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A new method of boundary treatment in Heat conduction problems with finite element method

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Abstract

In regards to the defect of respectively handling three conditions, this paper puts forward a new unified expression fitting for different boundary conditions. It also obtains the parameter values of expression in different boundary conditions. Take the infinite square cylinder putting into cold liquor as an example, it works out a program using the improved new boundary treatment to simulate cylinder temperature field. This paper gets distributions of temperature field at four different times. The results of simulation not only meet the physical situation, but also have high precision compared with the theoretical solutions. Since being easy to deal with complex boundary conditions, the new way makes the FEM useful again in engineering calculation and research.

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Keywords: finite element method; boundary treatment; square cylinder; unsteady temperature field

1. Introduction

Courant at first put forward the concept of element in 1943, and then Finite Element Method was applied in solid mechanics problem in the 1950s. Along with the development of computer technology, FEM has substantial theory basis and comprehensive applied prospect in various kinds physics field of engineering [1]. The method of finite element is essentially a mathematical method. The finite element method is a discrete process to approximately solve continuation function in connected areal based on

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variation principle. It also means that using finite combination of small simple shapes to replace complex areal which need to solve. The finite element method as a numerical analysis tool which has solid theoretical basis has comprehensive application in solving various kinds of physical fields in recent twenty years.

The finite element method has a lot of advantages, such as having direct physical significance, suiting for complex boundary conditions, having high precision and so on. But the final finite element equations don't have unified expression equation and need to respectively manage three class boundary conditions. This makes it hard for programmer to develop software to solve practical problems and wasting computer resources. Aiming at defects of finite element method handling boundary conditions, this paper put forward improved method and makes finite element method have more comprehensive application in engineering computing and scientific research.

Nomenclature

α	convective heat transfer coefficient	T_f	temperature of fluid
T_i, T_j	temperature of point i and j		

2. Improvement of boundary treatment

Temperature field has three types of boundary conditions. The first type boundary condition already known is its node temperature value, so it needn't to establish node energy conservation equations. The second type boundary condition is known as heat-flow density of boundary and the third type is known as the relationship between heat-flow density and temperature. Reference [3] has made deep research of the finite element method application in heat transfer problems. It has resolved plane transient temperature field using weighted residual method. The computing area is dissect by quadrilateral and obtained basic computing equation of plane temperature field.

This paper emphasizes on the second type and third type boundary conditions. It analyzes nodes energy conservation equation when they belong to second type boundary, third type boundary or combine point of the two types. As shown in Fig.1 (a), node i is related to one unit and two boundaries, so it can establish energy conservation equations on the area of quadrilateral ABCi. As shows in Fig.1 (b), node i is related to two units and two boundaries, so it can establish energy conservation equations on the area of quadrilateral ABCDE where boundary AE is divided to two sections Ei and Ai. As shows in Fig.1 (c), node i is related to three units and two boundaries, so it can establish energy conservation equations on the area of quadrilateral ABCDEFGi.

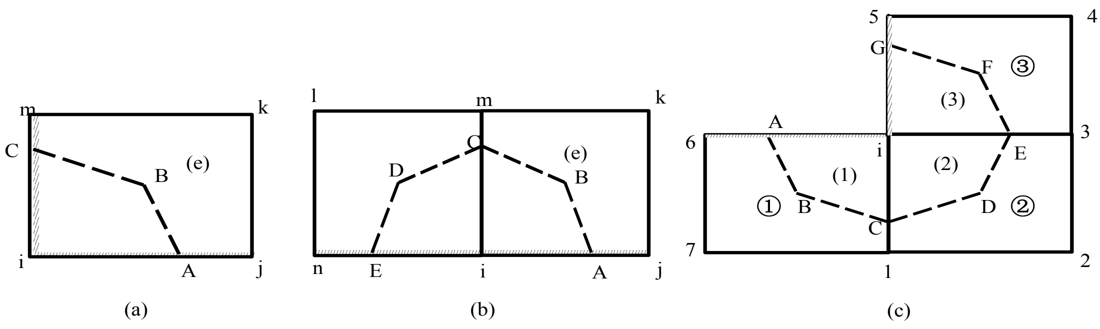

Fig.1 (a) One unit has two sides on boundary; (b) The first type of one unit has one side on boundary; (c) The second type of one unit has one side on boundary

