Multiproduct, multistage machine requirements planning models

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This paper proposes mathematical programming models for machine requirements planning in a multiproduct, multimachine, multistage manufacturing environment. Two models are developed, a mixed-integer linear programming model that takes into consideration factors of production such as resource and budget constraints, and a goal programming model that takes conflicting goals into account. Validation of the models is demonstrated by way of application to a computer hard disc manufacturing plant. The efficacy of the models and their results are discussed in a comparative manner.

Keywords: mixed-integer programming, goal programming, machine requirements planning, computer hard disc manufacturing

Introduction

An important problem that confronts a manufacturing manager is to develop an understanding of the operations in sufficient depth so as to make good planning and operating decisions. Modeling of manufacturing systems has proven to be a valuable means for gaining this understanding.

However, with the complicated nature of manufacturing systems, the task of optimal planning is far more varied, complex, and interactive than ever before. There are many planning and design decisions that have significant effects on both the time and costs associated with manufacturing a product.

Facility design is one aspect of planning that includes varied decision problems. Take, for instance, determining the type of facilities to utilize (e.g., equipment selection), their operating characteristics (e.g., conveyor speed), and their locations. One of the more common facility design issues, yet one which has received relatively little attention in research literature, is the question of how many machines to have on hand.

There are a limited number of methods for analyzing the machine requirements problem. Shublin and Madeheim[1] suggested the use of the relationship $x = (t/60) * (p/hu)$ for calculating the desired number of machines for a single work center with one product and one operation associated with the center. Here, x is the desired number of machines in the production period, t is the standard time for the operation in minutes, p is the total number of production units required per day, h is the standard number of hours available per day per machine, and u is the efficiency or "use" factor for the center. Similar deterministic, static approaches that use variants of Shublin and Madeheim's model have been reported by Apple,[2] Ireson,[3] Muther,[4] Johnson,[5] Reed,[6] Moore,[7] and Francis and White.[8] Most of these approaches are concerned with a single work center, single product, single operation situation as analyzed by Shublin and Madeheim. Until the late 1970s, the models formulated were confined to descriptive ones, not considering any constraints.

Later developments were of the constrained mathematical programming type. Miller and Davis[9] analyzed the machine requirements problem as a resource allocation model involving allocation of limited floor space, capital budget, and available overtime among various types of machines. Hayes, Davis, and Wysk[10] also addressed the problem of a serial, multistage machining system with a different processing operation occurring at each machining center (or stage) using a dynamic programming formulation in the context of cost minimization following the fundamental framework employed by Davis and Miller.[11] Other related works are those of Davis and Miller[12] and Davis and Kennedy,[13] both using a Markovian approach to the problem, and that of Nnaji and Davis.[14]

In this paper, the model of Hayes, Davis, and Wysk[10] is used as the general framework. Interactions with the planning issues of the production environment such as budget, operating cost, and machine availability as indicated by the plant capacity are incorporated in the model. The model determines the number of machines

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and the total processing time necessary to meet the required output in each work center.

Development of models

The general framework of Hayes, Davis, and Wysk[10] is used in the proposed models. In their model, a single product, serial, multistage machining system with a different processing operation occurring at each stage was analyzed using the dynamic programming approach. This paper expands coverage of the problem to a multiproduct, multimachine, multistage manufacturing system. Mixed-integer linear programming and goal programming formulations are used to determine the optimum number of machines required in each work center for each type of product.

The manufacturing system addressed in this paper may be characterized by the following: There are N products and M stages; some of the products may go through fewer than M stages; some machines may be used in different stages; a day is broken down into shifts of operation; there is a certain percentage of defective units generated by the processes at any stage of each of the products; and there is a given daily production requirement for individual products.

Mixed-integer linear programming (MILP) model

The mixed-integer linear programming model presented below is valid based on the following assumptions: Setup time and travel time of works-in-process between stages are relatively short and thus negligible; the sequence of operations (technological order) of all the products through all the stages is known; the technological order may be different for each product; the processing times for each machine and for each product are deterministic and known; machines in each stage or work center are of the same type; each work center must have at least one machine; and demand is less than capacity.

