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Geons of galileons

Thomas L. Curtright [a,*], David B. Fairlie [b]

[a] Department of Physics, University of Miami, Coral Gables, FL 33124-8046, USA
[b] Department of Mathematical Sciences, Durham University, Durham, DH1 3LE, UK

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ABSTRACT

We suggest that galileon theories should have an additional self-coupling of the fields to the trace of their own energy–momentum tensor. We explore the classical features of one such model, in flat 4D spacetime, with emphasis on solutions that are scalar analogues of gravitational geons. We discuss the stability of these scalar geons, and some of their possible signatures, including shock fronts.

Galileon theories are a class of models for new scalar fields whose Lagrangians involve multilinears of first and second derivatives, but whose nonlinear field equations are nonetheless still only second-order. They may be important for the description of large-scale features in astrophysics as well as for elementary particle theory [1,7]. Hierarchies of such Lagrangians giving rise to such field equations were first discussed mathematically in [8–10,14]. The simplest example involves a single scalar field.

This galileon field is usually coupled to all *other* matter through the trace of the energy–momentum tensor, $\Theta^{(\text{matter})}$, and is thus gravitation-like by virtue of the similarity between this universal coupling and that of the metric $g_{\mu\nu}$ to $\Theta_{\mu\nu}^{(\text{matter})}$ in general relativity. Indeed, some galileon models have been obtained from limits of higher dimensional gravitation theories [5].

But *surely*, in a self-consistent theory, for the coupling to be truly universal, the galileon should also be coupled to its own energy–momentum trace, even in the flat spacetime limit. Some consequences of this additional self-coupling are considered in this Letter.

The action for the lowest non-trivial member of the galileon hierarchy can be written in various ways upon integrating by parts. Perhaps the most compact and memorable of these is

$$A_2 = \frac{1}{2}\int \phi_\alpha \phi_\alpha \phi_{\beta\beta}\, d^n x. \tag{1}$$

where ϕ is the scalar galileon field, $\phi_\alpha = \partial\phi(x)/\partial x^\alpha$, etc., and where repeated indices are summed using the Lorentz metric $\delta_{\mu\nu} = \operatorname{diag}(1, -1, -1, \dots)$.

It is straightforward to include in A_2 a covariant coupling to a background spacetime metric and hence to deduce a symmetric energy–momentum tensor. In the flat-space limit, the result is

$$\Theta_{\mu\nu}^{(2)} = \phi_\mu \phi_\nu \phi_{\alpha\alpha} - \phi_\alpha \phi_{\alpha\nu} \phi_\mu - \phi_\alpha \phi_{\alpha\mu} \phi_\nu + \delta_{\mu\nu} \phi_\alpha \phi_\beta \phi_{\alpha\beta}. \tag{2}$$

This is seen to be conserved,

$$\partial_\mu \Theta_{\mu\nu}^{(2)} = \phi_\nu\, \mathcal{E}_2[\phi], \tag{3}$$

upon using the field equation that follows from locally extremizing A_2, $0 = \delta A_2/\delta\phi = -\mathcal{E}_2[\phi]$, where

$$\mathcal{E}_2[\phi] \equiv \phi_{\alpha\alpha}\phi_{\beta\beta} - \phi_{\alpha\beta}\phi_{\alpha\beta}. \tag{4}$$

An interesting wrinkle now appears: $\Theta_{\mu\nu}^{(2)}$ is not traceless. Consequently, the usual form of the scale current, $x_\alpha \Theta_{\alpha\mu}^{(2)}$, is not conserved [15]. On the other hand, the action (1) is homogeneous in ϕ and its derivatives, and is clearly invariant under the scale transformations $x \to sx$ and $\phi(x) \to s^{(4-n)/3}\phi(sx)$. Hence the corresponding Noether current must be conserved. This current is easily found, especially for $n = 4$, so let us restrict our attention to four spacetime dimensions in the following.

