Magnetic flux inversion in charged BPS vortices in a Lorentz-violating Maxwell–Higgs framework

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A B S T R A C T

We demonstrate for the first time the existence of electrically charged BPS vortices in a Maxwell–Higgs model supplemented with a parity-odd Lorentz-violating (LV) structure belonging to the CPT-even gauge sector of the standard model extension and a fourth order potential (in the absence of the Chern–Simons term). The modified first order BPS equations provide charged vortex configurations endowed with some interesting features: localized and controllable spatial thickness, integer flux quantization, electric field inversion and localized magnetic flux reversion. This model could possibly be applied on condensed matter systems which support charged vortices carrying integer quantized magnetic flux, endowed with localized flipping of the magnetic flux.

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Since the seminal works by Abrikosov [1] and Nielsen and Olesen [2] showing the existence of electrically neutral vortices in type-II superconducting systems and in field theory models, respectively, these nonperturbative solutions have been a theoretical issue of enduring interest. In the beginning of 90s, vortices solutions were studied in the context of planar theories including the Chern–Simons term [3] which turned possible the attaining of electrically charged vortices [4] also supporting BPS (Bogomol'nyi, Prasad, Sommerfield) solutions [5], related with the physics of anyons and the fractional quantum Hall effect [6]. In addition, charged BPS vortices also were found in the Maxwell–Chern–Simons model [7]. Further studies were performed involving nonminimal coupling [8] and new developments [9,10]. Generalized Chern–Simons vortex solutions were recently examined in the presence of noncanonical kinetic terms (high order derivative terms) [11] usually defined in the context of k-field theories [12–14]. These k-defects present a compact-like support [15], an issue of great interest currently [16]. Lately, in the context of effective field theories, it has also been demonstrated the existence of charged BPS vortices in a generalized Maxwell–Chern–Simons–Higgs model [17].

Lorentz-violating (LV) theories have been under attention in the latest years. The general theoretical framework for studying Lorentz-violation effects is the standard model extension (SME) [18] which encompasses Lorentz-violating terms in all sectors of the minimal standard model. In particular, the abelian gauge sector of this model is composed of two sectors, a CPT-odd and a CPT-even one. The CPT-even part is described by nineteen parameters enclosed in the rank 4 tensor, $(K_F)^{\mu\alpha\nu\beta}$, endowed with a double null trace and the symmetries of the Riemann tensor [19], being investigated in many respects [20]. Lorentz violation was also considered in connection with the formation of topological defects [21,22], with particular interest in the Higgs sector [23]. Recently, it has been investigated the formation of stable uncharged vortices in the context of the nonbirefringent Lorentz-violating and CPT-even Maxwell–Higgs electrodynamics, also including LV terms in the abelian Higgs sector of the SME, with new interesting results [24].

Up the moment it is known that abelian charged vortices are only defined in models endowed with the Chern–Simons term. This remains valid even in the context of highly nonlinear models, such as the Born–Infeld electrodynamics [25]. In this Letter we report for the first time the existence of abelian charged BPS vortices in a Maxwell–Higgs electrodynamics deprived of the Chern–Simons term and endowed with CPT-even LV terms. This achievement is ascribed to the CPT-even electrodynamics of the SME, whose parity-odd coefficients entwine the electric and magnetic sectors [26,27] in analogy to what happens in the models containing the Chern–Simons term. The BPS solutions are attained by considering a particular fourth order potential, and can be interpreted as vortex solutions in a dielectric medium [28]. The charged vortex solutions are localized, having spatial thickness controlled by the LV parameter, and presenting localized magnetic flux and electric field reversion in some radial region. This phenomenon could

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be of interest in condensed matter physics, mainly in connection with superconductivity, particularly with two-components super-conducting systems [29].

