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On the effective action of a space-like brane

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Abstract

Starting from the non-BPS D$(p+1)$-brane action, we derive an effective action in $(p+1)$ space dimensions by studying the fluctuations of various bosonic fields around the time-like tachyonic kink solution (obtained by Wick rotation of the space-like tachyonic kink solution) of the non-BPS brane. In real time this describes the dynamics of a space-like or Euclidean brane in $(p+1)$ dimensions containing a Dirac–Born–Infeld (DBI) part and an Wess–Zumino (WZ) part. The WZ part is purely imaginary and so the action is complex if it represents the source of the time-dependent background of type II string theory, i.e., S-brane. On the other hand, the WZ part as well as the action is real if it represents the source in type II* string theory, i.e., E-brane. The DBI part is the same as obtained before using different method. This is then further illustrated by considering brane probe in space-like brane background.

Non-BPS D$(p+1)$-branes exist in both type IIA (for $p=$ even) and type IIB (for $p=$ odd) string theory and are unstable due to the presence of open string tachyon in their world-volume [1]. The dynamics of the non-BPS branes can be described by an effective tachyon field theory [2–5] and if the tachyon depends on one of the space-like coordinates ($x^{p+1} \equiv x$ (say)) of the brane, it has an infinitely thin but finite tension kink solution [6]. It has been shown that for this solution the total energy of the brane is localized around $x=0$ with the tension $\mathcal{T}_p = \int V(T)\,dT$, T being the tachyon. Sen has shown [7] that even though the tachyon effective action is valid (where higher ($\geqslant 2$) derivatives of tachyon are neglected) for large T, the fluctuations of massless modes on the kink solution interpolating between the two vacua at $T=-\infty$ and $T=\infty$, passing through $T=0$, correctly reproduce the DBI action of the BPS Dp-brane without any higher derivative corrections. Also, the fluctuations are not assumed to be small in this derivation. The minimum of the potential $V(T)$ has been argued to describe the closed string vacuum [1].

In order to understand the decay of the non-BPS D$(p+1)$-brane, one should really consider the tachyon to be time dependent [8]. Thus the rolling of the tachyon is responsible for the decay of the non-BPS D$(p+1)$-brane to the closed string vacuum [9]. On the other hand, by a similar reasoning as given in the previous paragraph, the rolling of the tachyon can also be seen to be responsible for the appearance of

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space-like p-branes [10] (Sp-branes or Ep-branes) at the maximum of the tachyon potential $V(T)$ where $T \to 0$. Sp-branes (Ep-branes) are space-like topological defects localized in $(p+1)$-dimensional space-like hypersurface in type II (type II*)[1] string theory and appears when finely tuned incoming closed string radiation pushes the tachyon (at $x^0 = -\infty$) to the top of the potential from one side and then the tachyon rolls down to the other side (at $x^0 = \infty$) dissipating its energy back to radiation [12]. In this process, the space-like p-brane appears and exists only for a moment in time at $x^0 = 0$. These are the sources for the classical time-dependent solutions [13–16] of string effective action.[2]

In this Letter, we give a simple derivation of Sp-brane (or Ep-brane) action starting from the non-BPS D$(p+1)$-brane action with time-dependent tachyon. Here we make use of Sen's derivation [7] of codimension one BPS D-brane action as the space-like tachyonic kink solution of non-BPS D-brane action. When the tachyon is time dependent, the solution of the equation of motion can be obtained by Wick rotation of the corresponding solution with space-dependent tachyon. This solution may be regarded as the time-like tachyonic kink of the non-BPS D-brane action. We then study the fluctuations of various bosonic fields on the Euclidean world-volume around this solution and obtain an effective action. In real time, this effective action represents the Sp-brane (or Ep-brane) action describing the dynamics of the various fields living on the brane. We derive both the DBI and the WZ parts of the action, where the DBI part matches with the results obtained earlier [19] using different method. We will point out in what sense we call this an Sp-brane or an Ep-brane action. We also give an alternative argument to further support the form of the action that we have obtained. This is done by considering the brane probe in a non-extremal p-brane background. The p-branes turn into space-like branes as we consider the region beyond their horizons. The probe action, continued beyond the horizon, is found to take the form of the space-like brane action that we have been discussing so far.

