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Pricing Moving Window Parisian Option and Applications in Convertible Bonds

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Abstract

Parisian options are complex path-dependent options developed by barrier option. Moving window Parisian options are higher path-dependent options, which are widely used in the field of convertible bonds in recent years. In this work we propose to price moving window Parisian option by use of hitting time. A simulation algorithm of the pricing is presented. As an application, we provide the pricing equations of convertible bonds with reset clause. Furthermore our simulation method is applied to price convertible bonds with reset clause using the data in China mainland stock exchange. The results show that this algorithm can undoubtedly improve the accuracy of the convertible bonds pricing.

1. Introduction

Parisian option is a path-dependent option which is triggered by the first time the underlying asset price process spends a given time period below or above some level (barrier). The Parisian contract feature can be generally described in a consecutive or a cumulative way. In the consecutive case, the measurement of the duration below or above the barrier restarts from zero at each time the barrier is crossed, whereas these consecutive periods are summed in the cumulative case. The moving window Parisian option which is a hybrid variant of the cumulative and consecutive Parisian options, counts the cumulative time below or above the barrier some level over any pre-specified time length (window length). Once the moving window Parisian option is activated, its payoff at maturity equals that of the corresponding standard European option and it will expire instantly.

There have already existed a lot of studies on the consecutive Parisian option and cumulative Parisian option, such as [2]-[5], [9]-[19] and so on. Basically, the Parisian option can be priced either by Laplace transforms or partial differential equations (PDE for short). However, research on the moving window feature is little. To our knowledge, [7] computes the moving window Parisian option in discrete case using the forward shooting grid approach. Since the moving window Parisian option appears to exhibit a high path-dependence, it is difficult...
to handle the window feature effectively for pricing option under continuous-time framework. It is not quite straightforward to explicitly derive the partial differential equation or explicit formulas for the option price. So we could not use the classical methods of Parisian option to price the moving window Parisian option. In this paper we will evaluate this option by use of the method in [1]. We construct a simulation algorithm of the moving window Parisian option based on hitting time simulation.

Parisian option is not popularly exchange-traded, but there are various applications of Parisian optionality in the mathematical finance. These provisions based on the duration of breaching may be applied with the convertibility and callability features in convertible bonds i.e. [8]. Usually reset convertibles are usually issued when the prospect for the issuer is unfavorable. And recently convertible bonds with reset clause where the reset time is with the feature of moving window Parisian option are widely issued in Chinese capital market. Therefore, we will apply moving window Parisian option to price such convertible bonds with reset clause. Herein “reset” means that the conversion price is adjusted downwards if the underlying stock price satisfies pre-specified provisions. We derive the pricing equations of conversion option for the single reset and multiple resets, respectively. It generalizes the case of [6] by developing the formula for the conversion option with the reset clause under the condition of determinant reset time. Then basing on the derived formula we pricing six convertible bonds chosen from China mainland stock exchange which are Gehua, Chuantou, Guotou, Bohui, Chengxin and Yanjin convertible bonds. The simulation results show that the pricing error is much smaller compared with standard Monte Carlo method, which illustrates that the pricing method of convertible bonds by hitting time simulation we propose is effective.

Our work makes two contributions to the literature. First, this paper is one of few studies that consider the pricing of moving window Parisian option under continuous-time framework. For discretely moving window Parisian option, it is possible to solve the problem numerically because the describing PDE has finite dimensions. For continuously moving window Parisian option, the describing PDE has infinite dimensions and pricing is an extremely difficult task. And we provide an efficient way to approximate the price of continuously moving window Parisian option by hitting time simulation based on [1]. Second, our numerical results can be used to price convertible bonds with reset clause and the reset time is characterized by moving window Parisian option. And our pricing method of convertible bonds with reset clause are shown to be very efficient in applications. Especially, we apply our method to price convertible bonds with reset clause using the data in Chinese capital market. The results show that compared with standard Monte Carlo method, our method improves the accuracy of the convertible bonds pricing significantly.

The paper is organized as follows. In Section 2, we consider the valuation of the moving window Parisian option and present the result. In Section 3 we deduce the pricing equation of convertible bonds with reset clause. We empirically analyze convertible bonds price with reset clause in Section 4 as an application of moving window Parisian option. We conclude in last Section.

2. Moving window parisian option problem

We will describe the feature of moving window parisian option. We only focus on the down-and-in call in this section.