1. The theoretical framework

The theoretical environment in which our investigations are developed is a modified Maxwell–Higgs model defined by the following Lagrangian density

$$\mathcal{L}_{1+3} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\kappa^{\mu\nu}F_{\mu\rho}F_\nu{}^\rho + |D_\mu\phi|^2$$
$$+ \frac{1}{2}(1+\kappa_{00})\partial_\mu\Psi\partial^\mu\Psi + \frac{1}{2}\kappa^{\mu\nu}\partial_\mu\Psi\partial_\nu\Psi$$
$$- e^2\Psi^2|\phi|^2 - U(|\phi|,\Psi), \tag{1}$$

containing a convenient potential $U(|\phi|,\Psi)$. The two first terms in (1) define the nonbirefringent and CPT-even electrodynamics of the SME, whose nine LV nonbirefringent parameters are enclosed in the symmetric and traceless tensor, $\kappa^{\mu\nu}$, defined as

$$\kappa^{\mu\nu} = (K_F)^{\mu\alpha\nu}{}_\alpha, \tag{2}$$

where $(K_F)^{\mu\alpha\nu\beta}$ is the CPT-even gauge sector of the SME, whose properties were much investigated since 2001 [19,20]. The Higgs field, ϕ, is coupled to the gauge sector by the covariant derivative, $D_\mu\phi = \partial_\mu\phi - ieA_\mu\phi$. The neutral scalar field, Ψ, plays the role of an auxiliary scalar field, and is analogous to the one that appears in the planar Maxwell–Chern–Simons models endowed with charged BPS vortex configurations [7,10]. Note that the Lorentz-violating tensor, $\kappa^{\mu\nu}$, also modifies the kinetic term of the neutral field. The potential in Eq. (3) which assures the attainment of BPS first order equations is defined by

$$U(|\phi|,\Psi) = \frac{(ev^2 - e|\phi|^2 - \epsilon_{ij}\kappa_{0i}\partial_j\Psi)^2}{2(1-s)}. \tag{3}$$

Observe that it is a fourth order one. Here, v is the vacuum expectation value of the Higgs field, while $s = \mathrm{tr}(\kappa_{ij})$. The potential U possesses a nonsymmetric minimum, $\Psi = 0$, $|\phi| = v$, which is responsible for providing topological charged vortex configurations. We now analyze static solutions for the model projected in the xy-plane, where it recovers the structure of a $(1+2)$-dimensional theory. Remembering that in $(1+2)$-D it holds $F_{ij} = \epsilon_{ij}B$, $F_{0i} = E^i$ (the magnetic field becomes a scalar), we write the stationary Gauss's law of the planar model related to (1) by

$$L_{ij}\partial_i\partial_j A_0 + \epsilon_{ij}\kappa_{0i}\partial_j B = 2e^2 A_0|\phi|^2, \tag{4}$$

where

$$L_{ij} = (1+\kappa_{00})\delta_{ij} - \kappa_{ij}, \tag{5}$$

carries on the CPT-even and parity-even LV parameters. We note that is the parity-odd parameter κ_{0i} that couples the electric and magnetic sectors [26,27], making possible the existence of charged vortex configurations which are attained even in absence of the Chern–Simons term. Otherwise, for $\kappa_{0i} = 0$, the temporal gauge $A_0 = 0$ solves the Gauss's law, yielding compact-like uncharged vortex solutions [24].

2. BPS construction

In this section, we focus our attention on the development of a BPS framework [5] consistent with the second order differential equations obtained from the $(1+2)$-dimensional version of the Lagrangian (1). We begin writing the energy density $\mathcal{E}$ in the stationary regime as

$$\mathcal{E} = \frac{1}{2}(1-s)B^2 + U(|\phi|,\Psi) + |D_j\phi|^2$$
$$+ \frac{1}{2}L_{ij}(\partial_i A_0)(\partial_j A_0) + \frac{1}{2}L_{ij}(\partial_i\Psi)(\partial_j\Psi)$$
$$+ e^2 A_0^2|\phi|^2 + e^2\Psi^2|\phi|^2. \tag{6}$$