The dynamics of the non-BPS D$(p+1)$-brane is governed by the following tachyon effective action[3] [2–5],

$$S = -\int d^{p+2}x\, V(T)\sqrt{-\det(\eta_{\mu\nu} + \partial_\mu T \partial_\nu T)}, \qquad (1)$$

where T is the tachyon field and $V(T)$ is its potential, with $V(-T) = V(T)$ and $V(T)$ has a maximum at $T = 0$, while $V(T) \to 0$ as $T \to \pm\infty$. $\mu, \nu = 0, 1, \ldots, p+1$ are the world-volume indices. We have set the world-volume gauge fields and the transverse scalars to zero for simplicity and will include them later. Also, we assume that classically the tachyon is dependent on the time coordinate x^0. The equation of motion following from (1) is,

$$\partial_0\left[\frac{V(T)\partial_0 T}{\sqrt{1-(\partial_0 T)^2}}\right] + V'(T)\sqrt{1-(\partial_0 T)^2} = 0. \qquad (2)$$

Here 'prime' denotes the derivative of the function with respect to its argument. Now instead of solving this equation directly if we Wick rotate $x^0 \to i\tau$, then in terms of τ coordinate (2) can be rewritten as

$$\partial_\tau\left[\frac{V(T)\partial_\tau T}{\sqrt{1+(\partial_\tau T)^2}}\right] - V'(T)\sqrt{1+(\partial_\tau T)^2} = 0. \qquad (3)$$

The solution for this equation has the form $T = f(a\tau)$, where a is a parameter which will be taken to infinity at the end. The function f satisfies $f(-u) = -f(u)$, $f(\pm\infty) = \pm\infty$ and $f'(u) > 0$ for all u, otherwise it is an arbitrary function. This solution is obtained by Sen [7] and is used to derive the codimension one BPS D-brane action as the space-like tachyonic kink solution of non-BPS D-brane action. Now we note that if $f(a\tau)$ is a solution to Eq. (3), then $f(-iax^0)$

[1] Type II* string theory is related to type II string theory by a time-like T-duality [11]. So, for example, type IIA (type IIB) string theory compactified on a time-like circle of radius R is dual to type IIB* (type IIA*) string theory compactified on a dual time-like circle of radius $1/R$. Space-like p-branes in type II theory are the Sp-branes and the space-like branes in type II* theory are the Ep-branes.

[2] Actually here we are considering space-like p-brane solutions without a time reversal symmetry. It is also possible to consider solutions having a time reversal symmetry. A world-sheet construction for a particular boundary interaction (corresponding to $\lambda = 1/2$) [8] describing the tachyon on an unstable D-brane reveals that there is an array of space-like p-branes situated on the imaginary time axis at $x^0 = im\pi$, for odd integers m. In real time this has been interpreted as closed string radiation [12,17,18].

[3] We are using the convention where $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, \ldots, 1)$ and $\alpha' = 1$.

is a solution to Eq. (2). We point out that for real x^0 although $f(-iax^0)$ is a solution to Eq. (2), it is unphysical as it does not satisfy the proper boundary condition of the time-dependent tachyon. Namely, from the conservation of energy momentum tensor following from (1), $\partial_0 T_{00} = 0$, where

$$T_{00} = \frac{V(T)}{\sqrt{1-(\partial_0 T)^2}} \tag{4}$$

we find $\partial_0 T \to 1$ as $x^0 \to \pm\infty$ and $\partial_0 T = 0$ at $x^0 = 0$, i.e., at the top of the potential where the tachyon starts rolling. However, we can still formally use the solution $T = f(-iax^0)$ and mention how we can finally obtain the action in real time.