Define C the length of the moving window. The moving window parisian option (down-and-in call) is knocked in if the underlying asset price lies below the barrier total D units of time within any length of window C. Obviously, D ≤ C ≤ T.

To describe the stopping time with window parisian feature at (or away from) barrier L for an Itô process S. We define
\[ \tau = \inf\{t \geq C, \int_{t-C}^{t} I_{\{S_u \leq L\}} \, du = D\} \]

Let \( \{S_t, t \in [0, T]\} \) be the risk-neutral asset price process given by
\[ S_t = S_0 + \int_0^t r \, ds + \int_0^t \sigma dB_s \]  

(1)

where \( S_0 > L \) the initial value of the stock, \( r \) the risk-free interest rate and \( \sigma \) the volatility of the asset. And \( r, \sigma \) are assumed to be positive constants. \( B_t \) is standard Brownian Motion under the probability measure \( P \). It
follows that

\[ S_t = S_0 e^{(r - \frac{1}{2} \sigma^2) t + \sigma B_t}. \]

Then we introduce the following notations \( \bar{B}_t = mt - B_t, m = \frac{r - \mu}{\sigma}, l = \frac{1}{\sigma} \ln \frac{S_0}{S_t}. \) We know from Gisanov’s Theorem that there exists an equivalent measure \( Q \) such that \( \bar{B}_t \) is a standard Brownian motion under \( Q \) and \( \frac{dP}{dQ} = \exp\{m\bar{B}_t - \frac{m^2}{2} t\} \), thus \( S_t \) rewrite \( S_t = e^{\bar{B}_t} \). Similarly in [1], the moving window parisian option value can be written in the following:

**Theorem 1.** Let \( V \) be the moving window parisian option value with exercise price \( K \) at time \( t = 0 \). For \( K > L \), then

\[ V = e^m Q[\tau_H \leq T] \int_0^\infty e^{-(t + \sigma^2)} BS(L, T - t, K) Q[\tau_t \in dt | \tau_H \leq T] \]  

(2)

where \( \tau_H = \inf \{t > \tau, \bar{B}_t = l\} \), \( BC(L, T, K) \) is the Black-Scholes pricing formula for European call option with initial value \( L \), the maturity \( T \) and the exercise price \( K \).

**Proof of Theorem 1** By risk-neutral pricing theory, we obtain

\[ V = E[e^{-\tau T}(S_T - K)^+ I_{[\tau \leq T]}]. \]  

(3)

Since the stock price \( S_t \) hits the level \( L \) can be transformed to \( \bar{B}_t \) hits the level \( l = \frac{1}{\sigma} \ln \frac{S_0}{S_t} \), then we denote by \( \tau_H \) the hitting time

\[ \tau_H = \inf \{t > \tau, \bar{B}_t = l\}. \]

Using the fact \( K > L \), we have

\[ [S_T > K] \cap \{\tau \leq T\} = [S_T > K] \cap \{\tau_H \leq T\} \quad a.s. \]

Thus for (3), using the Gisanov Theorem we derive that

\[ V = E[e^{-\tau T}(S_0 e^{(r - \frac{1}{2} \sigma^2) t + \sigma B_t} - K)^+ I_{[\tau_H \leq T]} \]

\[ = E_Q[e^{m\bar{B}_t} (e^{\bar{B}_t} - K)^+ I_{[\tau_H \leq T]} \]

\[ = E_Q[e^{m\bar{B}_t} (e^{\bar{B}_t} - K)^+ I_{[\tau_H \leq T]} | \bar{F}_{\tau_H}] \]

\[ = E_Q[e^{m\bar{B}_t} (e^{\bar{B}_t} - K)^+ I_{[\tau_H \leq T]} | \bar{F}_{\tau_H}] \]

\[ = E_Q[e^{m\bar{B}_t} (e^{\bar{B}_t} - K)^+ I_{[\tau_H \leq T]} | \bar{F}_{\tau_H}] \]

\[ = E_Q[e^{m\bar{B}_t} (e^{\bar{B}_t} - K)^+ I_{[\tau_H \leq T]} | \bar{F}_{\tau_H}] \]

\[ = E_Q[e^{m\bar{B}_t} (e^{\bar{B}_t} - K)^+ I_{[\tau_H \leq T]} | \bar{F}_{\tau_H}] \]

\[ = E^m Q[\tau_H \leq T] \int_0^\infty \exp\{m\bar{B}_t - \frac{m^2}{2} t\} BC(L, T - t, K) Q[\tau_t \in dt | \tau_H \leq T]. \]

**Remark 1.** When the window \( C \) takes different values, the moving window parisian option can be reduced to different parisian options, i.e. \( C \) takes \( T, D \), respectively, the moving window parisian option becomes cumulative parisian option, consecutive parisian option respectively. In this sense, the moving window Parisian option we consider generalizes the case in [1].