In order to attain the first order differential equations, we first impose the following condition on the neutral field Ψ:

$$\Psi = \mp A_0, \tag{7}$$

which is similar to the ones appearing in the BPS vortex configurations of the Maxwell–Chern–Simons model [9,10]. By substituting (7) in Eq. (6), we attain

$$\mathcal{E} = \frac{1}{2}(1-s)B^2 + \frac{(ev^2 - e|\phi|^2 \pm \epsilon_{ij}\kappa_{0i}\partial_j A_0)^2}{2(1-s)}$$
$$+ |D_j\phi|^2 + L_{ij}(\partial_i A_0)(\partial_j A_0) + 2e^2 A_0^2|\phi|^2. \tag{8}$$

After converting the two first terms in quadratic form and by using the identity

$$|D_j\phi|^2 = |D_\pm\phi|^2 \pm e|\phi|^2 B \pm \frac{1}{2}\epsilon_{ab}\partial_a J_b, \tag{9}$$

where J_b is the spatial component of the current $J^\mu = i[\phi(D^\mu\phi)^* - \phi^* D^\mu\phi]$, the energy density takes on the form

$$\mathcal{E} = \frac{1}{2}(1-s)\left[B \mp \frac{ev^2 - e|\phi|^2 \pm \epsilon_{ij}\kappa_{0i}\partial_j A_0}{(1-s)}\right]^2$$
$$+ |D_\pm\phi|^2 \pm ev^2 B \pm \frac{1}{2}\epsilon_{ab}\partial_a J_b$$
$$+ L_{ij}(\partial_i A_0)(\partial_j A_0) + B\epsilon_{ij}\kappa_{0i}\partial_j A_0 + 2e^2 A_0^2|\phi|^2. \tag{10}$$

Now, we use the Gauss's law to transform the last three terms in a total derivative, rewriting the energy density as

$$\mathcal{E} = \frac{1}{2}(1-s)\left[B \mp \frac{ev^2 - e|\phi|^2 \pm \epsilon_{ij}\kappa_{0i}\partial_j A_0}{(1-s)}\right]^2$$
$$+ |D_\pm\phi|^2 \pm ev^2 B + \partial_a\mathcal{J}_a, \tag{11}$$

where

$$\mathcal{J}_a = \pm\frac{1}{2}\epsilon_{ab}J_b + L_{ab}A_0\partial_b A_0 + \epsilon_{ba}\kappa_{0b}BA_0. \tag{12}$$

This energy density (11) is minimized by requiring that the squared terms be null, establishing the two BPS conditions of the model:

$$|D_\pm\phi| = 0, \tag{13}$$
$$B = \frac{\pm(ev^2 - e|\phi|^2) + \epsilon_{ij}\kappa_{0i}\partial_j A_0}{(1-s)}. \tag{14}$$

Under these BPS conditions, we attain the BPS energy density,

$$\mathcal{E}_{\mathrm{BPS}} = \pm ev^2 B + \partial_a\mathcal{J}_a, \tag{15}$$

implying the total BPS energy,

$$E_{\mathrm{BPS}} = \pm ev^2 \int d^2r\, B = \pm ev^2\Phi, \tag{16}$$

which is proportional to the magnetic flux. It is worthwhile to note that the second term in (15) does not contribute to the total BPS energy, once the fields go to zero at infinity.

3. Charged vortex configurations

Specifically, we look for radially symmetric solutions using the standard static vortex ansatz

$$A_\theta = -\frac{a(r) - n}{er}, \qquad \phi = v g(r) e^{in\theta}, \qquad A_0 = \omega(r), \qquad (17)$$

that allows to write the magnetic field as

$$B = -\frac{a'}{er}. \qquad (18)$$

The scalar functions $a(r)$, $g(r)$ and $\omega(r)$ are regular at $r = 0$ and $r = \infty$, satisfying the appropriate boundary conditions:

$$g(0) = 0, \qquad a(0) = n, \qquad \omega'(0) = cte, \qquad (19)$$

$$g(\infty) = 1, \qquad a(\infty) = 0, \qquad \omega(\infty) = 0, \qquad (20)$$

with n being the winding number of the vortex solution. The boundary conditions above will be demonstrated explicitly in the remain of the manuscript.

