Detection of Sub-surface Delamination based on the Spatio-temporal Gradient Analysis over the A0-mode Lamb wave Fields

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Abstract

This study proposes a novel method of local phase velocimetry based on the spatio-temporal gradient analysis. Structural flaws such as sub-surface delamination, corrosion and fatigue cracks represent changes in effective thickness and local properties of materials, and therefore, measurement of phase velocity changes in elastic wave propagation can be employed to assess the integrity of material structures. Due to the dispersive character of the A0-mode Lamb wave, the phase velocity of Lamb wave is varied depends on the thickness of the plate. Hence, a measure of the local phase velocity of the A0-Lamb wave yields a measure of the sub-surface delaminations of the plate. Theoretical analysis and several numerical experiments show the principle of the proposed method in this paper.

Keywords: spatio-temporal gradient analysis; guided wave; non-destructive inspection; ultrasonic testing; A0-mode Lamb wave; wave frontal analysis;

1. Introduction

Sub-surface delamination is a separation of adjacent sub-surface laminate without any obvious visual evidence on the surface. Most common failure in composite laminates is sub-surface delamination which is difficult to be detected and characterized. As the use of composite laminates has been increasing in structures, the location and identification of damage in such kind of materials is currently a topic of considerable interest and many techniques are being developed and applied in this area. Recent studies showed that the elastic guided wave have the ability to travel along the structures without loss of energy [1]. This remarkable characteristic of guided wave is advantageous for non-destructive inspection of structures. This paper focuses on traveling of A0-mode Lamb wave. The A0-mode Lamb wave is sensitive to the delamination at all through thickness locations[2]. In addition, A0-mode Lamb wave has shorter wavelength than that of S0-mode at the same frequency and that why it is potentially more sensitive to delamination damage. Because of this sensitivity, A0-mode Lamb wave measurement has been utilized as one of the

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promising structural health monitoring technique for detecting hidden damage in composites[3]. Beside this above merit, the variation of phase velocity cause difficulties in interpretation of observed signals. Therefore, it is important to establish the delamination detection criterion independent of local wave numbers.

The aim of this paper is to propose a local phase velocimetry based on the analysis of the spatio-temporal gradient constraint equation which governs the propagation of A0-mode Lamb wave. The spatio-temporal gradient analysis is based on the constraint equation relating the spatial gradient to the temporal derivative for a moving object. When the Lamb wave propagates across the delaminated region, the phase velocity of A0-mode Lamb wave is drastically varied. Therefore, the proposed local velocimetry has an ability to detect the delamination in the structure.

This paper is organized as follows. We simplify the problem of the measuring phase velocity of a propagating Lamb-wave in a plate including a sub-surface delamination in Sec.2. The details of the numerical investigations are given in Sec.3. Section 4 presents the results of experimental verification of the ability of the local phase velocimetry.

2. Problem Formulation

Assuming single non-directional narrow-band sound source at the origin on the plate, the normal displacement, \( f(x, y, t) \), caused by the A0-mode Lamb wave field can be approximated at the point, \((x, y)\) which exists on the surface in the far-field as follows:

\[
f(x, y, t) = \frac{1}{(x^2 + y^2)^{1/4}} w(t - \frac{\sqrt{x^2 + y^2}}{v(\omega_0)}).
\]  

Here, \( w(t) \) denotes the narrow-band transmitted signal:

\[
w(t) = \int_{\omega_0}^{\omega_0 + \Delta \omega} A(\omega) e^{j\omega t} d\omega.
\]

The phase velocity of the Lamb-wave, \( v(\omega_0) \), is assumed to be constant within the band, \((\omega_0 - \Delta \omega, \omega_0 + \Delta \omega)\), where \( \omega_0 \) is the center frequency of the band. The above far-field condition is defined as follows:

\[
\frac{\omega_0 \sqrt{x^2 + y^2}}{v(\omega_0)} \gg 1.
\]

The vertical component of the particle velocity at \((x, y)\) can be obtained by differentiating \( f(x, y, t) \) for time as:

\[
f_y(x, y, t) = \frac{\partial}{\partial t} f(x, y, t) = \frac{1}{(x^2 + y^2)^{1/4}} \ddot{w}(t - \frac{\sqrt{x^2 + y^2}}{v(\omega_0)}).
\]

Here, \( \ddot{w}(t) \) is the time-differential of \( w(t) \) denoted as:

\[
\ddot{w}(t) = \int_{\omega_0 - \Delta \omega}^{\omega_0 + \Delta \omega} j\omega A(\omega) e^{j\omega t} d\omega.
\]