Notations used in the model are as follows: i is product type or product model; j is stage of the work center; k is work shift, either first, second, or third; l is type of machine used in the work center; $t_{i,j,k}$ is number of machine hours of operation of product i at stage j during shift k; $n_{l,i,j}$ is number of type l machines required by product i at stage j; $r_{i,j}$ is operating rate of product i at stage j; $b_{i,j}$ is percentage of defective units incurred by product i at stage j; $CO_{i,j,k}$ is the manufacturing cost for one hour of production of product i at stage j during shift k; CF_l is the fixed cost of a machine l per day; $S_{i,j}$ is demand of product i at stage j (per day); $S_{i,M}$ is demand of product i at the last stage, M; $w_{i,j}$ is total available working hours in each shift for stage j; $e_{j,l}$ is efficiency of machine l at stage j; PC_l is total number of machines of the same type l.

The objective to be achieved is to minimize the total cost of production per period. It is desired to find the values of $t_{i,j,k}$ and $n_{l,i,j}$ that minimize the following daily operating cost function:

Minimize total cost: Processing cost + equipment cost

$$\text{Total cost} = \sum_i \sum_j \sum_k (CO_{i,j,k} * t_{i,j,k}) + \sum_i \sum_j (CF_l * n_{l,i,j}) \quad \text{for all } l \quad (1)$$

Subject to the restrictions stated below.

First, the quantity of output of product (i.e., units started less fraction defective) from some stage j must be greater than or equal the quantity processed by the next stage ($j + 1$).

$$(r_{i,j} * e_{j,l} * (1 - b_{i,j})) * \sum_k t_{i,j,k} \geq S_i(j+1) \quad \text{for all } i \text{ and } j < M \quad (2)$$

For the final stage this restriction takes the following form:

$$(r_{ij} * e_{j,l} * (1 - b_{i,j})) * \sum_k t_{i,j,k} \geq S_{i,M} \quad \text{for all } i \text{ and } j = M \quad (3)$$

Second, the number of units assigned for processing in each stage cannot use more processing time than is available on the number of machines allocated to that stage.

$$t_{i,j,k} \leq w_{i,j} * n_{l,i,j} \quad \text{for all } i, j, k, l \quad (4)$$

Third, the total number of machines of the same type allotted to the different stages and products should not go beyond plant capacity. Because the same type of machine can be used by the different work centers for any of the products, or by any of the stages of the same product, the total allocation must not exceed what is available in the plant.

$$\sum_l n_{l,i,j} \leq PC_l \quad \text{for all } i, j, l \quad (5)$$

Fourth, it is desired to have integers for the number of machines allocated. Also, each work center should have at least one machine.

$$n_{l,i,j} \geq 1, \text{ integers} \quad \text{for all } i, j, l \quad (6)$$

Last, nonnegativity constraints are included.

$$t_{i,j,k} \geq 0 \quad \text{for all } i, j, k \quad (7)$$

The above model is fundamentally a mixed-integer linear program requiring an integer domain for the subset of variables associated with the number of machines and allowing the other decision variables to have continuous solutions.

Goal programming (GP) model

One major assumption of the mixed-integer programming model is that demand is less than capacity. However, in the actual environment, in solving the mixed-integer linear programming model, it is possible that there would be instances wherein the constraint on raw material availability or capacity limitations would be exceeded. It might sometimes be worthwhile or necessary to buy extra raw materials or capacity at a high price. In such a case, solving the problem as a mixed-integer linear programming model would obviously lead to an infeasible result as all constraints cannot be satisfied simultaneously.

Two approaches could be taken to resolve this infeasibility. One approach is to increase the upper bound on raw materials and capacity constraints; another is to use the goal programming technique.

Increasing the upper bound on capacity and raw materials would require *a posteriori* calculations to determine the added amount necessary to fulfill the constraints. Another shortcoming of this approach is that one might not be definite about the logical amount by which the capacity or raw materials could be increased. Incrementing the limit by trial and error would ultimately lead to a feasible result; however, this would not be efficient.

Another approach is to use "soft" constraints. That is, the constraint can be violated at a certain cost. Add deviational surplus and slack variables to make an equality constraint. By using these deviational variables, it is possible to allow the right-hand side coefficient to be overachieved or underachieved by the amount assumed by the deviational variables. The objective in this situation is to satisfy all the constraints as closely as possible. Such is the technique of goal programming (see e.g., Tabucanon[15]).