We now introduce the dimensionless variable $t = evr$ and implement the following changes:

$$g(r) \to \bar{g}(t), \qquad a(r) \to \bar{a}(t), \qquad \omega(r) \to v\bar{\omega}(t),$$
$$B \to ev^2 \bar{B}(t), \qquad \mathcal{E} \to v^2 \bar{\mathcal{E}}(t). \qquad (21)$$

Thereby, the BPS equations (13), (14) and the Gauss's law (4) are rewritten in a dimensionless form

$$\bar{g}' = \pm \frac{\bar{a}\bar{g}}{t}, \qquad (22)$$

$$\bar{B} = -\frac{\bar{a}'}{t} = \frac{\pm(1 - g^2) - \kappa\bar{\omega}'}{(1 - s)}, \qquad (23)$$

$$(1 + \lambda_r)\frac{(t\bar{\omega}')'}{t} - \kappa\frac{(t\bar{B})'}{t} - 2\bar{g}^2\bar{\omega} = 0, \qquad (24)$$

where $s = \mathrm{tr}(\kappa_{ij}) = \kappa_{rr} + \kappa_{\theta\theta}$ and we have defined $\kappa = \kappa_{0\theta}$ and $\lambda_r = \kappa_{00} - \kappa_{rr}$. Also, the signal $+$ corresponds to $n > 0$ and $-$ to $n < 0$. We can also observe from Eqs. (22)–(24) that under the change $\kappa \to -\kappa$, the solutions go as $\bar{g} \to \bar{g}$, $\bar{a} \to \bar{a}$, $\bar{\omega} \to -\bar{\omega}$.

We now discuss the magnetic flux quantization. We first rewrite the BPS energy density (15) in terms of the ansatz (17), that is,

$$\bar{\mathcal{E}}_{\mathrm{BPS}} = \pm\bar{B} \pm \frac{(\bar{a}\bar{g}^2)'}{t} + (1 + \lambda_r)\frac{(t\bar{\omega}\bar{\omega}')'}{t} - \kappa\frac{(t\bar{\omega}\bar{B})'}{t}. \qquad (25)$$

The first term is the magnetic field whose integration under boundary conditions (19), (20) gives the magnetic flux contribution to the BPS energy. The remaining three terms are total derivatives whose integration under boundary conditions (19), (20) gives null contribution to the total BPS energy. Thus,

$$\bar{E}_{\mathrm{BPS}} = \pm \int d^2t\, \bar{\mathcal{E}}_{\mathrm{BPS}} = \pm \int d^2t\, \bar{B}(t),$$
$$= \pm 2\pi \int_0^\infty dt\, t\left(-\frac{\bar{a}'}{t}\right) = \pm 2\pi n. \qquad (26)$$

This shows that the BPS vortex solutions present energy or magnetic flux quantization. Next, by using the BPS equations (22) and the Gauss's law (24), the BPS energy density (25) takes the suitable form

$$\bar{\mathcal{E}}_{\mathrm{BPS}} = (1 - s)\bar{B}^2 + 2\frac{\bar{a}^2\bar{g}^2}{t^2} + 2\bar{g}^2\bar{\omega}^2 + (1 + \lambda_r)(\bar{\omega}')^2, \qquad (27)$$

which is positive-definite for

$$s < 1, \qquad \lambda_r > -1. \qquad (28)$$

3.1. Asymptotic behavior

The asymptotic behavior for $t \to 0$ is obtained solving Eqs. (22)–(24) using power-series method. Thus, we attain

$$\bar{g}(t) = Gt^n + \cdots, \qquad (29)$$

$$\bar{a}(t) = n - \frac{1}{2}\frac{(1 + \lambda_r)}{(1 - s)(1 + \lambda_r) + \kappa^2}t^2 + \cdots, \qquad (30)$$

$$\bar{\omega}(t) = \omega_0 + \frac{\kappa}{(1 - s)(1 + \lambda_r) + \kappa^2}t + \cdots. \qquad (31)$$