In that case the trace is obviously a total divergence:

$$\Theta^{(2)} \equiv \delta_{\mu\nu}\Theta_{\mu\nu}^{(2)} = \partial_\alpha(\phi_\alpha\phi_\beta\phi_\beta). \tag{5}$$

That is to say, for $n = 4$ the virial is the trilinear $V_\alpha = \phi_\alpha\phi_\beta\phi_\beta$. So a conserved scale current is given by the combination,

$$S_\mu = x_\alpha \Theta_{\alpha\mu}^{(2)} - \phi_\alpha\phi_\alpha\phi_\mu. \tag{6}$$

Interestingly, this virial is not a divergence modulo a conserved current, so this model is *not* conformally invariant despite being scale invariant. Be that as it may, it is not our principal concern here.

Our interest here is that the nonzero trace suggests an additional interaction where ϕ couples directly to its own $\Theta^{(2)}$. This is

* Corresponding author.
E-mail address: curtright@miami.edu (T.L. Curtright).

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similar to coupling a conventional *massive* scalar to the trace of its own energy–momentum tensor [11]. In that previously considered example, however, the consistent coupling of the field to its trace required an iteration to all orders in the coupling. Upon summing the iteration and making a field redefinition, the Nambu–Goldstone model emerged. But, for the simplest galileon model in four spacetime dimensions, (1), a consistent coupling of field and trace is much easier to implement. *No iteration is required*. The first-order coupling alone is consistent, after integrating by parts and ignoring boundary contributions, so that [16]

$$-\frac{1}{4}\int \phi\, \partial_\alpha(\phi_\alpha\phi_\beta\phi_\beta)\, d^4x = \frac{1}{4}\int \phi_\alpha\phi_\alpha\phi_\beta\phi_\beta\, d^4x. \tag{7}$$

(Similar quadrilinear terms have appeared previously in [2,3], only multiplied there by scalar curvature R so that they would drop out in the flat spacetime limit that we consider.) Consistency follows because (7) gives an additional contribution to the energy–momentum tensor which is *traceless*, in 4D spacetime:

$$\Theta^{(3)}_{\mu\nu} = \phi_\mu\phi_\nu\phi_\alpha\phi_\alpha - \frac{1}{4}\delta_{\mu\nu}\phi_\alpha\phi_\alpha\phi_\beta\phi_\beta, \qquad \Theta^{(3)} = 0. \tag{8}$$

Of course, coupling ϕ to its own trace may impact the Vainstein mechanism [20] by changing the effective coupling of $\Theta^{(\text{matter})}$ to both backgrounds and fluctuations in ϕ. We leave this as an exercise for the reader.

Based on these elementary observations, we consider a model with action

$$A = \int \left(\frac{1}{2}\phi_\alpha\phi_\alpha - \frac{1}{2}\lambda\phi_\alpha\phi_\alpha\phi_{\beta\beta} - \frac{1}{4}\kappa\phi_\alpha\phi_\alpha\phi_\beta\phi_\beta\right) d^4x, \tag{9}$$

where for the Lagrangian L we take a mixture of three terms: the standard bilinear, the trilinear galileon, and its corresponding quadrilinear trace-coupling. The quadrilinear is reminiscent of the Skyrme term in nonlinear σ models [19] although here the topology would appear to be always trivial.

The second and third terms in A are logically connected, as we have indicated. But why include in A the standard bilinear term? The reasons for including this term are to soften the behavior of solutions at large distances, as will be evident below, and also to satisfy Derrick's criterion for classical stability under the rescaling of x. Without the bilinear term in L the energy within a spatial volume would be neutrally stable under a uniform rescaling of x, and therefore able to disperse [4,6].

Similarly, for positive κ, the last term in A ensures the energy density of static solutions is always bounded below under a rescaling of the field ϕ, a feature that would not be true if $\kappa = 0$ but $\lambda \neq 0$. So, we only consider $\kappa > 0$ in the following. But before discussing the complete $\Theta_{\mu\nu}$ for the model, we note that we did *not* include in A a term coupling ϕ to the trace of the energy–momentum due to the standard bilinear term, namely, $\int \phi\Theta^{(1)}\, d^4x$, where

$$\Theta^{(1)}_{\mu\nu} = \phi_\mu\phi_\nu - \frac{1}{2}\delta_{\mu\nu}\phi_\alpha\phi_\alpha, \qquad \Theta^{(1)} = -\phi_\alpha\phi_\alpha. \tag{10}$$

We have omitted such an additional term in A solely as a matter of taste, thereby ensuring that L is invariant under constant shifts of the field. Among other things, this greatly simplifies the task of finding solutions to the equations of motion.