Whereas linear programming deals with minimization or maximization of objective functions, goal programming is concerned with the condition of achieving prespecified targets or goals.

Once individual goals have been stated, the objective of goal programming is to achieve the goal portfolio as closely as possible, i.e., to minimize the set of deviations or "distances" from the goals. All goals can be considered simultaneously or they can be taken one by one according to the priority structure.

All notations previously defined in the mixed-integer linear programming model are used in this goal programming formulation, with additional deviational variables denoted as d^- (for underachievement) and d^+ (for overachievement).

For the machine requirements planning model, the goals are established as follows: (1) Demand should be met at all times; (2) Time employed in each shift should be utilized as fully as possible; (3) Production cost should not exceed allotted operating expenses; and (4) Total machines allocated should not exceed plant capacity. These goals may be associated with first to fourth priorities, respectively. It is desired to achieve these goals as fully as possible.

Meeting the demand, as dictated by the mother company is the primary objective of the machine requirements planner. Thus, the production goal can be expressed as follows:

$$r_{i,j,} * (1 - b_{i,j,}) * e_{j,l} * \sum t_{i,j,k} + d_{1,i}^- - d_{1,i}^+ = S_M \quad \text{for all } i \text{ and } j = M \quad (8)$$

The objective is to minimize underachievement of the production goals $d_{l,i}^-$, for all product types.

A certain number of machines are allocated to each stage per shift and it is desired to utilize these machines fully. Hence, the goal that the units being processed at a stage must use all the processing time available on the number of machines allocated to that stage can be written as:

$$t_{i,j,k} + d_{2,i,j,k}^- - d_{2,i,j,k}^+ = W_{i,j} * n_{l,i,j} \quad \text{for all } i, j, k, l \quad (9)$$

The objective is to minimize both underachievement and overachievement of time utilization goals in all shifts.

Two factors comprise the cost function—the operating cost and the fixed cost incurred by the machines used. An operating budget (Y) is set as the allotted daily expense, and this should not be exceeded unnecessarily. Thus, it is desired to achieve the following:

$$\sum_i \sum_j \sum_k (CO_{i,j,k} * t_{i,j,k}) + \sum_i \sum_j (CF_l * n_{l,i,j}) + d_3^- - d_3^+ = Y \quad \text{for all } l \quad (10)$$

The objective is to minimize the excess expense of the cost goal, d_2^+. Y is the budget for daily operating expenses.

A specific number of machines are available in the plant. Additional machines may be acquired from facilities of sister companies. Hence, it is desired to minimize exceeding plant capacity. This is expressed as:

$$\sum_l n_{l,i,j} + d_{4,l}^- = d_{4,l}^+ = PC_l \quad \text{for all } i, j, l \quad (11)$$

The objective is the minimization of exceeding plant capacity, $d_{4,1}^+$, for all types of machines.

Production of each product goes through a series of fixed stages. The output at one stage serves as the input to the succeeding stage. This structural constraint is expressed as:

$$r_{i,j} * (1 - b_{i,j}) * e_{i,j} * \sum_k t_{i,j,k} = S_{i,}(j + 1) \quad \text{for all } i, j < M \quad (12)$$

If the goals are prioritized according to the structure set forth earlier, then the objective function is expressed as follows:

$$\text{Minimize } Z = P_1 * \left(\sum_i d_{1,i}^-\right) + P_2 K * \left(\sum_i \sum_j \sum_k d_{2,i,j,k}^- + \sum_i \sum_j \sum_k d_{2,i,j,k}^+\right) + P_3 * d_3^+ + P_4 * \left(\sum_1 d_{4,l}^+\right) \quad (14)$$

The highest priority is given to minimize all $d_{1,i}^-$, which is to ensure that demand for all products is met. The $d_{1,i}^+$ are not considered in the objective function because production can exceed demand. Both the positive and negative deviations are included in the objective function at second priority, as the exact utilization of time available based on the number of machines allocated is desired. Only d_3^+ is included in the third priority because management would be more than happy if cost could be reduced. The last goal is to ensure that production is within the capacity of the plant. Hence, only $d_{4,l}^+$ is included in the objective function—the idea being to minimize the excess capacity that would be required to meet demand. All decision and deviational variables are restricted to be nonnegative. A change in the priority structure would require corresponding changes in equation (4).

