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Assisting Primary School Children to Progress through Their van Hiele’s Levels of Geometry Thinking Using Google SketchUp

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Abstract

This research aims to study the effectiveness of newly-developed learning modules built to assist Malaysian primary students to progress through the first three levels of widely accepted van Hiele’s model of geometry thinking. The first part of the paper reviews the background of study focusing on the problems encountered by Malaysian mathematics teachers to practice this model in the classroom using Geometer’s Sketchpad. It also includes brief essential information about the learning modules which was built using Google SketchUp. The second part describes methodology adopted in the research which involved quantitative approach of a pre versus post quasi-experimental design ran on a group of forty Year 6 primary school children in Malaysia. The results of comparative single-group mean analysis of the test scores suggest that the learning modules were potential in assisting students to progress through the respective van Hiele’s levels of geometry thinking, with all of them demonstrated progression of at least within level.

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1. Introduction

Geometry is one of the fields that are formally introduced in the Malaysian mathematics school curriculum right from the early primary education. Its emphasis increases as students progress to secondary education, where about forty percent of the sixty topics in the five-year secondary mathematics curriculum comprise of Geometry content (Malaysian Ministry of Education, 2007). A deep conceptual
understanding and the ability to visualize geometrical properties at the elementary level are most needed for the students to progress to the higher level learning of Geometry.

2. Background of Problems

When it comes to the learning of Geometry at schools, many mathematics educators associate the development of thinking with a famous learning model called van Hiele’s model of geometry thinking which was first proposed by Dina van Hiele-Geldof and Pierre van Hiele. They identify five differentiated levels of thinking that a student must progress sequentially from one level of thinking to the next without skipping any level. These are Level 0 or L0 (Recognition), Level 1 or L1 (Analysis), Level 2 or L2 (Informal Deduction), Level 3 or L3 (Deduction) and Level 4 or L4 (Rigor). The progression from level to level is more dependent on the content and method of instruction (van Hiele, 1986). Since then, the model has been creating a growing interest among researchers worldwide including Malaysia to investigate the nature and extent of students’ level of thinking in learning Geometry (Usiskin, 1982; Wu & Ma, 2005; Ding & Jones, 2006; Saifulnizan, 2007; Noraini, 2007). The van Hiele’s model of geometry thinking, which was first introduced more than three decades ago, remains applicable until now and still gaining popularity among mathematics educators.

In recent years, the technology-related applications have been widely developed and used to assist learners to learn mathematics more meaningfully and effectively. GSP in particular, offers advantages in assisting learners to draw and visualize in both 2D and 3D dynamic construction. There are numerous studies which revealed that, when properly used, the GSP could help learners improve the progression of their van Hiele’s levels of geometry thinking and thus improved the learning of Geometry (see for example Saifulnizan, 2007; Noraini, 2007).

In Malaysia, all secondary schools were supplied with GSP (in limited number) by the Ministry of Education with intention to assist both teachers to teach and students to learn Geometry and Geometry-related subjects more effectively in the classroom lessons. Every year, a substantial number of resource mathematics teachers were trained to use this van Hiele-based dynamic software in order to produce effective teaching and learning, especially in Geometry. However, the GSP-assisted learning in the classrooms has not been practiced widely mainly due to the limited availability among students to practice outside the classroom. The problem was worsened by the lack of expertise to use among teachers as well as the lacking of clear ready-to-use learning modules associated with each of the related geometry topic (Azlina & Lok, 2009). The use of GSP in the primary school classrooms has been much rarer as the supply of GSP was limited to secondary schools only whilst the number of primary school mathematics teachers who received training to practice GSP-assisted learning was much lesser. Based on a close scrutiny on the GSP, series of discussions with fellow mathematics educators and our own teaching experiences in mathematics, the researchers feel that the complexity to use GSP among young learners at primary schools may have aggravated the problems.

Speaking about of the use of computer software in teaching and learning of geometry learning, the Google Sketchup (GSU) software seems to offer several advantages as compared with GSP. First, it is an easy and intuitive to use for learning purposes especially on Geometry. Second, the standard version is downloadable from Google free of charge for students to use it on their own computers. Third, it carries features that help learners to motivate themselves to clear up spatial problems by creating corresponding virtual models by themselves. The virtual models should help students to understand that all drawings are representations of spatial problems which have to be solved by spatial thinking (Leopold, 2008). Younger students can easily create the 2D and 3D objects, since only a few simple tools are required. Older
students who are learning about equations of curves can also take advantage of GSU’s text tool, to see whether their curves follow the correct paths.