From Eqs. (30) and (18), the magnetic field in the origin ($t = 0$) is given by

$$\bar{B}(0) = \frac{(1 + \lambda_r)}{(1 - s)(1 + \lambda_r) + \kappa^2}, \qquad (32)$$

while Eq. (31) yields the electric field $\bar{\omega}'$ at $t = 0$,

$$\bar{\omega}'(0) = \frac{\kappa}{(1 - s)(1 + \lambda_r) + \kappa^2}, \qquad (33)$$

which establishes the second boundary condition for the field $\bar{\omega}(t)$. We should note that the denominator in the last two equations is positive-definite due to the energy positivity conditions established in Eq. (28): $(1 - s)(1 + \lambda_r) + \kappa^2 > 0$. Hence, the physical fields are well defined in the origin whenever conditions (28) are satisfied.

In the sequel we study the asymptotic behavior for $t \to +\infty$, for which it holds $\bar{g} = 1 - \delta g_1$, $\bar{a} = \pm\delta a_1$, $\bar{\omega} = \pm\delta\omega_1$, with δg_1, δa_1, $\delta\omega_1$ being small corrections to be computed. After replacing such forms in Eqs. (22)–(24), and solving the linearized set of differential equations

$$\delta g_1 \sim t^{-1/2}e^{-\beta t} \sim \delta\omega_1, \qquad \delta a_1 \sim t^{1/2}e^{-\beta t}, \qquad (34)$$

where β is given as

$$\beta = \sqrt{\frac{2 + \lambda_r - s \pm \sqrt{(\lambda_r + s)^2 - 4\kappa^2}}{(1 - s)(1 + \lambda_r) + \kappa^2}}. \qquad (35)$$

Here, $(+)$ correspond to $\lambda_r + s > 0$ and $(-)$ to $\lambda_r + s < 0$, such that in the limit $\kappa = 0$ we get the asymptotic behavior for the BPS uncharged vortex, $\beta = \sqrt{2/(1 - s)}$, see Ref. [24]. We now analyze the β parameter. First, the condition $2 + \lambda_r - s > 0$ is guaranteed because of the energy positivity condition (28). The same holds for the denominator $(1 - s)(1 + \lambda_r) + \kappa^2 > 0$. On the other hand, the term inside the square root in the numerator, $(\lambda_r + s)^2 - 4\kappa^2$, is not definite-positive. Thus, we have a region $|\lambda_r + s| \geqslant 2\kappa$ where β is a positive real number, yielding an exponentially decaying asymptotic behavior. On the other hand, in the region $|\lambda_r + s| < 2\kappa$ the parameter β becomes a complex number with positive real part, which implies a sinusoidal behavior modulated by an exponentially decay factor.

3.2. Numerical solutions

We now analyze the case where β is a real number by setting $\lambda_r = 0$ and $s = 2\kappa$, so that $\beta = \sqrt{2/(1 - \kappa)}$. Consequently, the only free parameter is κ, whose values assuring a positive-definite energy density are $\kappa < 1/2$, in accordance with the condition (28).

In Figs. 1–5 we present some profiles (for the winding number $n = 1$) generated by numerical integration of Eqs. (22)–(24) using the Maple 13 libraries for solving the coupled nonlinear differential equations. In all figures the value $\kappa = 0$ reproduces the profile of the Maxwell–Higgs vortex [2] which is depicted by a solid black line. The legends given in Fig. 1 hold for all figures.