Now we compute the moving window Parisian option value in equation (2) by constructing a simulation algorithm. We will use the method in [1] to approximate directly hitting time \( \tau_H \) by the simulation of hitting times. Then we need the distribution function of \( \tau_l = \inf \{t \geq 0, B_t = l\} \), which is given by:

\[ F_{\tau_l}(t) = P(\tau_l \leq t) = 2P(B_l \leq l) = 2\Phi\left(\frac{1}{\sqrt{l}}\right), \]  

(4)
where $\Phi$ is standard normal distribution function. In order to draw the samples, we get the inverse distribution function:

$$F_{\tau_l}^{-1}(y) = \left( \frac{l}{\Phi^{-1}(\frac{y}{2})} \right)^2$$  \hspace{1cm} (5)

In order to compute the value in (2), we mainly simulate the hitting time with the nature of moving window Parisian time. The approximation of $\tau_H$ will be done in the following way. Similarly in [1], Let’s fix an $\epsilon > 0$ and take $l'(\epsilon)$ such that hitting the level $L - \epsilon$ by $B$ is equivalent to hitting the level $l'$ by $\bar{B}$. Now we simulate $\tau_1$ from $F_{0,l'}$, the distribution of hitting times of $\bar{B}$ hitting $l'$ starting in 0. Then we simulate $\tau_2$ from $F_{l,l'}$, the distribution of hitting times of $\bar{B}$ hitting $l$, starting in $l'$, and $\tau_3$ from $F_{l',l}$, the distribution of hitting times of $\bar{B}$ hitting $l'$, starting in $l$. It is noted that $F_{l,l'} = F_{l',l} = F_{0,l-l'}$, since $\bar{B}$ is symmetric and stationarity, which is the standard Brownian motion under $Q$. We also introduce the extra variable $h_1, h$, that is initially zero respectively. Setting $h_1 = h_1 + \tau_3, h = h + \tau_2$, we have four possibilities:

1. If $\tau_1 + \tau_2 > T$, we throw away the sample.

2. If $h > H$, we have a sample of $\tau_H$. And we stop, keeping the sample.

3. If $h < H$, and $h_1 < C - H$, we need to get back at $l'$ again. Then we simulate $\tau_4$ from $F_{l,l'}$ the distribution of hitting times of $\bar{B}$ hitting $l$, starting in $l'$. Letting $\tau_1 = \tau_1 + \tau_2 + \tau_3$. We keep on repeating this until we either have to throw away the sample or we can keep it.

4. If $h < H$, and $h_1 \geq C - H$, we need to start to move backward two periods and keep on repeating the above process until we either have to throw away the sample or we can keep it.

Transforming this in a simulation algorithm and using the hitting time distribution in (4) and its inverse in (5), we obtain the following algorithm on moving window Parisian option.

* take $\alpha = F_{\tau_l}(T), h = 0, h_1 = 0, Res = 0$

* Repeat $N$ times:

  Sample U from $U[0, \alpha]$, obtain the sample $\tau_1$

  while ($\tau_1 < T$) and ($h < H$)

    sample w from $U[0, 1]$

    obtain $\tau_2 = F_{\tau_0,l}^{-1}(w)$;

    $\tau_1 = \tau_1 + \tau_2$;

    $h = h + \tau_2$;

  while $h < H$

    sample w from $U[0, 1]$

    obtain $\tau_3 = F_{\tau_0,l}^{-1}(w)$;

    $\tau_1 = \tau_1 + \tau_3$;

    $h_1 = h_1 + \tau_3$;

  if $h_1 > (C - H)$

    the initial point of $h, h_1$ move backward one period respectively, break

  if else

    sample w from $U[0, 1]$

    obtain $\tau_4 = F_{\tau_0,l}^{-1}(w)$;

    $\tau_1 = \tau_1 + \tau_4$;