However, as far as the van Hiele’s model of geometry learning is concerned, GSU possesses a drawback as it is designed for architectural applications rather than pedagogical. Thus, it is equipped with much less pedagogically oriented instructions as compared with GSP. Nevertheless, with its built-in features, the researchers strongly believe that GSU can be utilized to provide an alternative learning tool especially for the young learners to accomplish effective learning of Shapes and Spaces especially by following the levels and learning phases prescribed by van Hiele’s model of geometry thinking.

Having accepted the important roles of van Hiele’s model of thinking in Malaysian mathematics learning, of Geometry, the pedagogical problems related to the GSP-assisted learning in Malaysian context mentioned above have triggered the researchers to investigate the following question - Beside GSP, can we build a pedagogically oriented, free access and relatively simple to use computer-based learning tool that would assist primary school children to progress through van Hiele’s level of thinking in the learning of Geometry?

3. Background Information of the Study

The full study was conducted in two stages. First, it involved the design and development of newly-developed learning modules built using Google SketchUp. For simplicity, these learning modules are called GSU-based Learning Modules. These learning modules were built to assist Malaysian primary students to progress through the first three levels of van Hiele’s geometry thinking. In the second stage, the newly-developed GSU-based Learning Modules were tried out to a sample of the targeted group. A quantitative investigation was then conducted to study how well did these learning modules assist the learners to progress through the first three levels of van Hiele’s geometry thinking, especially in the learning of Shapes and Space in Malaysian mathematics curriculum.

The main focus of this paper is to reports the study conducted in the second stage. Brief descriptions pertaining to the first stage are included in the following section to provide readers with a clear background of the whole study.

3.1 About the Newly-Developed GSU-based Learning Modules

The newly-developed GSU-based Learning Modules consisted of carefully crafted learning activities designed to assist primary school children to progress through the first three levels of van Hiele’s geometry thinking, namely L0, L1 and L2. It was designed and developed in accordance of procedures prescribed by ADDIE model concentrating on the learning of geometry topics of Shapes and Spaces. It consisted of three sub-modules, namely Module 1 (Triangles), Module 2 (Squares and Cubes) and Module 3 (Rectangles and Cuboids). Each of these modules incorporated relevant content and learning activities designed to be executed with the help of visualization-oriented features available in the GSU. For each module, students would have to execute specific tasks in specific order aiming to assist them to attaining the respective level-to-level progression of the van Hiele’s levels of geometry thinking. The restriction of emphasis to the first three van Hiele’s levels was mainly due to established research findings conducted elsewhere which revealed that the first three levels played much more important role in producing better conceptual understanding of elementary Geometry of Year 6 (Knight, 2006). As experienced mathematics teachers, the researchers strongly believe that it would be extremely important for learners of this age group to acquire these first three levels of thinking if they were to succeed in the learning of higher level of Geometry.
Each of the GSU-based Learning Modules was designed and developed based on the levels and learning phases described by the van Hiele’s model of geometry thinking concentrating on learning properties as emphasized by Crowley (1987), namely,

a. Students must proceed through the levels in order.
b. Students move through the levels without skipping any level. Their progress from level to level is more dependent on the content and method of instruction than on age.
c. For learning to occur, instruction must occur at the level of the student. If instruction is delivered at a higher level than the learner, the student will have difficulty following the thought processes used.

The learning activities incorporated in the GSU-based Learning Modules were designed and developed by following closely the van Hiele’s school of thought that students pass through numerous levels of geometry thinking as they progress from merely recognizing geometry shapes to being able to construct a formal geometry proof (van Hiele, 1999; Clements, 2004).