The field equation of motion for the model is $0 = \delta A/\delta\phi = -\mathcal{E}[\phi]$, where

$$\mathcal{E}[\phi] \equiv \phi_{\alpha\alpha} - \lambda(\phi_{\alpha\alpha}\phi_{\beta\beta} - \phi_{\alpha\beta}\phi_{\alpha\beta}) - \kappa(\phi_\alpha\phi_\beta\phi_\beta)_\alpha. \tag{11}$$

As expected, this field equation is second-order, albeit nonlinear. Also note, under a rescaling of both x and ϕ, nonzero parameters λ and κ can be scaled out of the equation. Define

$$\phi(x) = \frac{\lambda}{\kappa}\,\psi\left(\sqrt{\frac{\kappa}{\lambda^2}}x\right). \tag{12}$$

Then the field equation for $\psi(z)$ becomes

$$\psi_{\alpha\alpha} - (\psi_{\alpha\alpha}\psi_{\beta\beta} - \psi_{\alpha\beta}\psi_{\alpha\beta}) - (\psi_\alpha\psi_\beta\psi_\beta)_\alpha = 0, \tag{13}$$

where $\psi_\alpha = \partial\psi(z)/\partial z^\alpha$, etc. In effect then, if both λ and κ do not vanish, it is only necessary to solve the model's field equation for $\lambda = \kappa = 1$.

Though $\mathcal{E}$ is nonlinear, it is nevertheless still true that some plane waves are exact solutions. For "light-ray" plane waves, $\mathcal{E}[A\exp(ik_\alpha x_\alpha)] = 0$ for constant A and k_α, if $k_\alpha k_\alpha = 0$ with A arbitrary. In this case, each of the terms in $\mathcal{E}$ vanish separately. In fact, light-ray plane waves are only one among many possible solutions for which both $\phi_{\alpha\alpha} = 0$ and $\phi_\beta\phi_\beta = 0$. On the other hand, for massive plane waves, $\mathcal{E}[A\exp(ik_\alpha x_\alpha)] = 0$ if $1/k_\alpha k_\alpha = -3\kappa A^2 < 0$. The latter "tachyonic" solutions would seem to be less interesting for real physics.

For static, spherically symmetric solutions, $\phi = \phi(r)$, the field equation of motion becomes

$$0 = \frac{1}{r^2}\frac{d}{dr}\left(r^2\left(\phi' + \lambda\frac{2}{r}(\phi')^2 + \kappa(\phi')^3\right)\right), \tag{14}$$

where $\phi' = d\phi/dr$. This is immediately integrated once to obtain a cubic equation,

$$r^2\phi' + 2\lambda r(\phi')^2 + \kappa r^2(\phi')^3 = C, \tag{15}$$

where C is the constant of integration. Now, without loss of generality (cf. (12) and (13)) we may choose $\lambda > 0$. Then, if $C = 0$, either ϕ' vanishes, or else there are two solutions that are real only within a finite sphere of radius $r = \sqrt{\lambda^2/\kappa}$. These two "interior" solutions are given exactly by

$$\phi'_\pm = -\frac{1}{r\kappa}\left(\lambda \pm \sqrt{\lambda^2 - r^2\kappa}\right). \tag{16}$$

Note that these solutions always have $\phi' < 0$ within the finite sphere.

Otherwise, if $C \neq 0$, then examination of the cubic equation for small and large $|\phi'|$ determines the asymptotic behavior of ϕ' for large and small r. In particular, there is only one type of asymptotic behavior for large r:

$$\phi' \underset{r\to\infty}{\sim} \frac{C}{r^2} \quad \text{for either sign of } C. \tag{17}$$