Now we consider the dynamics of the translational zero mode along x^0 direction and it corresponds to the fluctuation of the tachyon as,

$$T\left(x^0, \xi^\alpha\right) = f\left(-ia\left(x^0 - X^0\left(\xi^\alpha\right)\right)\right), \tag{5}$$

where ξ^α, with $\alpha = 1, \ldots, p+1$ are the world-volume coordinates excluding time and $X^0(\xi^\alpha)$ is a scalar living in $(p+1)$-dimensional Euclidean world-volume associated with the translational zero mode of the time-like kink along x^0. Using (5) we find,

$$\begin{aligned}
&\sqrt{-\det(\eta_{\mu\nu} + \partial_\mu T \partial_\nu T)} \\
&\quad = \sqrt{1 + \eta_{\mu\nu}\partial_\mu T \partial_\nu T} \\
&\quad = \left[1 + a^2 f'^2\left(1 - \delta^{\alpha\beta}\partial_\alpha X^0 \partial_\beta X^0\right)\right]^{1/2}.
\end{aligned} \tag{6}$$

Now substituting (6) into (1) we find for $a \to \infty$,

$$\begin{aligned}
S &= -\int dx^0 \int d^{p+1}\xi\, V(f) a f' \\
&\quad \times \left[1 - \delta^{\alpha\beta}\partial_\alpha X^0 \partial_\beta X^0\right]^{1/2} \\
&= -i \int V(y)\, dy \int d^{p+1}\xi\, \sqrt{\det\left(\delta_{\alpha\beta} - \partial_0 X^0 \partial_\beta X^0\right)}.
\end{aligned} \tag{7}$$

In writing the second line in (7), we have made a change of variable $f(-ia(x^0 - X^0(\xi^\alpha))) = y$. We notice that the integral $-i\int V(y)\,dy$ is nothing but the action S per unit $(p+1)$-dimensional Euclidean world-volume with the tachyon taking its classical value $T_{\text{cl}} = f(-iax^0)$. However, in real time the integral does not make sense as we have mentioned before and we have to really 'undo' the effect of Wick rotation of x^0 coordinate by replacing $f(-iax^0) \to T_{\text{cl}}(x^0)$. So, we have to replace,

$$\begin{aligned}
-i\int V(y)\, dy &= -\int dx^0\, V(f) a f' \\
&= -\int dx^0\, V(f)\sqrt{1 - (-ia)^2 f'^2} \\
&\to -\int dx^0\, V(T_{\text{cl}})\sqrt{1 - (\partial_0 T_{\text{cl}})^2} \equiv S_0.
\end{aligned} \tag{8}$$

So, in real time the action would take the form,

$$S = S_0 \int d^{p+1}\xi\, \sqrt{\det\left(\delta_{\alpha\beta} - \partial_\alpha X^0 \partial_\beta X^0\right)}, \tag{9}$$

where $S_0 = -\int dx^0\, V(T_{\text{cl}})\sqrt{1 - (\partial_0 T_{\text{cl}})^2}$. Since the expression inside the square root in (9) is already expressed in real scalar X^0, this is the DBI action of the space-like p-brane. Note that the kinetic term of the scalar X^0 has a wrong sign since this is the zero mode associated with the time translation.

Following Sen [7], it is not difficult to include world-volume gauge fields and other transverse scalars into the action and we will use the same procedure as discussed above. The action now takes the form,

$$S = -\int d^{p+2}x\, V(T)\sqrt{-\det(a_{\mu\nu})}, \tag{10}$$

where

$$\begin{aligned}
&a_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu T \partial_\nu T + \partial_\mu x^I \partial_\nu x^I + f_{\mu\nu} \quad \text{with} \\
&f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu.
\end{aligned} \tag{11}$$

Here, $\mu, \nu = 0, 1, \ldots, p+1$ are the world-volume indices of non-BPS D$(p+1)$-brane and $I = p+2, \ldots, 9$ are the transverse space indices. a_μ is the world-volume gauge field and x^I are the scalars corresponding to the transverse coordinates. We assume for simplicity that classically both the gauge fields and the scalars on the world volume vanish and the fluctuations of various fields take the forms,

$$\begin{aligned}
&T\left(x^0, \xi^\alpha\right) = f\left(-ia\left(x^0 - X^0\left(\xi^\alpha\right)\right)\right), \\
&a_0\left(x^0, \xi^\alpha\right) = 0, \qquad a_\alpha\left(x^0, \xi^\alpha\right) = A_\alpha\left(\xi^\alpha\right), \\
&x^I\left(x^0, \xi^\alpha\right) = X^I\left(\xi^\alpha\right).
\end{aligned} \tag{12}$$