Figs. 1 and 2 depict the numerical results obtained for the Higgs field and vector potential, whose profiles are drawn around the
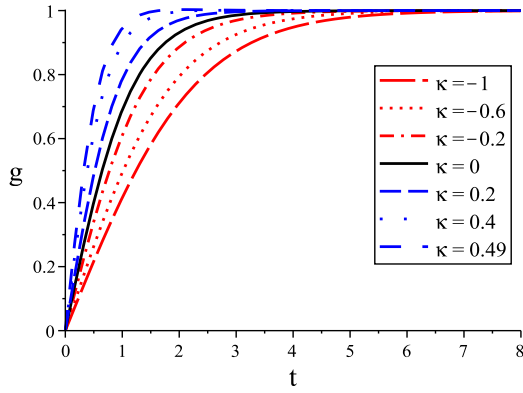
Fig. 1. Scalar field $\bar{g}(t)$ (solid black line, $\kappa = 0$, is the BPS solution for the Maxwell–Higgs model).
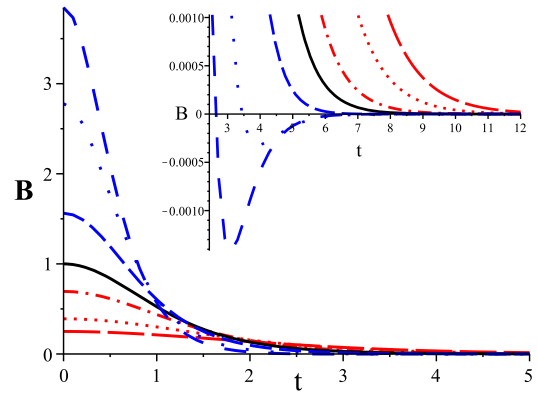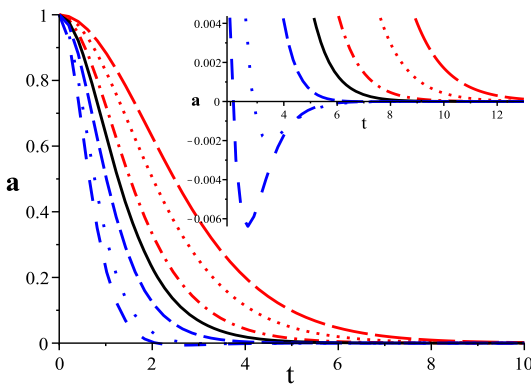

Fig. 2. Vector potential $\bar{a}(t)$.

Fig. 3. Magnetic field $\bar{B}(t)$.

Fig. 4. Electric field $\bar{\omega}'(t)$.

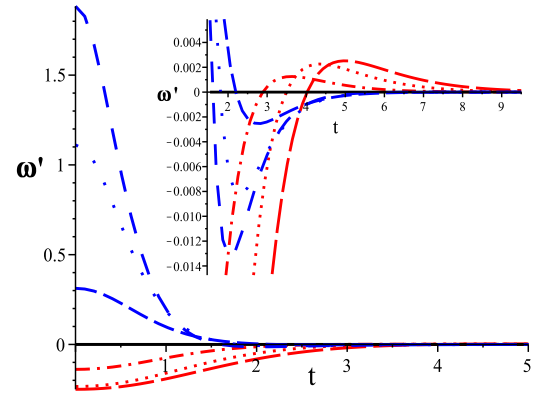**Fig. 5.** Energy density $\bar{\mathcal{E}}(t)$.

ones corresponding to the Maxwell–Higgs model. These profiles become wider and wider for $\kappa < 0$ and increasing $|\kappa|$, reaching more slowly the respective saturation region. Otherwise, for $0 < \kappa < 1/2$ it continuously shrinks approaching the minimum thickness when κ approaches to $1/2$. Moreover, the vector potential profile displays a novelty: for $0 < \kappa < 1/2$ it assumes negative values over a small region of the radial axis (see insertion in Fig. 2), which is obviously associated with a localized magnetic flux inversion. This inversion becomes more pronounced for κ values near to $1/2$. Also, the region presenting localized magnetic flux inversion is a little shifted to the origin when κ tends to $1/2$.

Fig. 3 depicts the magnetic field behavior. The profiles are lumps centered at the origin whose amplitudes are proportional to $(1 - \kappa)^{-2}$. For $\kappa < 0$ and increasing values of $|\kappa|$, the magnetic field profile becomes increasingly wider with continuously diminishing amplitude. For $0 < \kappa < 1/2$, the profile becomes narrower and higher for an increasing κ, reaching its maximum value for $\kappa = 1/2$. A close zoom on the profiles corresponding to κ closer to $1/2$ (see insertion in Fig. 3) reveals that the magnetic field flips its signal, showing explicitly localized magnetic flux reversion.