However, there are two types of behavior for large $|\phi'|$, corresponding to small r. Either

$$r = \frac{-2\lambda}{\phi'\kappa}\left(1 + O\left(\frac{1}{\phi'}\right)\right) \tag{18}$$

provided $\phi' < 0$, but with either sign of C; or else

$$r = \frac{1}{\phi'^2}\left(\frac{C}{2\lambda} + O\left(\frac{1}{\phi'}\right)\right) \tag{19}$$

provided $C > 0$, but with either sign of ϕ'. The corresponding real solutions behave as

$$\phi' \underset{r\to 0}{\sim} \frac{-2\lambda}{\kappa r} \quad \text{for either sign of } C, \quad \text{or} \tag{20}$$

$$\phi' \underset{r\to 0}{\sim} \pm\sqrt{\frac{C}{2\lambda r}} \quad \text{provided } C > 0. \tag{21}$$

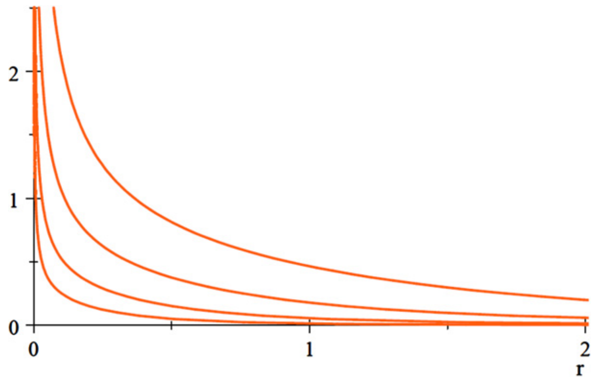

Fig. 1. $\psi'(r)$ for $C = +1/4^N$, with $N = 0, 1, 2, 3$ for top to bottom curves, respectively.

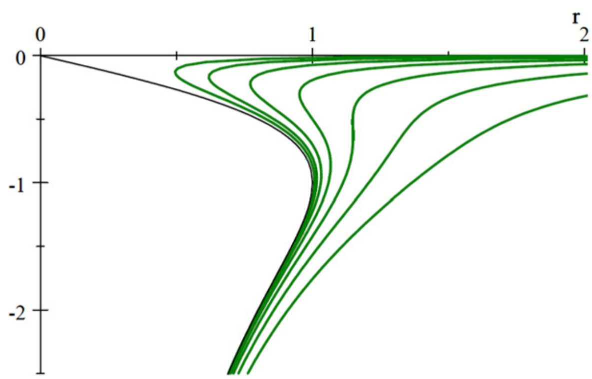

Fig. 2. $\psi'(r)$ for $C = -1/2^N$, with $N = 6, 5, 4, 3, 2, 1, 0$ from left to right, respectively. The thin black curve is a union of the two $C = 0$ solutions in (16).

Comparison of the small r behavior to the large r asymptotics shows that in half these cases the solutions would require zeroes to be real and continuous for all r. But such zeroes do not occur. Instead, half of the cases provide real solutions only over a finite interval of r, somewhat similar to the $C = 0$ solutions in (16), but not so easily expressed, analytically.

The solutions which are real for all $r > 0$ boil down to two cases, with small and large r behavior given by either

$$\phi' \underset{r\to 0}{\sim} \sqrt{\frac{C}{2\lambda r}} \quad \text{and} \quad \phi' \underset{r\to\infty}{\sim} \frac{C}{r^2} \quad \text{for } C > 0, \tag{22}$$

or else

$$\phi' \underset{r\to 0}{\sim} \frac{-2\lambda}{\kappa r} \quad \text{and} \quad \phi' \underset{r\to\infty}{\sim} \frac{C}{r^2} \quad \text{for } C < 0. \tag{23}$$

From further inspection of the cubic equation to determine the behavior of ϕ' for intermediate values of r, when $C > 0$ it turns out that ϕ' is a single-valued, positive function for all $r > 0$, joining smoothly with the asymptotic behaviors given in (22). However, it also turns out there is an additional complication when $C < 0$. In this case there is a critical value $(\kappa^{3/2}/\lambda^2)C_{\text{critical}} = -4\sqrt{3}/27 \approx -0.2566$ such that, if $C \leqslant C_{\text{critical}}$ then ϕ' is a single-valued, negative function for all $r > 0$, while if $C_{\text{critical}} < C < 0$ then ϕ' is triple-valued for an open interval in $r > 0$. It is not completely clear to us what physics underlies this multivalued-ness for some negative C. But in any case, when $C < 0$ it is also true that ϕ' joins smoothly with the asymptotic behaviors given in (23). All this is illustrated in Figs. 1 and 2, for $\lambda = \kappa = 1$.

