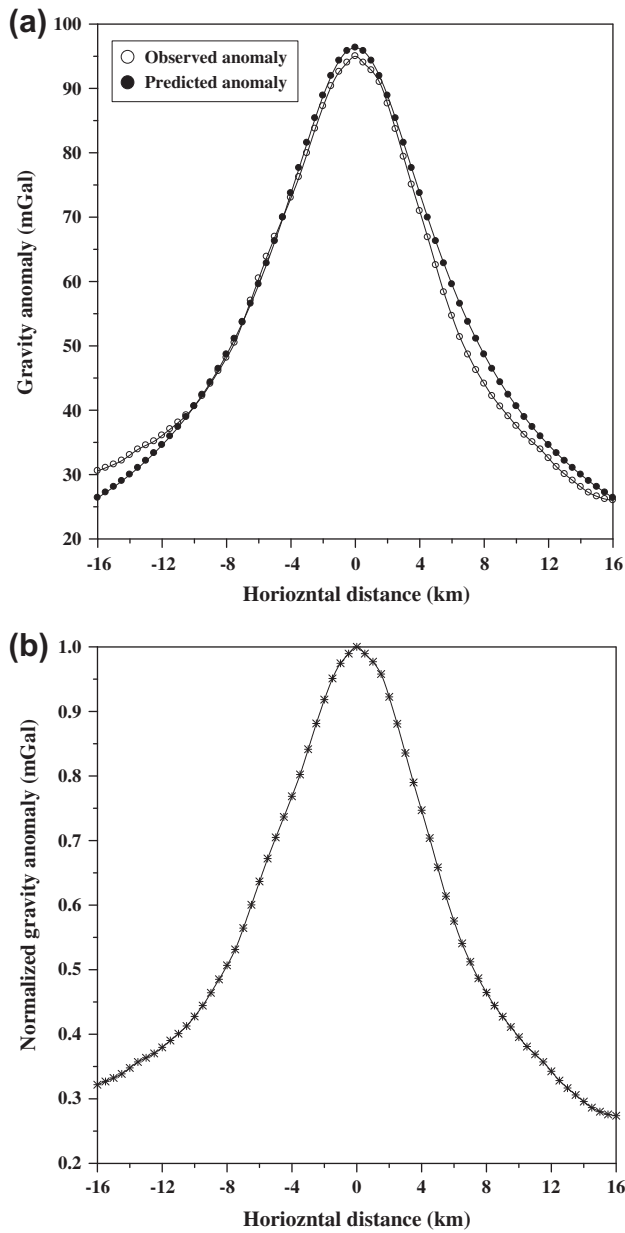


Fig. 5 (a) The observed gravity anomaly over the Louga area, west coast of Senegal, West Africa. (b) Normalized gravity anomaly data over the Louga area, west coast of Senegal, West Africa.