    $h = h + \tau_4$;
end
if \( \tau_1 < T \)
\[
res = res + \exp(-(r + m^2/2) \tau) \star BS(L, T - \tau_1, K); \\
* res = (\frac{1}{T}) \star \exp(m \star l) \star \alpha \star res.
\]

**Remark 2.** As is analyzed in [1], the accuracy of the approximation can be controlled. In fact there are two aspects affecting the option price, on the one hand, sample paths that stay long enough below the barrier \( L - \epsilon \) are thrown away, on the other hand, the time from the last exit of the level \( L \) up to the first hitting of level \( L - \epsilon \) is not taken into account. The error of the first factor introduced to a particular choice of \( \epsilon \) is negligible, and the expected value of error in variable can be \( \frac{1}{3}(\frac{\ln(L - \epsilon / \sigma)}{\sigma})^2 \). And the quality of this approximation is a trade-off between accuracy and computation time.

### 3. Pricing convertible bonds with reset clause

In this section we consider the convertible bond which is non-callable and non-convertible until maturity \( T \).

For a class of non-callable convertible bonds with the reset clause, under the assumptions of no dividends on the underlying stock and the flat term structure of the risk-free interest rate, no conversion occurs prior to maturity, then the conversion option must necessarily be the European type. Hence, convertible bond price \( V_{CB} \) = straight bond value \( V_B \) + conversion option value \( V_{CO} \).

Suppose \( K \) is the original conversion price, and let \( \tau \) be the downward reset time with characteristics of moving window Parisian option. Then, the actual conversion price of the reset convertible is changed to \( bK \) \((b < 1)\) if the stock price satisfies downward reset clause of the convertible bonds, or it remains \( K \) otherwise. This implies that the conversion price is adjusted downwards from \( K \) to \( bK \) when \( \tau < T \). In other words, the conversion ratio is adjusted upwards from \( B/K \) to \( B/(bK) \) when \( \tau \leq T \), where \( B \) is the par value of the convertible bond.

As a basic framework for pricing, we use the bond plus option valuation. We mainly focus on the price of conversion option, which is essential for analyzing the price of convertible bonds under the constant interest rate assumption. Assume that the convertible bond receives a coupon of amount \( C(>0) \) at time \((i = 1, ..., m)\) where \( 0 < T_1 < T_2 < \ldots < T_m < T \). For the straight bond value \( V_B \), we immediately have

\[
V_B = C \sum_{i=1}^{m} e^{-rT_i} + Be^{-rT}.
\]

On the other hand, for the conversion option price \( V_{CO} \), we have

**Theorem 2.** Let \( V_{CO} \) be the conversion option value of the credit-riskless, non-callable, convertible bond with the downward reset clause at time \( t = 0 \). \( \tau \) is reset time with characteristics of moving window Parisian option, and \( aK \) is the predetermined stock price, \( 1 > a > b \). Then,

\[
V_{CO} = e^{ml}Q[\tau_H \leq T] \int_0^\infty e^{-(r + \frac{\nu^2}{2})} BS(aK, T - t, bK)Q[\tau_H \in dt|\tau_H \leq T] \\
- e^{ml}Q[\tau_H \leq T] \int_0^\infty e^{-(r + \frac{\nu^2}{2})} BS(aK, T - t, K)Q[\tau_H \in dt|\tau_H \leq T] \\
+ BS(S_0, T, K)
\]

where \( l = \frac{1}{\sigma} \ln \frac{bK}{aK}, \ \tau_H = \inf\{t > \tau, \overline{B}_t = l\} \).

**Proof of Theorem 2.** From the definition of downward reset clause and risk-neutral pricing theory, we obtain

\[
V_{CO} = E[e^{-rT} (S_T - bK)^+ I_{[\tau_1 \leq T]}] + E[e^{-rT} (S_T - K)^+ I_{[\tau > T]}]
\]

Using Theorem 1 and fact \( K > aK > bK \), the result follows. \( \square \)
The reset right embedded in a derivative refers to the feature that the issuer can alter certain terms in the derivative contract according to some preset rules. In the following, we consider options that allow the issuer to adjust the conversion price by number of times according to predetermined rules during the life of the convertible bond. It is noted that the necessary assumption for Theorem 3 to hold is that whenever the reset clause is satisfied, the conversion option price cannot be adjusted downward until the stock price goes back the predetermined price.