A test particle coupled by $\phi\Theta^{(\text{matter})}$ to any of these galileon field configurations would see an effective potential which is not $1/r$, for intermediate and small r. Therefore its orbit would show deviations from the usual Kepler laws, including precession at variance with that predicted by conventional general relativity. It would be interesting to search for such effects, say, by considering stars orbiting around the galactic center.

For the solutions described by (22) and (23), the total energy outside any large radius is obviously finite for both $C > 0$ and $C < 0$. And if $C > 0$, the total energy within a small sphere surrounding the origin is also manifestly finite. But if $C < 0$ the energy within that same small sphere could be infinite *unless* there is a cancellation between the galileon term and the trace interaction term. Remarkably, this cancellation does occur [17]. So both $C > 0$ and $C < 0$ types of static solutions for the model have finite total energy.

Complete information about the distribution of energy is provided by the model's energy–momentum tensor,

$$\Theta_{\mu\nu} = \Theta^{(1)}_{\mu\nu} - \lambda\Theta^{(2)}_{\mu\nu} - \kappa\Theta^{(3)}_{\mu\nu}. \tag{24}$$

As expected, this is conserved, given the field equation $\mathcal{E}[\phi] = 0$, since

$$\partial_\mu\Theta_{\mu\nu} = \phi_\nu\mathcal{E}[\phi]. \tag{25}$$

The energy density for *static* solutions differs from the canonical energy density for such solutions (namely, $-L$) by a total spatial divergence that arises from the galileon term:

$$\Theta_{00} = -L|_{\text{static}} - \frac{1}{2}\lambda\vec{\nabla}\cdot\big((\nabla\phi)^2\vec{\nabla}\phi\big). \tag{26}$$

This divergence will not contribute to the total energy for fields such that $\lim_{r\to\infty}(\phi/\ln r)$ exists. Assuming that is the case, Derrick's scaling argument for static, finite energy solutions of the equations of motion [4] shows the energy is just twice that due to the bilinear $\Theta^{(1)}_{00}$. Thus,

$$E = \int \Theta_{00}\, d^3r = \int (\vec{\nabla}\phi)^2\, d^3r. \tag{27}$$

For the spherically symmetric static solutions of (15), this becomes an expression of the energy as a function of the parameters and the constant of integration C:

$$E[\lambda,\kappa,C] = 4\pi\int_0^\infty (\phi')^2 r^2\, dr. \tag{28}$$

Again without loss of generality, consider $\lambda = \kappa = 1$. Then for either $C > 0$ or for $C < C_{\text{critical}} < 0$ [18], change integration variables from r to $s \equiv \phi'$ to find:

$$E(C \gtrless 0) = I(|C|) \mp \left(|C| + \frac{1}{2}\pi\right), \tag{29}$$

$$I(C > 0) \equiv \frac{1}{2}\int_0^\infty \frac{P(s,C)\, ds}{(s^2+1)^4 R(s,C)}, \tag{30}$$

where $R(s,C) = \sqrt{s^4 + s(s^2+1)C}$ and where the numerator of the integrand is an eighth-order polynomial in s, namely, $P(s,C) = 8s^8 + 12Cs^7 + (3C^2 - 8)s^6 + 8Cs^5 + 7C^2s^4 - 4Cs^3 + 5C^2s^2 + C^2$. Thus, $I(C)$ is an elliptic integral. But rather than express the final result in terms of standard functions, it suffices here just to plot $E(C)$, in Fig. 3. Note that E increases monotonically with $|C|$.