Note from above that we are assuming that the $(p+2)$-dimensional fields a_μ and x^I do not depend on time x^0 and the fluctuations away from the time-like tachyonic kink are arbitrary. This makes sense for the Wick rotated (or Euclideanized) theory and the justification can be found in Ref. [7]. Now for this field configuration we can compute various components of $a_{\mu\nu}$ as

$$a_{00} = -1 - a^2 f'^2, \qquad a_{0\alpha} = a_{\alpha 0} = a^2 f'^2 \partial_\alpha X^0,$$
$$a_{\alpha\beta} = \left(1 - a^2 f'^2\right)\partial_\alpha X^0 \partial_\beta X^0 + A_{\alpha\beta}, \quad \text{where}$$
$$A_{\alpha\beta} = \delta_{\alpha\beta} - \partial_\alpha X^0 \partial_\beta X^0 + \partial_\alpha X^I \partial_\beta X^I + F_{\alpha\beta} \tag{13}$$

with $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$. Now using (13) we find,

$$\sqrt{-\det(a_{\mu\nu})} = af'\sqrt{\det(A_{\alpha\beta})} \quad \text{for } a \to \infty. \tag{14}$$

Substituting (14) into the action (10) we get,

$$S = -\int dx^0 \int d^{p+1}\xi\, V(f) af' \sqrt{\det(A_{\alpha\beta})}$$
$$= -i \int V(y)\, dy \int d^{p+1}\xi \sqrt{\det(A_{\alpha\beta})}, \tag{15}$$

where $A_{\alpha\beta}$ is given in Eq. (13). Again in the last line we have introduced the variable $y = f(-ia(x^0 - X^0(\xi^\alpha)))$. As before we can write the action in real time by replacing

$$-i \int V(y)\, dy$$
$$\to -\int dx^0\, V(T_{\mathrm{cl}})\sqrt{1 - (\partial_0 T_{\mathrm{cl}})^2} \equiv S_0, \tag{16}$$

where S_0 is the action per unit $(p+1)$-dimensional Euclidean volume evaluated with the classical values of the fields. Therefore, we get

$$S = S_0 \int d^{p+1}\xi \sqrt{\det(A_{\alpha\beta})}. \tag{17}$$

This is the form of the DBI part of the space-like p-brane action. We note that the first three terms in $A_{\alpha\beta}$ given in (13), i.e., $\delta_{\alpha\beta} - \partial_\alpha X^0 \partial_\beta X^0 + \partial_\alpha X^I \partial_\beta X^I$ is the pull-back of the space–time metric on the $(p+1)$-dimensional Euclidean world-volume of the space-like p-brane in the static gauge and so, the DBI action really has the form

$$S = S_0 \int d^{p+1}\xi \sqrt{\det(g_{\alpha\beta} + F_{\alpha\beta})}. \tag{18}$$

Taking into account the closed string background in the NSNS sector the action would take the form,

$$S = S_0 \int d^{p+1}\xi\, e^{-\phi}\sqrt{\det(g_{\alpha\beta} + B_{\alpha\beta} + F_{\alpha\beta})}, \tag{19}$$

where ϕ is the dilaton, $g_{\alpha\beta}$ and $B_{\alpha\beta}$ are respectively the pull-backs of the space–time metric and the anti-symmetric tensor to the world-volume of the space-like p-brane.