The GSU-based Learning Modules carried a special importance of geometrical structures on learning mathematics, i.e. specific visualization-oriented learning activities via the utilization of special features offered by the GSU. These visualization-oriented activities formed an important component of building concrete or at least semi-concrete of our mental representation of a concept (Konyalioglu, Ipek, & Isik, 2003). They involved several types of geometrical representations of diagrams, pictures and shapes for visualization of the abstract concepts in Geometry. In particular, these visualization-oriented activities focused on assisting learners to:

- recognize shapes
- make associations about shapes and developing visual thinking by means of presenting illustrations from a variety of positions.
- establish geometry concepts through activities involving analyzing shapes through geometric manipulations which were made simpler by the use of GSU.
- develop informal deductive thinking using problem-solving conditions which guide them in discovering issues and eventually stimulate geometry thinking.

The full details of the design and development of the GSU-based Learning Modules are explained in Tan (2011).

4. About The Study

4.1 Objectives of Study

This study aims to investigate the effectiveness of the GSU-based Learning Modules in assisting primary school students to progress through the first three van Hiele’s levels of geometry thinking in the learning of Shapes and Spaces.

4.2 Methodology

This small-scale study adopted a pre and post quasi-experimental design using a single group of subjects. No control group was used in this study as the researchers were only interested to see the progression of van Hiele’s levels of geometry thinking ‘within the subjects under study’. As such, this study performed no comparative investigation between groups of students who used the GSU-based Learning Modules and those who did not. The effectiveness of GSU-based Learning Modules was evaluated via the investigation on the progression of geometry thinking made by the subjects after the use
of these learning modules. The data of the study were gathered and analyzed using simple quantitative approach.

4.3 Sample

The study was conducted to forty Year 6 students from a primary school at which one of the researchers was teaching. They were identified and selected using specially adapted test called Wu’s Geometry Test (or simply WGT) created by Wu et al. (2005a). The sample comprised of three categories of students, each represented a particular cohort of students with specific van Hiele’s levels of geometry thinking of below BL0, L0 and L1 respectively. The number of students for each cohort were 4, 9, and 27 respectively. These numbers were based mainly on ‘what were available’ and no stratification method has been applied in the sampling method.

4.4 Instrument

In short, the adapted version of WGT consisted of three sub-tests (namely Test 1, Test 2 and Test 3), each to measure students’ van Hiele’s levels of geometry thinking associated with each of the respective three GSU-based learning sub-modules described earlier. The adaptation involved only the exclusion of items from the original version of WGT (Wu et al., 2005a) which were not covered by the Malaysian primary mathematics curriculum. Each sub-test consisted of 25 items and it has been designed to measure the student’s van Hiele’s levels of geometry thinking as to whether his or her level of thinking was at BL0, L0, L1 or L2. The full details of the test items and the validation process adopted in the adaptation are explained in Tan (2011). It must be noted that the same WGT was used to measure the students’ van Hiele’s levels of geometry thinking before (pre) and after (post) the use of GSU-based Learning Modules.

4.5 The Fieldwork

Once selected using the (pre) WGT, each of the school children was asked to undergo a series of learning sessions on Shapes and Spaces, this time with the assistance of the newly-developed GSU-based Learning Modules. Each of the learning sessions took one hour and was conducted in the evening (i.e. after the normal morning classes). It ran four times a week for three consecutive weeks.

During the learning session, each student was asked to perform self-instructed learning activities prescribed in the respective module at their own preferential learning speed in specific sequences starting from Module 1 and then followed by Module 2 and Module 3. It must be noted that during the whole learning sessions, the teacher did not perform the teaching of Shapes and Spaces at all. Instead his role was to provide instructions to students ‘what to do’ as well as to provide minimal help to clarify the learning activities to be performed. These help were given only as necessary.

The students’ van Hiele’s levels of thinking were measured again using the (post) WGT upon the completion of the whole intervention program at the end of third week.

4.6 Test Scoring and Data Analysis

The adapted version of WGT preserved the scoring principle set by Usiskin (1982) in measuring the van Hiele’s levels of geometry thinking, i.e. for each learning module,

a. Ones are considered to have attained a particular van Hiele’s level of geometry thinking if they manage to answer correctly at least 60% of the total items designated for each particular level associated with the respective module, and
b. One’s van Hiele’s level of geometry thinking is defined as the highest level attained by him/her.

Simple comparative analyses of WGT scores were performed to investigate the increases (or decreases) of post-test scores versus the pre-test scores for both the over-all and individual cases of the students.

No inferential analysis of means (ANOVA or alike) was performed due to the small size of sample as well as the possibility that the sampling criteria used in the study might not meet the assumptions required by inferential statistics.