For other values of λ and κ with the constant of integration C specified as in (15), the energy of the solution is given in terms of the function defined by (29), (30):

$$E[\lambda,\kappa,C] = \left(\lambda^3/\kappa^{5/2}\right) E\left(\kappa^{3/2}C/\lambda^2\right). \tag{31}$$

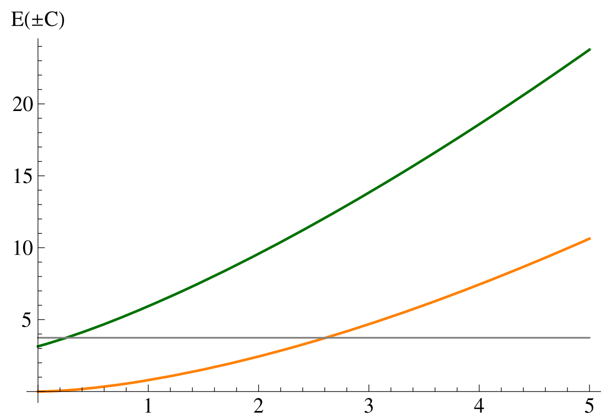

Fig. 3. $E(\pm C)$ versus $C \geqslant 0$ as lower/upper curves (the horizontal line is $E(C_{\text{critical}}) \approx 3.7396$).

The energy curves indicate double degeneracy in E, for different values of $|C|$, when $E[\lambda, \kappa, C] > \pi\lambda^3/\kappa^{5/2}$. Also, for a given $|C|$ the negative C solutions are *higher* in energy, with $E[\lambda, \kappa, -|C|] - E[\lambda, \kappa, |C|] = \pi\lambda^3/\kappa^{5/2} + 2|C|\lambda/\kappa$. Or at least this is true for all $|C| \geqslant |C_{\text{critical}}|$ in which case $E[\lambda, \kappa, C] \geqslant \frac{\lambda^3}{\kappa^{5/2}} E(\frac{\kappa^{3/2}}{\lambda^2} C_{\text{critical}}) \approx 3.7396\ \lambda^3/\kappa^{5/2}$ [18].

Finite energy classical solutions of gravity-like theories bring to mind the "geons" proposed long ago by Wheeler [21]. These were envisioned in their purest form as distributions of only gravitational energy held together solely by gravitational interaction. Combinations of electromagnetic energy and gravity were also considered, as were systems containing neutrinos. Wheeler argued that such configurations would be *relatively* stable, if they existed, but would eventually dissipate due to a variety of both classical and quantum effects, including light–light scattering, as well as production and absorption of quanta. While plausible distributions were sketched, and decay rates were estimated, *exact* classical solutions were not found.

The same mechanisms would seem to apply to any hypothetical classical galileon distributions such as those discussed here, the main difference being that analytic spherically symmetric solutions might still be obtainable even if conventional gravitational effects were included. Perhaps these gravitational effects would not alter the qualitative features of the static pure ϕ configurations given above. Should they really exist, presumably these galileon geons could also be dissipated by various classical and quantum effects. All this is far beyond our current abilities and the scope of this Letter, of course, but the general ideas suggest some interesting possibilities.

Whatever the cause, if the configuration's energy loss were gradual, as a first step it might suffice to model the time-dependent system quasi-statically, as a continuous flow from one static solution to another. That is to say, perhaps a good approximation would be to take $C(t)$, with $|C|$ and $E(C)$ decreasing monotonically with time. For the positive C case, this would be more or less uneventful as the whole configuration would just slowly disappear without any abrupt changes. But for the negative C case, as t increased C_{critical} would be reached, beyond which the solution would begin to fold over, exhibiting the multivalued features shown in Fig. 2. But this is just the usual picture for the formation of a shock front. These particular galileon shocks would implode, converging towards the origin, as shown in http://server.physics.miami.edu/~curtright/PsiWave.gif. We believe this is a plausible scenario and a reasonable physical interpretation of the model's multivalued solutions. Moreover, it would seem to provide a signature for their existence.

As is clear from Fig. 2, the shock front would form when $d\phi'/dr = \infty$. For the $C < 0$ static solutions of (15) it is not difficult to determine the locus of such singular points. It is given by the intersection of the solutions, for various C, and the curve $(1 + 3\kappa\phi'^2)r = 4\lambda\phi'$. As usual for singular points in the development of a shock, almost certainly there is some physics missing from the equations. Since ϕ'' is large, the obvious modification would be to include higher derivative terms in the action, which is tantamount to attempting an ultraviolet completion of the model. This is an open question. Perhaps higher terms in the galileon hierarchy would be natural candidates to be included.