Theorem 3. Let $V_{MCO}$ be the conversion option value of the credit-riskless, non-callable, convertible bond with the multiple downward reset clause at time $t = 0$. For $\tau_i$, $i = 1, \cdots, n$ are reset times with characteristics of moving window Parisian option, $b_1 K < b_2 K < \cdots < b_n K$ are conversion prices, $a_1 K < a_2 K < \cdots < a_n K$ are the predetermined stock prices, $1 > b_i > a_i$, respectively. Let’s define $\tau_{hi} = \inf\{t > \tau_i | \widetilde{B}_t = l_i\}$, Suppose the counting of $\tau_i$ starts after $\tau_{h_{i-1}}$, $i = 2, \cdots, n$, Then

$$V_{MCO} = \sum_{i=1}^{n} e^{m \cdot Q[\tau_{H_i} \leq T]} \int_0^{\infty} e^{-\left(r+\frac{\sigma^2}{2}\right) T} BS(a_i K, T - t, b_i K) Q[\tau_{H_i} \in dt | \tau_{H_i} \leq T]$$

$$- \sum_{i=1}^{n-1} e^{m \cdot Q[\tau_{H_{i+1}} \leq T]} \int_0^{\infty} e^{-\left(r+\frac{\sigma^2}{2}\right) T} BS(a_{i+1} K, T - t, b_i K) Q[\tau_{H_{i+1}} \in dt | \tau_{H_{i+1}} \leq T]$$

$$- e^{m \cdot Q[\tau_{H_1} \leq T]} \int_0^{\infty} e^{-\left(r+\frac{\sigma^2}{2}\right) T} BS(a_1 K, T - t, K) Q[\tau_{H_1} \in dt | \tau_{H_1} \leq T]$$

$$+ \sum_{i=1}^{n-1} BS(S_0, T, b_i K) + BS(S_0, T, K)$$

(8)

where $l_i = \frac{1}{\sigma^2} \ln \frac{a_i K}{S_0}$, $BC(S_0, T, K)$ is the Black-Scholes pricing formula for European call option with initial value $S_0$, the maturity $T$ and the exercise price $K$.

Proof of Theorem 3. From the definition of multiple downward reset clause and risk-neutral pricing theory, we obtain

$$V_{MCO} = \sum_{i=1}^{n-1} E[e^{-rT}(S_T - b_i K)^+ I_{[\tau_i \leq T \cap [\tau_{i+1} > T]}] + E[e^{-rT}(S_T - K)^+ I_{[\tau_i > T]}]$$

$$+ E[e^{-rT}(S_T - b_n K)^+ I_{[\tau_n \leq T]}]$$

$$= \sum_{i=1}^{n-1} E[e^{-rT}(S_T - b_i K)^+ I_{[\tau_i \leq T]}] + \sum_{i=1}^{n-1} E[e^{-rT}(S_T - b_i K)^+]$$

$$- \sum_{i=1}^{n-1} E[e^{-rT}(S_T - b_i K)^+ I_{[\tau_{i+1} \leq T]}] + E[e^{-rT}(S_T - K)^+]$$

Using similar method in Theorem 1, we get the result. □

4. Simulating the price of convertible bonds with reset clause

In this section, we will apply hitting time simulation method into convertible bonds with reset clause in China mainland stock exchange. The reset clause can be triggered only one time or many times and reset time is with the feature of moving window Parisian option in the market. Therefore we classified them into single-triggered and multi-triggered types.

In order to get the price of convertible bonds with reset clause, we mainly compute the conversion options price. Then we employ our pricing method to calculate the price of single-triggered and multi-triggered conversion options by Theorem 2 and Theorem 3. For the first type, if the reset clause is triggered, the actual conversion price will be turn down, or or it remains unchanged. And for the second type, if the reset clause is triggered for
many times the actual conversion price will be adjusted downward successively for every trigger, or it remains unchanged.

In fact, all convertible bonds are listed in Shanghai Stock Exchange and Shenzhen Stock Exchange in China market. Up to now, the issues of convertible bonds were 76 and some convertible bond had matured and the listed convertible bonds are 35. We will choose some convertible bonds which satisfy the conditions described in Theorem 2,3. They are Gehua(No.1) convertible bond, Chuantou(No.2) convertible bond, Guotou(No.3) convertible bond, Chengxin(No.4) convertible bond and Yanjin(No.5) convertible bond. Table 1 lists the necessary information and parameters for empirical analysis.