Apart from the DBI part the Sp-brane (or Ep-brane) action should also contain a Wess–Zumino term. The Wess–Zumino term of a non-BPS D$(p+1)$-brane has the form,

$$S_{\mathrm{WZ}} = \int W(T)\, dT \wedge c \wedge e^f, \tag{20}$$

where $W(T)$ is an even function of T, which vanishes as $T \to \pm\infty$. $f = f_{\mu\nu}\, dx^\mu \wedge dx^\nu$ and $c = \sum_{q\geqslant 0} c^{(p+1-2q)}$, where $c^{(p+1-2q)}$ are the pull-backs of the RR $(p+1-2q)$-form fields on to the world-volume. Note that here we are considering only the bosonic sector and in the absence of the RR background the WZ term would vanish.[4] Now to evaluate (20) we first find,

$$f = f_{\mu\nu}\, dx^\mu \wedge dx^\nu$$
$$= 2 f_{0\alpha}\, dx^0 \wedge d\xi^\alpha + F_{\alpha\beta}\, d\xi^\alpha \wedge d\xi^\beta. \tag{21}$$

For the fluctuation (12) the above simply reduces to F. Now since $dT = -iaf'\, du$, where $u = x^0 - X^0(\xi^\alpha)$, we can write

$$c = \sum_m c^{(m)}$$
$$= \sum_m \left(m c^{(m)}_{0\alpha_2\cdots\alpha_m} \partial_{\alpha_1} X^0 + c^{(m)}_{\alpha_1\cdots\alpha_m}\right)$$
$$\times\, d\xi^{\alpha_1} \wedge \cdots \wedge d\xi^{\alpha_m}. \tag{22}$$

The term in the r.h.s. of (22) is the pull-back of an m-form onto the $(p+1)$-dimensional Euclidean world-volume of the space-like p-brane and can be identified with $C^{(m)}$. Using these relations the WZ term in (20) simplifies to

$$S_{\mathrm{WZ}} = \int W(f)(-iaf')\, du \wedge C \wedge e^F$$
$$= \int W(y)\, dy \int C \wedge e^F, \tag{23}$$

where in writing the last expression we have introduced a new variable $y = f(-iau)$. If we now identify

[4] In fact if we include the fermionic sector c also has a contribution from the fermions and so the WZ term would be non-vanishing even if the RR background is zero [4].

$\int W(y)\,dy = \int V(y)\,dy$ in analogy with the space-like tachyonic kink solution then

$$S_{\mathrm{WZ}} = i\left(-i\int V(y)\,dy\right)\int C\wedge e^{F}. \tag{24}$$

Now as argued before the integral in the parenthesis makes sense only in the Wick rotated theory, i.e., when x^0 is imaginary. However, if we want to write it in real x^0 the integral $-i\int V(y)\,dy$ should be replaced by $S_0 = -\int dx^0\, V(T_{\mathrm{cl}})\sqrt{1-(\partial_0 T_{\mathrm{cl}})^2}$. Also if we include the closed string NSNS field B, then F should be replaced by $F+B$, where $B_{\alpha\beta}$ should be the pull-back of the space–time field onto the world-volume of space-like p-brane. So, the full space-like p-brane action containing the DBI part and the WZ part is given by

$$\begin{aligned}
&S_{\mathrm{DBI}} + S_{\mathrm{WZ}} \\
&\quad = S_0\int d^{p+1}\xi\, e^{-\phi}\sqrt{\det(g_{\alpha\beta}+B_{\alpha\beta}+F_{\alpha\beta})} \\
&\qquad + iS_0\int C\wedge e^{F+B}.
\end{aligned} \tag{25}$$

Thus we find that in type II theory where the RR form fields Cs are real, the above action is complex. So, the Sp-brane action obtained this way has a complex structure. Note that the action in (25) differs from the usual BPS D-brane action by an overall factor of i and therefore the equations of motion remain the same. This is important for their solutions to give a consistent background preserving the conformal symmetry of the open string world-sheet. On the other hand, if we interpret the action (25) as that of an Ep-brane in type II* theory then the RR form fields C is replaced by $C\to C' = -iC$ [11] and the action is real. The DBI part of the above action matches exactly with the Sp-brane action obtained earlier [19] using different method. However, since in that derivation the closed string background was not taken into account, the WZ term was absent and it was not clear whether the action really corresponded to an Sp-brane or an Ep-brane action. Here, we observe that if we insist on the reality of the full action, then the action in Ref. [19] should be considered as an Ep-brane action.