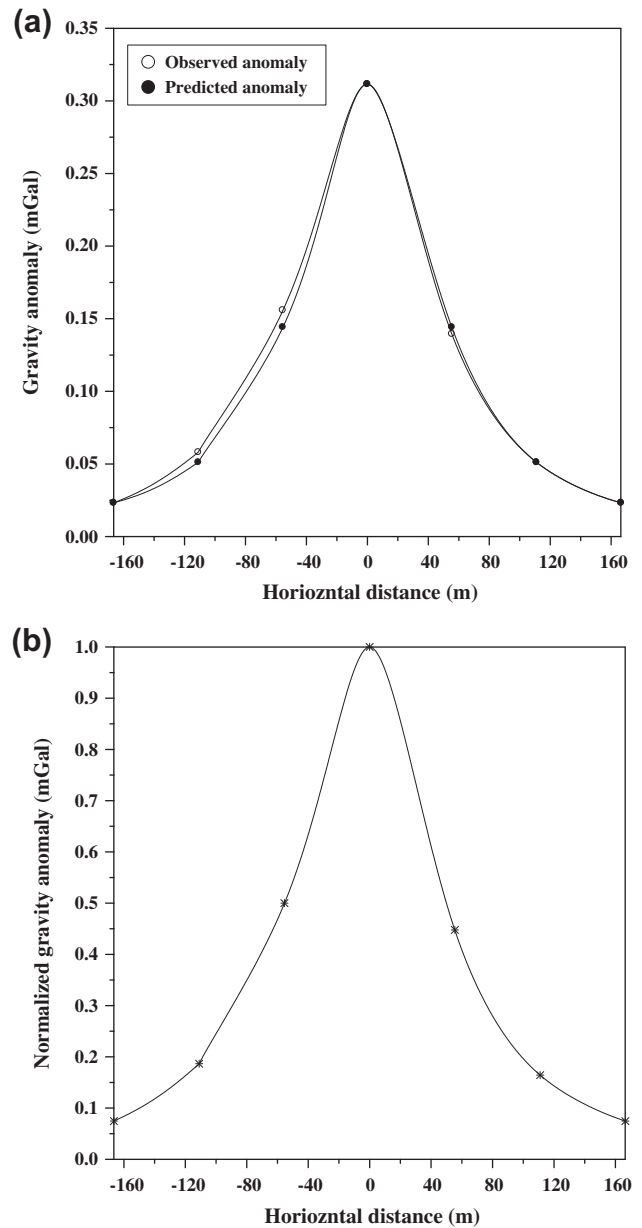


Fig. 6 (a) The residual gravity anomaly over a manganese deposit near Nagpur, India. (b) Normalized gravity anomaly data over a manganese deposit near Nagpur, India.

Table 2 Numerical results for the Louga area, west coast of Senegal, West Africa.

N (km)	q -Parameter	z -Parameter (km)	A -parameter (mGal \times km)	μ (mGal)
1.0	0.50	4.59	481.60	5.35
1.5	0.56	5.31	632.75	3.34
2.0	0.53	4.94	545.68	2.48
2.5	0.51	4.72	503.73	4.07
3.0	0.50	4.57	477.43	5.64
3.5	0.49	4.47	462.11	6.66
4.0	0.49	4.44	456.78	7.07
4.5	0.49	4.37	445.82	8.05
5.0	0.47	4.18	420.41	10.39
Average (m)	0.51	4.62	491.81	5.62

Table 3 Numerical results for the Manganese deposit near Nagpur, India.

N (km)	q -Parameter	z -Parameter (m)	A -parameter (mGal \times m)	μ (mGal)
55.5	1.092	53.234	16.460	0.007
111.0	1.149	56.779	17.813	0.005
166.5	1.000	47.224	14.715	0.011
Average (m)	1.080	52.412	16.329	0.007

semi-infinite vertical cylinder model buried at a depth of 4.94 km.

Manganese deposit body anomaly

The residual gravity anomaly over a manganese deposit near Nagpur, India [37] was shown (Fig. 6a). This profile has a length of 333 m, and the gravity curve was digitized at intervals of 55.50 m. The proposed inverse technique has been applied to the observed data to estimate the q -parameter, z -parameter and A -parameter using the normalized field of the observed gravity data (Fig. 6b). Then we computed the standard error (μ) between the observed values and the values computed from estimated parameters q , z and A for each N -value. The results are shown in Table 3 for the three cases of different N -values. Also we computed the set of mean values and the optimum set ($\mu = 0.005$ mGal) is given at $N = 111.00$ m. The best-fit-model parameters are $q = 1.15$, $z = 56.78$ m and $A = 17.81$ mGal \times m (Fig. 6a). This suggests that the shape of the buried structure resembles a 2-D horizontal cylinder model buried at a depth of 56.8 m. The shape and the depth to the center of the ore body obtained by the present method agree very well with those obtained from other methods [38].

Conclusion

A fast least-squares approach has been developed to estimate the appropriate nature of the source (q -parameter), the depth (z -parameter) and the amplitude coefficient (A -parameter) of a buried structure from the normalized gravity anomaly data of a long profile. The inverse fast algorithm has been derived for fast gravity quantitative interpretation for geometrically simple anomalous bodies, such as a 3D semi-infinite vertical cylinder, a 2D infinitely long horizontal cylinder, and a 3D sphere.

The suggested method is automatic and it can use all the observed gravity data in estimating these three parameters. Previous techniques have typically used only a few points, distances, standardized curves, and nomograms. The suggested algorithm is found to be stable and can estimate the gravity parameters with a reasonable accuracy even when the observed data is contaminated with noise, and the origin of the gravity structure is approximately determined. The method has been successfully tested on synthetic examples with and without random errors, and successfully applied to field examples from Senegal and India. The estimated gravity inverse parameters are found in a good agreement with the known published values.

Conflict of interest

The authors have declared no conflict of interest.

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