Orthogonal pair of out-of-plane shear strains are derived by spatial differentiation as follows:

\[
\frac{\partial}{\partial x} f(x, y, t) = -\frac{x}{v(\omega_0)}(x^2 + y^2)^{3/4} \ddot{w}(t - \frac{\sqrt{x^2 + y^2}}{v(\omega_0)}) - \frac{x}{2(x^2 + y^2)^{5/4}} w(t - \frac{\sqrt{x^2 + y^2}}{v(\omega_0)})
\]

\[
\frac{\partial}{\partial y} f(x, y, t) = -\frac{y}{v(\omega_0)}(x^2 + y^2)^{3/4} \ddot{w}(t - \frac{\sqrt{x^2 + y^2}}{v(\omega_0)}) - \frac{y}{2(x^2 + y^2)^{5/4}} w(t - \frac{\sqrt{x^2 + y^2}}{v(\omega_0)})
\]

Substituting the far-field condition, eq.(3), into (6) and (7), we can approximate the pair of shearing strains as follows:

\[
f_x(x, y, t) = -\frac{x}{v(\omega_0)}(x^2 + y^2)^{3/4} \ddot{w}(t - \frac{\sqrt{x^2 + y^2}}{v(\omega_0)})
\]

\[
f_y(x, y, t) = -\frac{y}{v(\omega_0)}(x^2 + y^2)^{3/4} \ddot{w}(t - \frac{\sqrt{x^2 + y^2}}{v(\omega_0)})
\]
Therefore, when unique plane wave propagates on the plate, it is clear from the above equations (8) and (9), that the orthogonal pair of out-of-plane strains are linearly dependent on the corresponding normal-particle velocities, which is denoted in eq.(4).

Focusing on the auto and cross correlations for temporal and spatial gradients, following relations are obtained:

\[
\phi_{tt}(x,y) = \lim_{T \to \infty} \frac{1}{T} \int f_t(x,y,t) f_t^*(x,y,t) \, dt,
\]

\[
\phi_{xx}(x,y) = \lim_{T \to \infty} \frac{1}{T} \int f_x(x,y,t) f_x^*(x,y,t) \, dt = \frac{x^2}{v^2(\omega_0)(x^2 + y^2)} \phi_{tt}(x,y),
\]

\[
\phi_{yy}(x,y) = \lim_{T \to \infty} \frac{1}{T} \int f_y(x,y,t) f_y^*(x,y,t) \, dt = \frac{y^2}{v^2(\omega_0)(x^2 + y^2)} \phi_{tt}(x,y),
\]

\[
\phi_{xt}(x,y) = \lim_{T \to \infty} \frac{1}{T} \int f_x(x,y,t) f_t^*(x,y,t) \, dt = \frac{-x}{v(\omega_0) \sqrt{x^2 + y^2}} \phi_{tt}(x,y),
\]

\[
\phi_{yt}(x,y) = \lim_{T \to \infty} \frac{1}{T} \int f_y(x,y,t) f_t^*(x,y,t) \, dt = \frac{-y}{v(\omega_0) \sqrt{x^2 + y^2}} \phi_{tt}(x,y).
\]

Therefore, the local phase velocity at \((x,y)\) can be obtained by the following equation as:

\[
v(\omega_0) = \frac{\sqrt{x^2 + y^2} \phi_{xt}(x,y) \phi_{yt}(x,y)}{x \phi_{xx}(x,y) \phi_{yt}(x,y) + y \phi_{yy}(x,y) \phi_{xt}(x,y)}
\]

3. Numerical Experiments

3.1. Numerical model

A 3D elastic CIP method is used to simulate an two-layers isotropic laminate with a delamination [4, 5, 6]. A schematic diagram of the numerical model used in the elastic CIP simulation is shown in Fig.1(a) and (b). Table 1 shows the elastic constants and geometrical specifications which are used in the numerical experiments.

3.2. Local phase velocimetry

In the numerical experiments, the material is considered to be two-layered carbon steel plate (S45C) as shown in Fig.1. The figures 2 (a) and (b) show the vertical component of the particle velocity on the cross section of the lamina.

Fig. 1. (a) A schematic diagram and (b) stress free boundaries of the numerical model
Table 1. Elastic constants and geometrical specifications of numerical model

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<th>density</th>
<th>Young’s modulus E [GPa]</th>
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<td>height [mm]</td>
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<td>6.0</td>
</tr>
</tbody>
</table>

center frequency of incident signal

f₀ [kHz]

200

specimen and, corresponding phase velocity of the Lamb-wave field on the surface. The major wave fronts yield the phase velocity to be 2600.0 m/s, which corresponds to the phase velocity of A0-mode Lamb wave which is propagating on the S45C steel plate with 2 mm in thickness. On the delamination region, the phase velocity is decreased to be 2000.0 m/s. This result coincides with the dispersive characteristics of the A0 mode Lamb wave.

4. Concluding Remarks

This paper proposes a local phase velocimetry and presents simple digital signal processing algorithm that characterizes the Lamb wave field over the multilayer material. For measurement of the local phase velocity, spatio-temporal gradient analysis based on the linear dependency among the vertical displacement, the vertical particle velocity, and a pair of shear strains is used. By analyzing the orthogonal pair of out-of-surface strains and vertical particle velocities, it is found that the local phase velocity of the Lamb wave is reduced within the delaminated region.

References