<table>
<thead>
<tr>
<th>NO</th>
<th>Conversion ratio</th>
<th>T</th>
<th>σ</th>
<th>Trigger conditions</th>
<th>Discount rate(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.67</td>
<td>6</td>
<td>0.3777</td>
<td>10/20,90%</td>
<td>2.5</td>
</tr>
<tr>
<td>2</td>
<td>5.79</td>
<td>6</td>
<td>0.3006</td>
<td>10/20,90%</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>14.04</td>
<td>6</td>
<td>0.2326</td>
<td>10/20,90%</td>
<td>2.75</td>
</tr>
<tr>
<td>4</td>
<td>9.48</td>
<td>5</td>
<td>0.5084</td>
<td>10/20,90%</td>
<td>2.79</td>
</tr>
<tr>
<td>5</td>
<td>4.61</td>
<td>5</td>
<td>0.3621</td>
<td>20/30,85%</td>
<td>2.5</td>
</tr>
</tbody>
</table>

In table 1, trigger condition such as 10/20, 90% means that if the underlying asset price lies below 90%K beyond 10 days within any length of window 20 days, the conversion price is adjust downward (such as 95%K). And in order to determine the interest rate, we choose the interest rate for three-month bank deposit within the convertible bond maturity. Without loss of generality, we choose the historical data of 100 days from initial time to estimate the volatility.

Table 2 shows the simulation results of five convertible bonds. Because the listed company which issued convertible bond seldom issue dividend, we neglect its effect on convertible bond price. And coupon rate of convertible bond is extremely low, we also neglect its effect on convertible bond price. Face value of the convertible bond is 100. In this paper, the convertible bond price = face value of convertible bond* e^{-rT} + convertible ratio*option price.

<table>
<thead>
<tr>
<th>NO</th>
<th>Option Price</th>
<th>Bonds Price</th>
<th>Bonds Price on First Listing Day</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.5085</td>
<td>122.8125</td>
<td>127</td>
<td>-3.3%</td>
</tr>
<tr>
<td>2</td>
<td>5.5318</td>
<td>115.5561</td>
<td>116.02</td>
<td>-0.4%</td>
</tr>
<tr>
<td>3</td>
<td>2.2974</td>
<td>117.0449</td>
<td>118.11</td>
<td>-0.9%</td>
</tr>
<tr>
<td>4</td>
<td>5.9767</td>
<td>143.6306</td>
<td>140</td>
<td>2.6%</td>
</tr>
<tr>
<td>5</td>
<td>8.3046</td>
<td>127.4559</td>
<td>130</td>
<td>-1.9%</td>
</tr>
</tbody>
</table>

1 they are gotten by simulation.
2 (Bonds price - Bonds Price on First Listing Day ) / Bonds Price on First Listing Day.

According to information in market, Gehua(No.1) convertible bond and Chuantou(No.2) convertible bond are only triggered once, so we use Theorem 2 to calculate conversion price. And Guotou(No.3) convertible bond and Yanjin(No.5) convertible bond are triggered twice while Chengxin(No.4) convertible bond is triggered six times, then we use Theorem 3 to calculate conversion price.

From table 2 we can find that the pricing error is very small. Therefore accurate pricing results illustrates that the pricing method of convertible bonds by hitting time simulation we propose is effective.

Furthermore, we compare the bond prices calculated by hitting time method and the standard Monte Carlo method. The following table shows the results of the comparison.

Table 3 shows that the bond prices calculated by standard Monte Carlo method are systematically larger than the bond prices calculated by hitting time method. And the error using hitting time simulation is much less than the error using standard Monte Carlo method, which means our method can efficiently improve the accuracy of the convertible bonds pricing.
Table 3. The comparison between the bond prices simulated by hitting time method and standard Monte Carlo method.

<table>
<thead>
<tr>
<th>NO</th>
<th>Bonds Price(^1)</th>
<th>Bonds Price(^2)</th>
<th>Bonds Price on First Listing Day</th>
<th>Error(^3)</th>
<th>Error(^4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>122.8125</td>
<td>138.8976</td>
<td>127</td>
<td>-3.3%</td>
<td>9.3%</td>
</tr>
<tr>
<td>2</td>
<td>115.5561</td>
<td>126.4309</td>
<td>116.02</td>
<td>-0.4%</td>
<td>9.0%</td>
</tr>
<tr>
<td>3</td>
<td>117.0449</td>
<td>128.4324</td>
<td>118.11</td>
<td>-0.9%</td>
<td>8.7%</td>
</tr>
<tr>
<td>4</td>
<td>143.6306</td>
<td>149.2783</td>
<td>140</td>
<td>2.6%</td>
<td>6.6%</td>
</tr>
<tr>
<td>5</td>
<td>127.4559</td>
<td>137.6272</td>
<td>130</td>
<td>-1.9%</td>
<td>5.9%</td>
</tr>
</tbody>
</table>