The case study

The company under consideration manufactures computer hard discs. Its production system is divided into six main sections: Subassembly, Clean Room, Final Assembly I (FAI), Final Assembly II (FAII), Final Test (FT), and Button-Up (BU).

The Subassembly area prepares the different components for the Clean Room. It also includes some minor operations such as base casting and bar code labeling. It is here that the disc head is assembled. After processing, the units are transported by conveyor to the major assembly and test sections (FAI, FAII, and FT) and finally to the BU section. This study considers only these last four sections, as these are the stages where major machines are used. Other stages consist mostly of manual assembly operations.

The system manufactures 12 different product types (A to L, in capital alphabetic coding) and 15 different machine types (a to o, in lower case alphabetic coding). Each product would require a certain combination of machines. These are given in *Table 1*. The problem then is to decide on the number of machines of each type to be allocated to a particular production stage whenever a certain product type is manufactured.

Application of the models

The two models were applied to the case study presented to test their validity. The company's existing approach in solving the machine requirements problem gives little consideration to the interrelationships between the number of machines used and such system components as in-process inventories, scheduling rules, product quality, available space, and production time. Another shortcoming of the existing description model is the lack of consideration of scarce resources involved in the manufacturing system as well as of the competition among various work centers for these resources. These create interdependence among the work centers that should necessarily be accounted for.

Initially, the approach taken to analyze the machine requirements problem was by the mixed-integer linear programming method. However, certain difficulties were encountered. Initially, because of the assumption that demand is less than capacity, infeasibility of solution results when this constraint is violated. This limits the model to a narrow application because in the actual manufacturing environment, the competition for scarce resources is normally involved. To resolve the infeasibility, the upper bounds of the plant capacity were increased. Note that this would require *a posteriori* calculations in order to determine the number of machines needed that are in excess of plant capacity.

Second, although integer solutions were obtained, sensitivity analysis could not be performed without resolving the whole problem. Integer programming models have no shadow prices or dual variables with an interpretation comparable to linear programming.

In the algorithm used to evaluate integer programming models, neither the optimal value nor the optimal solution need be continuous as a function of the coefficients defining the constraints. As such, integer programming models can behave in an erratic manner because of the presence of multiple discontinuities caused by (necessary discrete) changes of the value for the integer variables. Ergo, dual variables of the mixed-

Table 1. Machine-type (coded)* requirements of products in various production stages.

Product type	Production stage 1	2	3	4	5	6	7	8	9
A	d	n	g	f	i	e	i	o	–
B	d	n	g	f	i	e	i	o	o
C	a	e	n	f	k	e	e	o	–
D	a	e	n	f	k	e	e	o	–
E	a	d	n	f	i	d	i	o	–
F	a	d	n	f	h	i	e	e	o
G	a	c	n	f	h	i	e	e	o
H	a	o	n		m	m	o	–	–
I	a	d	n	f	i	d	i	o	–
J	a	c	n	f	h	i	e	e	o
K	a	c	n	f	h	i	d	d	o
L	a	d	n	f	i	d	d	o	–

* Data are coded to maintain confidentiality.

integer linear programming models could not be interpreted in the same manner as linear programming models.

To determine the influence of varying a resource level on the optimal solution, in general, one must resolve the mixed-integer linear programming problem with alternate resource levels. This is allowable in the model analyzed, because the average time to obtain results is only about 24 sec.

Comparing the two cases of the mixed-integer linear programming formulation, wherein one yielded an integer solution while the other case did not, it was observed that the continuous solution does not differ much from the integer end results. Rounding up or down would yield almost similar results. However, such subjective rounding off of solutions is not always practical. Rounding up has the consequence of creating excess or idle machine capacity and its associated opportunity cost. Rounding down creates the necessity for subcontracting or overtime, which normally incurs a penalty cost above routine operations. Moreover, simple rounding of a fractional number of machines at each work center can lead to an infeasible solution when the actual production system is considered with its inherent limited resources and other pragmatic constraints. This is especially true when the decision variables involve small values; in such cases adding or decreasing a unit by rounding up or down has a significant effect.

As a result of these limitations, another approach was used to address the machine requirements problem—goal programming.

When a problem involves a number of constraints, not all of which can be satisfied simultaneously, a conventional model would obviously be infeasible. In such a case, it is better to restrict our aim to satisfying all the constraints as closely as possible. This is one of the advantages of goal programming.