To further illustrate the possible nature of the world-volume action of the previously discussed time-dependent configurations, let us consider the following scenario. Consider the static black p-brane solutions of type II supergravities. These solutions are typically singular and the singularity is hidden behind the horizon. Due to the presence of the horizon, one can associate a non-zero temperature with these solutions. As a result, they completely break the original supersymmetry of the type II theories. It is well known [20] that, as one crosses the horizon of such a solution, the role of time and space gets interchanged. Consequently, a static metric turns into a time-dependent metric as long as we restrict our attention inside the horizon. This time-dependent metric is typically the S-brane metric that we have been discussing so far. In the following, we would like to consider Dp-brane probes in such a p-brane background. As we will see, the probe brane action can be interpreted as a space-like p-brane action once it crosses the horizon of the background geometry. However, the action turns out to be complex as before. To get a real action, one needs to make the corresponding RR form of the corresponding Dp-brane purely imaginary. This, in turn, turns the background to a solution of a type II* theory along with the probe action to the one of Ep-brane action. In the following, we discuss this in some detail.

The static non-extremal p-brane solutions of our interest are given in [21]. In D space–time dimension, they have the form

$$\begin{aligned}
ds^2 &= e^{2A}\left(-e^{2f}\left(dx^0\right)^2 + dx^i\,dx^i\right) \\
&\quad + e^{2B}\left(e^{-2f}\,dr^2 + r^2\,d\Omega^2\right),
\end{aligned} \tag{26}$$

where (x^0, x^i) parametrize the $(p+1)$-dimensional world-volume of the p-brane. The coordinates transverse to the brane are r and the $(D-p-2)$ coordinates on the unit sphere $d\Omega$. The functional form of A, B and f can be found in [22]. For our purpose, we only need to know that they depend solely on the radial coordinate r. This solution has a singularity at $r=0$. Furthermore, e^{2f} becomes zero at finite non-zero value of r. The location represents the horizon. Beside the metric, there is a non-trivial dilaton ϕ which also is a function of r. The other non-trivial field is a form field whose field strength is given by $F = \lambda\epsilon_{D-p-2}$. Here ϵ_{D-p-2} is the volume form on the unit sphere $d\Omega$. The solution (26) then corresponds to a solitonic p-brane with magnetic charge λ.

As discussed in [22], the metric in (26) becomes a time-dependent one once we consider interior region of the p-brane. Inside the horizon, the function

$e^{2f} \to -e^{2f}$. Consequently, as can be seen from (26), the time-like coordinate x^0 becomes space-like and the radial coordinate r acquires a new interpretation as the time-like coordinate. Renaming, therefore, the new space-like coordinate x^0 as z and the time-like coordinate r as τ, we get the metric as

$$ds^2 = e^{2A}\big(e^{2f}\,dz^2 + dx^i\,dx^i\big) - e^{2B-2f}\,d\tau^2 + e^{2B}\tau^2\,d\Omega^2, \tag{27}$$

where A, B and f are all now functions of time τ. The functional form of the dilaton field ϕ remains the same as before except that it now acquires a time dependence due to the replacement of r by τ. The above configuration can be identified as a space-like brane configuration with $(p+1)$-dimensional world-volume parametrized by (z, x^i). However, this brane is anisotropic on the world-volume due to the appearance of e^{2f} only infront of z coordinate. However, this will not be of importance on what we discuss in the following.