To get a handle on such terms, and for purposes of comparison to the model in (9), consider briefly another model somewhat similar in form, but whose Lagrangian consists only of terms taken from the galileon hierarchy, without any coupling to Θ. After rescaling the field and coordinates to achieve a standard form, this alternate model may be defined by

$$A_{\text{self-dual}}[\psi] = \int \left(\frac{1}{2}\psi_\alpha\psi_\alpha - \frac{1}{4}\psi_\alpha\psi_\alpha\psi_{\beta\beta} + \frac{1}{12}\psi_\alpha\psi_\alpha(\psi_{\beta\beta}\psi_{\gamma\gamma} - \psi_{\beta\gamma}\psi_{\beta\gamma}) \right) d^4x. \tag{32}$$

The difference with (9) lies in the last term, which is quadrilinear in the field, as before, but now has two fields with second derivatives.

As the name suggests, this model is self-dual, in the following sense: The action retains its form under a Legendre transformation [9] (also see [12]) to a new field Ψ and new coordinates X, as defined by:

$$\psi(x) + \Psi(X) = x_\alpha X_\alpha. \tag{33}$$

Thus $A_{\text{self-dual}}[\psi] = A_{\text{self-dual}}[\Psi]$, provided integrations by parts give no surface contributions. This identity suggests that there are interesting properties for the quantized model, such as its ultraviolet behavior, but that is outside the scope of the present discussion.

Here it suffices to compare the classical physics following from (32) with that following from (9). Upon integrating once the classical equations of motion for static, spherically symmetric solutions of the field equations for (32), the result is again a cubic equation,

$$r^2\psi' + r(\psi')^2 + \frac{1}{3}(\psi')^3 = C, \tag{34}$$

but the $(\psi')^3$ term is no longer weighted by r^2 as it was in (15). Thus the small and large r behaviors are now given by

$$\psi' \underset{r\to 0}{\sim} (3C)^{1/3} \quad \text{and} \quad \psi' \underset{r\to\infty}{\sim} \frac{C}{r^2}, \tag{35}$$

for either sign of the constant of integration, C. These static solutions have finite total energy for either sign of C, as before, only now ψ' is always bounded. Moreover, upon inspection of the behavior of ψ' for intermediate r, and various C, unlike the previous model the solutions are now always single-valued for either $C > 0$ or $C < 0$. Thus there are no multivalued solutions like those shown in Fig. 2. However, each of the $C < 0$ static solutions does have a single point for which $d\psi'/dr = \infty$, namely, $r = (3|C|)^{1/3}$. So there is still a reason to expect the existence of shock fronts for quasi-static time-dependent fields in this alternate model. Finally, again for $C < 0$, to have ϕ' real for all $r > 0$, it is necessary to join together "interior" and "exterior" solutions at $r = (3|C|/2)^{1/3}$.

It remains to investigate the stability of these spherically symmetric solutions under perturbations, especially to check for the existence of superluminal modes, along the lines of [13]. Evidently, superluminal modes are a possible feature for models of this type.

In conclusion, it would be interesting to search for evidence of geons containing galileons at all distance scales, including galactic and sub-galactic, as well as cosmological. Perhaps a combination of trace couplings and various galileon terms, such as those in (9) and (32), will ultimately lead to a realistic physical model.

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[16] Also note that A_2 follows from coupling ϕ to the trace of the manifestly chargeless tensor $(\partial_\mu \partial_\nu - \delta_{\mu\nu} \partial_\alpha \partial_\alpha)\phi_\beta \phi_\beta$.
[17] For $C < 0$, to see cancellation between the individually divergent galileon and trace interaction energies for small r requires leading and next-to-leading terms in the expansion: $\phi' \underset{r \to 0}{\sim} \frac{-2\lambda}{\kappa r} + \frac{\kappa C}{4\lambda^2} + O(r)$.
[18] The multivalued behavior of any solution for $C_{\text{critical}} < C < 0$ makes the determination of the total energy ambiguous, at best, for these cases. This is an unresolved issue.
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