5.0 The Findings and Discussions

Two layers of analyses were performed in the full study, first based on the whole GSU-based Learning Modules followed by analysis on each of the three sub-modules. Due to close consistency of findings throughout the sub-modules as well limitation of the size of article, this paper focuses only on the first.

Using the procedures and conventions described earlier, the WGT scores gathered before and after the use of GSU-based Learning Modules were analyzed and summarized in Table 1. The information clearly suggested that before the intervention, all students were found to have achieved van Hiele’s levels of geometry thinking of L1 or below with four of them felt within BL0 which indicated that they did not even attain the threshold score of L0. This situation might be considered as ‘relatively unsatisfactory’ if they were to meet the van Hiele’s requirements to learn Geometry effectively as it indicated most students only operated on the lower thinking levels of recognition and analysis during the learning of Shapes and Spaces. If the test scores could be taken as an indication of their level of understanding in Shapes and Spaces, it might then be concluded that their understanding was rather low as reflected by the low mean scores of less than 56%.

These information, coupled with summary of essential information related to the progression of van Hiele’s levels of geometry thinking after the use of GSU-based Learning Modules (Table 2) seemed to insinuate significant changes of scenarios. These information showed that all of the students have managed to progress at least within the level (i.e. a situation where individual showed increase of test scores within a particular level but did not reach a level-by-level progression as defined by Usiskin’s criteria). Among them, more than half have managed to perform between levels progression. Again, if the test scores could be taken as an indication of their level of understanding in Shapes and Spaces, there were quite marked improvement of understanding made by the students as reflected by the increase of means from less than 56% to around 70%. These findings appeared to indicate that the newly developed GSU-based Learning Modules possessed high potential to assist students to progress through the respective van Hiele’s levels of geometry thinking.

The progression of van Hiele’s levels of geometry thinking was investigated further by plotting a graph to scrutinize the progression made by each subject under study (Figure 1). This plot appeared to suggest that:

a. The use of the GSU-based Learning Modules has managed to assist majority of the students to progress through their van Hiele’s level of geometry thinking.

b. With little exception, the progression occurred in the manner as described by van Hiele, that, the progressions of levels of thinking were sequential where learners passed through the levels in order as their understanding increased.
Information elucidated in Figure 1 and Table 3 showed that there were four students who did not seem to ‘follow’ the van Hiele’s rule of progression of levels of geometry thinking (referred to as ‘jump phenomenon’) where they have progressed through van Hiele’s levels of geometry thinking without following strictly the sequential order. In particular, three students demonstrated a progression from BL0 to L1 whilst one of them progressed through from L0 to L2. It was unclear if these non-sequential progressions indicated a ‘break of rules’ of the van Hiele’s model of geometry thinking but the researchers feel that these might have not been the case if the progression were measured twice or more during the whole of the intervention period. Having said that, these jump phenomenon appeared to suggest the potential of the newly designed GSU-based Learning Modules in assisting weak learners to learn Shapes and Spaces in the Malaysian primary mathematics of Geometry.

It must be noted that the above claims made about the effectiveness of the newly-developed GSU-based Learning Modules may be refutable as the experimental design employed in this study may have permitted other factors such as test retention and repeated learning to have contributed to the progression of the van Hiele’s levels of geometry thinking. However, it must also be stressed again that the students received no ‘second time or additional teaching lessons of Shapes and Spaces’ during the whole experimental sessions. Viewed from the design and development of the GSU-based Learning Modules, these were designed and developed using thorough principles focusing on the van Hiele’s model of geometry thinking. In particular, a great emphasis were given on the visualization-based learning activities and hands-on explorations to develop geometry thinking that would actively engage the students in the learning processes and enhance students’ conceptual understanding of geometry concepts. Thus, any progressions of the van Hiele’s levels of thinking occurred are thought to be likely resulted by the active engagement of students in the learning processes.

As mentioned earlier, similar analyses on each of the GSU-based Learning Modules were performed in the study. With very little exceptions, these analyses revealed a more or less consistent pattern of findings throughout all modules. The details of such findings are reported in Tan (2011).