Let us now consider a probe brane of $(p+1)$-dimensional world-volume in the background geometry given in (26). The action of the probe brane will have a DBI part and the WZ part given by

$$S = -\mathcal{T}_{p+1}\int d^{p+1}\xi\, e^{-\phi}\sqrt{-\det(g_{\mu\nu})} + \mathcal{T}_{p+1}\int_{\mathcal{M}_{p+1}} C_{p+1}, \tag{28}$$

where μ, ν run over the $p+1$ world-volume coordinates $\xi^0, \xi^1, \ldots, \xi^p$ and C_{p+1} is the usual RR-form associated with the brane. In the above, the RR form is integrated over the $(p+1)$-dimensional world-volume. $\mathcal{T}_{p+1}$ is related to the tension of the brane. We will now explicitly evaluate the probe action (28) in the p-brane background given in (26). We will first consider the DBI part and later focus on the WZ part of the action. In the static gauge

$$\xi^0 = x^0 \quad \text{and} \quad \xi^i = x^i, \tag{29}$$

the components of the induced metric $g_{\mu\nu}$ take the form

$$\begin{aligned}
g_{00} &= -e^{2A+2f} + e^{2B-2f}\left(\frac{\partial r}{\partial \xi^0}\right)^2 + r^2 e^{2B}\frac{\partial\theta_a}{\partial\xi^0}\frac{\partial\theta^a}{\partial\xi^0},\\
g_{0i} &= e^{2B-2f}\frac{\partial r}{\partial\xi^0}\frac{\partial r}{\partial\xi^i} + r^2 e^{2B}\frac{\partial\theta_a}{\partial\xi^0}\frac{\partial\theta^a}{\partial\xi^i},\\
g_{ii} &= e^{2A} + e^{2B-2f}\left(\frac{\partial r}{\partial\xi^i}\right)^2 + r^2 e^{2B}\frac{\partial\theta_a}{\partial\xi^i}\frac{\partial\theta^a}{\partial\xi^i},\\
g_{ij} &= e^{2B-2f}\frac{\partial r}{\partial\xi^i}\frac{\partial r}{\partial\xi^j} + r^2 e^{2B}\frac{\partial\theta_a}{\partial\xi^i}\frac{\partial\theta^a}{\partial\xi^j},
\end{aligned} \tag{30}$$

where θ^a are the coordinates on $d\Omega$. Now, we would like to continue the probe action beyond the horizon, which occurs at the point where $e^{2f} = 0$. As discussed earlier, this is done by substituting

$$e^{2f} \to -e^{2f}, \quad x^0 \to z \quad \text{and} \quad r \to \tau \tag{31}$$

in (26). However, in order to maintain the previous gauge choice (29), we now need to have the following identifications of world-volume coordinates:

$$\xi^0 \to \xi^{p+1}, \quad \text{with}\ \xi^{p+1} = z,\ \xi^i = x^i, \tag{32}$$

so that the induced metric components on the world-volume are now

$$\begin{aligned}
g_{p+1,p+1} &= e^{2A+2f} - e^{2B-2f}\left(\frac{\partial\tau}{\partial\xi^{p+1}}\right)^2 + \tau^2 e^{2B}\frac{\partial\theta_a}{\partial\xi^{p+1}}\frac{\partial\theta^a}{\partial\xi^{p+1}},\\
g_{p+1,i} &= -e^{2B-2f}\frac{\partial\tau}{\partial\xi^{p+1}}\frac{\partial\tau}{\partial\xi^i} + \tau^2 e^{2B}\frac{\partial\theta_a}{\partial\xi^{p+1}}\frac{\partial\theta^a}{\partial\xi^i},\\
g_{ii} &= e^{2A} - e^{2B-2f}\left(\frac{\partial\tau}{\partial\xi^i}\right)^2 + \tau^2 e^{2B}\frac{\partial\theta_a}{\partial\xi^i}\frac{\partial\theta^a}{\partial\xi^i},\\
g_{ij} &= -e^{2B-2f}\frac{\partial\tau}{\partial\xi^i}\frac{\partial\tau}{\partial\xi^j} + \tau^2 e^{2B}\frac{\partial\theta_a}{\partial\xi^i}\frac{\partial\theta^a}{\partial\xi^j}.
\end{aligned} \tag{33}$$