\(^1\) they are gotten by hitting time simulation.
\(^2\) they are gotten by standard Monte Carlo simulation.
\(^3\) (Bonds price\(^1\) - Bonds Price on First Listing Day) / Bonds Price on First Listing Day.
\(^4\) (Bonds price\(^2\) - Bonds Price on First Listing Day) / Bonds Price on First Listing Day.

5. Conclusion remarks

Based on the above analysis, exploration on moving window parisian option pricing pushes forward application of such options. Applying the hitting time simulation algorithm into convertible bonds with reset clause, it can grasp the characteristics of these clauses more accurately, and undoubtedly contributes to improve the accuracy of the convertible bonds pricing.

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References

Appendix A. The Code in the Simulation Experiment

N=10000; ε=0.5; H=10/250; C=20/250; S₀=10.46; a₁=0.9; b=0.95; K=10.53; v=0.20716; T=6; r=0.0279;

aa = [a₁, a₁ * b, ..., a₁ * b¹⁹]; bb = [b, b², ..., b²⁰]; V=0; V₁=0; V₂=0; V₃=0; V₄=0;

B=aa*K; X=bb*K; l=log(B(1,1)/S₀)/v; l₀=log((B(1,1)-ε)/S₀)/v;

α=2*normcdf(l/sqrt(T),0,1); m = (r – 0.5 * (v²))/v;

for i=1:1:N
    τ=0; t=zeros(1,20); p=1;
    while(accumu(t,p)<T)
        l=log(B(1,p)/S₀)/v;
        l₀=log((B(1,p)-ε)/S₀)/v;
        a=zeros(1,N);
        j=0; τ₁=0; h=0; k=3; n=2; τ₂=0;
        u=unifrnd(0,α); h₁=(l/norminv(u/2,0,1))²;
        j=j+1; a(1,j)=h₁; τ=h₁;
        while (τ <T) && (h<H)
            w=rand(); h₂=((l₀-l)/norminv(w/2,0,1))²;
            j=j+1; a(1,j)=h₂; τ=τ+h₂;
            h=h+h₂;
        end
        while (h<H)&&(τ₁ > (C-H))
            w=rand();
            h₃=((l₀-l)/norminv(w/2,0,1))²; τ₁=τ₁+h₃;
            j=j+1; a(1,j)=h₃; τ=τ+h₃;
        end
        if τ₁ > (C-H)
            τ₁=τ₁+a(1,k);
            k=k+2; τ₂=τ₂+a(1,n);
            n=n+2; h=h-τ₂; τ₁=τ₁-τ₁; τ₁=0; τ₂=0;
            break
        end
        else
            w=rand();
            h₄=((l₀-l)/norminv(w/2,0,1))²;
            j=j+1; a(1,j)=h₄; h=h+h₄; τ=τ+h₄;
        end
    end
end

if accumu(t,p)<T
    p=p+1;
end
end
while(p)
if accumu(t,p)<T
    for i=1:p
        \[ V_1 = V_1 + \exp(m \cdot l) \cdot \alpha \cdot \exp(- (r + m^2 / 2)) \cdot BS('c', B(1, p), X(1, p), T - t(1, p), r, v); \]
    end
    for i=1:(p-1)
        \[ V_2 = V_2 + \exp(m \cdot l) \cdot \alpha \cdot \exp(- (r + m^2 / 2)) \cdot BS('c', B(1, p + 1), X(1, p), T - t(1, p), r, v); \]
    end
    \[ V_3 = V_3 + \exp(m \cdot l) \cdot \alpha \cdot \exp(- (r + m^2 / 2)) \cdot BS('c', B(1, 1), K, T - t(1, p), r, v); \]
    \[ V_4 = V_4 + BS('c', S_0, K, T, r, v); \]
    \[ V = V_1 - V_2 - V_3 + V_4; \]
    break
else
    p = p - 1;
end
end
end
\[ V = 1/N \cdot V. \]