6. Conclusion

As noted earlier, the dynamic graphical GSU software is neither built on van Hiele’s model of thinking nor specifically aimed to be used to assist learners to learn Geometry more effectively. Bearing in mind of the limitations and possible refutation mentioned above, this study seem provides evidences that, when incorporated with carefully designed learning activities, GSU can be used to assist primary school children to progress through the van Hiele’s levels of geometry thinking. It is hoped that this would lead to the full development of a practical computer-assisted learning alternative that eventually assists primary school children to learn Geometry more meaningfully and effectively.

7. Limitations and Future Studies

The results of this research should be interpreted with the precautions and arguments mentioned in the earlier section in mind, that some additional factors such as test retention, repeated learning, etc. might have contributed to the findings. The researchers are currently undertaking a series of qualitative studies to investigate these threats and will be reporting the results once full analysis are available.
References:


van Hiele, P. M. (1999). Developing geometric thinking through activities that begin with play. *Teaching Children Mathematics, 5*(6), 310-316.


**Appendix**

**Table 1**: The Number and Percentages of Students According to the van Hiele’s Levels of Geometry Thinking among Students Before and After the Use of the Whole GSU-based Learning Modules (n=40)

<table>
<thead>
<tr>
<th>van Hiele’s Levels of Geometry Thinking</th>
<th>Number of Students</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre</td>
<td>Post</td>
</tr>
<tr>
<td>BL0</td>
<td>4 (10.0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>L0</td>
<td>9 (22.5%)</td>
<td>1 (2.5%)</td>
</tr>
<tr>
<td>L1</td>
<td>27 (67.5%)</td>
<td>28 (70.0%)</td>
</tr>
<tr>
<td>L2</td>
<td>0 (0%)</td>
<td>11 (27.5%)</td>
</tr>
</tbody>
</table>

*Mean WGT Score* 55.9% 70.3%

**Note**: Numbers in brackets show the percentage of students for each category.

**Table 2**: Break-down of Number and Percentages of Students for Each Category of Progression of van Hiele’s Levels of Geometry Thinking Before and After the Use of the Whole GSU-based Learning Modules (n=40)

<table>
<thead>
<tr>
<th>Category of Progression of van Hiele’s Levels of Thinking</th>
<th>The Progression of van Hiele’s Levels of Thinking Involved</th>
<th>Jump Phenomena</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Levels</td>
<td>BL0 to L0</td>
<td>L0 to L1</td>
<td>L1 to L2</td>
</tr>
<tr>
<td></td>
<td>1 (2.5%)</td>
<td>8 (20.0%)</td>
<td>10 (25.0%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3 (7.5%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 (2.5%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>23 (57.5%)</td>
</tr>
<tr>
<td>Within Level</td>
<td>L0</td>
<td>L1</td>
<td>L2</td>
</tr>
<tr>
<td></td>
<td>0 (0%)</td>
<td>17 (42.5%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>No Progression at all</td>
<td>L0</td>
<td>L1</td>
<td>L2</td>
</tr>
<tr>
<td></td>
<td>0 (0%)</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
</tbody>
</table>

**Notes**: a. Numbers in brackets show the percentage of students for each category.

b. *Within Level progression is a situation where individual shows increase of WGT scores within a particular level but does not reach the threshold of the immediate next level of geometry thinking*
Table 3: Details of WGT Scores of Each Individual Who Demonstrated Jump Phenomenon (threshold score = 60%)

<table>
<thead>
<tr>
<th>Student</th>
<th>WGT Scores (in percentage)</th>
<th>The Jump Phenomenon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level L0</td>
<td>Level L1</td>
</tr>
<tr>
<td>S36</td>
<td>36</td>
<td>60</td>
</tr>
<tr>
<td>S37</td>
<td>52</td>
<td>76</td>
</tr>
<tr>
<td>S39</td>
<td>44</td>
<td>72</td>
</tr>
<tr>
<td>S31</td>
<td>64</td>
<td>68</td>
</tr>
</tbody>
</table>

Figure 1: Plot of Individual van Hiele’s Levels of Geometry Thinking Before and After the Use of the Whole GSU-based Learning Modules (n=40)

Notes:  
a. Dotted line represents the threshold score of 60% of the WGT for the respective level of geometry thinking  
b. $S_i$ denotes the $i^{th}$ Student ($i = 1,2,3, \ldots ,40$)