It can now easily be checked that the above components follow from the probe action

$$-i\mathcal{T}_{p+1}\int d^{p+1}\xi\, e^{-\phi}\sqrt{\det(g_{\alpha\beta})} \tag{34}$$

when evaluated on the background (27). Here, the indices α, β run over the Euclidean world-volume coordinates $\xi^1, \ldots, \xi^{p+1}$. This is precisely the form of space-like brane action as obtained before and also suggested in [19] if we identify $-i\mathcal{T}_{p+1} = S_0$, where S_0 is the action given before in (8). Note that with this identification we encounter two puzzles. First of all, S_0 as given in (8) is real, whereas $-i\mathcal{T}_{p+1}$ appears to be purely imaginary[5] since $\mathcal{T}_{p+1}$ is related to the tension of the brane given in (28). Secondly, S_0 is the action per unit $(p+1)$-dimensional Euclidean volume of a non-BPS brane with the tachyon and other fields taking their classical values, but in this derivation tachyon does not appear explicitly. The resolution of the puzzles can be understood as follows. Note that the action (28) can also be regarded as a non-BPS D$(p+1)$-brane action on its space-like tachyonic kink where the tachyon as well as the other backgrounds depend only on the brane direction $x^{p+1} = x$ (say). So, $\mathcal{T}_{p+1} = \int V(y)\,dy$ (as mentioned before) with $y = T = f(ax)$, with $a \to \infty$ and the function f is as defined after Eq. (3). Now as the probe brane is taken inside the horizon, $x \to \tau$ and $x^0 \to z$, i.e., the space-like coordinate x becomes time-like and so, $f(ax) \to f(a\tau)$, but it is no longer a solution to the tachyon equation of motion. It will be a solution if we Euclideanize τ, i.e., $f(-ia\tau)$ will be a solution. This is exactly the solution we used before. So, the factor in front of the DBI part of the action becomes $-i\mathcal{T}_{p+1} = -i\int V(y)\,dy$ and this when continued to the real time gives S_0 (see Eq. (8)).

Let us now look at the WZ part. We note that under (31) and (32), the WZ term in (28) takes the following form

$$\mathcal{T}_{p+1} \int_{\mathcal{M}_{p+1}} C_{p+1} \to \mathcal{T}_{p+1} \int_{\tilde{\mathcal{M}}_{p+1}} C_{p+1}. \tag{35}$$

Here, $\mathcal{M}_{p+1}$ is the world-volume of the static brane with coordinates $\xi^0, \xi^1, \ldots, \xi^p$ where as $\tilde{\mathcal{M}}_{p+1}$ is the world-volume of the space-like brane with coordinates $\xi^1, \ldots, \xi^p, \xi^{p+1}$. Here again we identify $\mathcal{T}_{p+1} = iS_0$ and consequently, the total world-volume action of the time-dependent configuration given in (27) is the sum of (34) and (35) exactly the same form as obtained earlier in (25). The action is clearly complex unless we make the RR field imaginary $C_{p+1} \to -iC_{p+1}$. As discussed before, under such a transformation, the solution (27) really corresponds to an Ep-brane solution of type II* theory.

[5] We would like to point out that here we are assuming that $\mathcal{T}_{p+1}$ will remain real as we interchange $r \to \tau$ and $x^0 \to z$, but this is not really true as we have seen in our earlier discussion.

To conclude, we have given a simple derivation of a space-like brane action including the DBI and the WZ parts starting from the non-BPS D-brane action. We have used the time-like tachyonic kink solution of the tachyon effective action describing the dynamics of the non-BPS D-brane by Wick rotation of the space-like tachyonic kink solution. The fluctuations of various bosonic fields on the Euclidean world-volume around this solution produce an effective action. In real time, this describes the low energy effective action of a space-like brane. We notice that if we start from type II theory where the closed string background has real RR form fields then the action is complex and it is the action of an Sp-brane. But, if we start from type II* theory where the RR form is still real but differ from those of type II theory by a factor of i then the action is real and it represents the action of an Ep-brane. We pointed out that by just studying the DBI part we can not conclude whether it represents the action of an Sp-brane or an Ep-brane. To further support the correctness of the form of the action we have also given an alternative argument.

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