In a first view, Fig. 3 seems to show that the magnetic flux varies with the value of κ. Note, however, that the magnetic flux is calculated by integrating $2\pi t \bar{B}(t)$, as showed in Eq. (26). An explicit plot of the function $2\pi t \bar{B}(t)$ clearly shows that for every $\kappa < 1/2$ the magnetic flux is the same independently of the localized magnetic field reversion. This result is in accordance with the properties of the BPS vortex solutions.

Fig. 4 shows that electric field profiles also are lumps centered at the origin with amplitudes proportional to $\kappa/(1 - \kappa)^2$, having a minimum value for $\kappa = -1$ and maximal value for $\kappa = 1/2$. As it occurs with the magnetic field, the electric profiles become
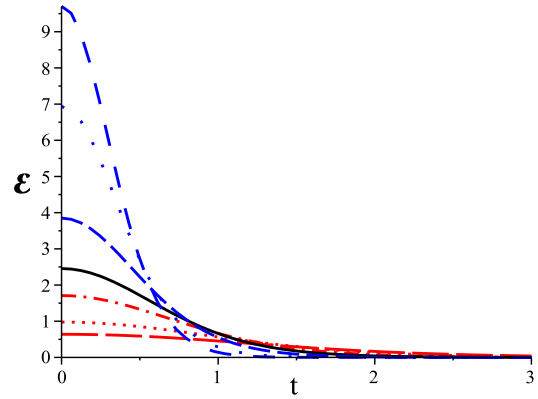
narrower and higher while κ increases tending to $1/2$. Now, the difference is that, for $\kappa < 0$, the profiles become negative (as predicted after BPS equations). A close zoom along the t-axis (see insertion in Fig. 4), however, reveals that the electric field undergoes inversion both for positive and negative values of κ. Such reversion is ubiquitous in all profiles.

The BPS energy density (27) is exhibited in Fig. 5, having profiles very similar to the magnetic field ones, being more localized and possessing a higher amplitude, however. As the BPS energy density is positive-definite, no inversion regions are observed, naturally.

By looking at the profiles of the BPS solutions it is observed that the spatial thickness is controlled by the Lorentz-violating parameter, allowing to obtain compact-like defects as in the un-

charged case analyzed in Ref. [24]. The present model is being regarded as an effective electrodynamics, which subjected to the usual vortex ansatz, provides vortex solutions in a dielectric continuum [28], as already mentioned in Ref. [24]. This interpretation allows to consider Lorentz-violating parameters with magnitude above the values stated by the known vacuum upper-bounds.

These charged vortex configurations are endowed with several interesting features, as space localization (exponentially decaying behavior), integer magnetic flux quantization, magnetic flux and electric field reversal. Specifically, the localized magnetic flux inversion is a very interesting phenomenon, with sensitive appeal in condensed matter superconducting systems. Recently, a magnetic inversion was reported in the context of fractional vortices in superconductors described the two-component Ginzburg–Landau (TCGL) model [29]. In it, the magnetic flux is fractionally quantized and delocalized, presenting a $1/r^4$ decaying behavior, and a subtle reversion. Such scenario, however, differs from the one described by our theoretical model, which provides a localized magnetic flux of controllable extent, exhibiting flux inversion just for the parameters that yields narrower (compact-like) profiles.

One should still note that the set of parameters here analyzed is only one of the many theoretical possibilities available. So, it can exist a set of parameters for which the magnetic flux might undergo a more accentuated reversion, reaching more appreciable flipping magnitudes. Such behavior could be associated with a complex β parameter, yielding oscillating solutions which become less localized (and more similar with the ones of Ref. [29]). Another interesting way is to investigate this theoretical system in the presence of Lorentz-violating terms in the Higgs sector. Such analysis are under progress with results being reported elsewhere.

Acknowledgements

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