What the mixed-integer linear programming model cannot handle, i.e., when demand exceeds capacity, goal programming solves through the use of deviational variables incorporated in the constraints. How many more machines are needed (in excess of plant capacity) can readily be seen in the positive deviational variables. The number of machines not being used are reflected in the negative deviational variables. The types of products that cannot be satisfied are also indicated. In addition, one can perform sensitivity analysis by restructuring the goals or changing the parameters.

In terms of computation time, the goal programming method consumed less time in obtaining its solution. The mixed-integer linear programming method requires a longer time because of its branch and bound algorithm. It is possible to encounter prohibition computation time for very large problems using the mixed-integer linear programming method. In the model studied, the goal-programming approach used about 8% of

Table 2. Comparison of results in terms of number of machines for each type using the base demand for a particular period.*

Product type	(Model)	Production stage 1	2	3	4	5	6	7	8	9
A	(I)	7	77	3	11	10	20	9	4	—
	(II)	4	77	2	12	11	24	9	4	—
	(III)	4	77	2	15	13	24	9	4	—
B	(I)	1	21	1	3	3	8	2	2	1
	(II)	1	21	1	3	3	8	2	2	1
	(III)	1	21	1	5	4	11	2	2	1
C		(No production during the period considered)								
D	(I)	2	1	18	3	8	12	2	3	—
	(II)	2	1	17	2	9	12	1	2	—
	(III)	2	1	17	3	14	12	1	2	—
E	(I)	8	12	163	38	32	97	31	7	—
	(II)	7	13	163	43	36	103	35	8	—
	(III)	7	13	164	50	40	104	35	8	—
F	(I)	1	1	9	5	2	2	5	1	1
	(II)	1	1	9	5	2	2	5	1	1
	(III)	1	1	9	3	2	2	5	1	1
G		(No production during the period considered)								
H	(I)	1	4	22	16	15	2	1	—	—
	(II)	1	4	21	18	16	2	1	—	—
	(III)	1	5	21	22	13	2	1	—	—
I	(I)	1	1	3	1	1	2	1	1	—
	(II)	1	1	2	1	1	1	1	1	—
	(III)	1	1	2	1	1	1	1	1	—
J		(No production during the period considered)								
K	(I)	1	1	13	1	2	2	5	1	1
	(II)	1	1	12	1	2	2	5	1	1
	(III)	1	1	12	1	3	3	5	1	1
L	(I)	1	2	8	1	2	3	1	1	—
	(II)	1	1	8	1	2	3	1	1	—
	(III)	1	1	8	1	2	3	1	1	—

* I = MIP model; II = GP models; III = existing procedure.

the time required by the mixed-integer linear programming approach.

A summary of comparative results using mixed-integer programming (MIP), goal programming (GP), and the existing procedure for a given set of demand data is shown in *Table 2*. It shows that the goal programming results are closely in agreement with the results based on the existing heuristics.

Concluding remarks

The advantage of using mathematical programming models over descriptive computation is that it seeks to optimize or at least satisfy management goals and production process limitations. Descriptive models define what is needed; mathematical programming models incorporate the "what if" options. In particular, goal programming models, through the restructuring of goal priorities and changes in the parameters, can perform experiments to evaluate different options. At the same time, goal programming gives the planner a better understanding of the relationships of his or her decisions to other planning and production decisions.

Both mathematical models have their advantages and are preferred over the descriptive model for the primary reason that the latter does not consider the limitations inherent in any production system. Furthermore, it does not provide the information regarding slack or surplus attainable by using mathematical models. As to choosing between the two mathematical models, mixed-integer linear programming models are preferred in situations where restrictions on capacity and raw materials are unlikely to be violated. Dealing with small units, e.g., choice of one or two machines and where integer end results are desired, mixed-integer linear programming is ideal.

Goal programming models answer the solution of infeasibility caused by violation of constraints in linear programming. It aims to satisfy the goals as much as possible and shows their underachievement or overachievement through its deviational variables. It is flexible, and provides the "satisficing" solution under varying inputs and goal structures. Furthermore, as a solution method, goal programming is simple and easy to understand.

Other than the advantages stated above, the goal programming method is preferred in this case study because it answers the question of the excess plant capacity needed, a drawback that the mixed-integer linear programming method cannot handle.

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