In a word, there have been unified expressions which are shown in equation (1) suitable for all kinds of boundary nodes.

$$\sum_{e}\left[\overline{q_{n(e)i}}F_{(e)i}-\overline{q_{v(e)i}}V_{(e)i}+\rho C_p\frac{\partial\overline{T_{(e)i}}}{\partial t}V_{(e)i}\right]+\sum_{b}\overline{q_{bi}}F_{bi}=0 \tag{1}$$

Where, summation symbol with subscript e stands for the sum of units related to node i; summation symbol with subscript b stands for the sum of boundaries related to node i.

From the view of unit related to boundary, it can always respective computing the contribution from boundaries to nodes, although the unit maybe has one side or two sides on the boundary.

The contribution computing process shown in equation (1) has no difference with internal unit computing process, the computing equations and results can find in reference [3]. So this paper emphasize on the contribution from boundaries to nodes. Take the contribution from side ij to node i as an example shows in Fig.1 (a). Assuming temperature has linear change on the boundary ij, because only section iA has effect on heat balance of this area, it can get equation (2) when the boundary belonging to second type and heat-flow density is constant.

$$\overline{q_{bi}}F_{bi}=\frac{2}{3}s_b q_2 \tag{2}$$

Where, s_b stands for the length of boundary ij; q_2 stands for heat-flow density on outer normal direction of boundary ij;

$$\overline{q_{bi}}F_{bi}=\frac{2}{3}s_b\left(\frac{2}{3}q_{2i}+\frac{1}{3}q_{2j}\right) \tag{3}$$

Where, q_{2i}, q_{2j} are heat-flow densities of i and j on boundary ij in the direction of outward normal.

When the boundary belongs to third type, it can get equation (4).

$$\overline{q_{bi}}F_{bi}=\frac{2}{3}s_b\alpha\left[\left(\frac{2}{3}T_i+\frac{1}{3}T_j\right)-T_f\right] \tag{4}$$

In the same way, it can compute the contribution from boundary ij to node j. Choosing the same symbol as used in finite element method, it can get equation (5).

$$\begin{Bmatrix}\dfrac{\partial J^b}{\partial T_i}\\[2mm]\dfrac{\partial J^b}{\partial T_j}\end{Bmatrix}=\begin{Bmatrix}\overline{q_{bi}}F_{bi}\\\overline{q_{bj}}F_{bj}\end{Bmatrix}=\begin{bmatrix}u_{ii} & u_{ij}\\u_{ji} & u_{jj}\end{bmatrix}\begin{bmatrix}T_i\\T_j\end{bmatrix}-\begin{Bmatrix}v_i\\v_j\end{Bmatrix} \tag{5}$$

When the boundary belongs to the second type and heat-flow density is constant, the parameter values of equation (5) are shown in equation (6).

$$u_{ii} = u_{ij} = u_{ji} = u_{jj} = 0; \quad v_i = v_j = -\frac{2q_2 s_b}{3} \tag{6}$$

When heat-flow density changes in linear, the parameter values of equation (5) are shown in equation (7).

$$u_{ii} = u_{ij} = u_{ji} = u_{jj} = 0; \quad v_i = v_j = -\frac{2}{3} s_b \left(\frac{2}{3} q_{2i} + \frac{1}{3} q_{2j} \right) \tag{7}$$

When the boundary belongs to the third type, the parameter values of equation (5) are shown in equation (8).

$$u_{ii} = u_{jj} = \frac{4\alpha s_b}{9}; \quad u_{ij} = u_{ji} = \frac{2\alpha s_b}{9}; \quad v_i = v_j = \frac{2\alpha s_b T_f}{3} \tag{8}$$

The final combining equation shows in equation (9).

$$\sum_e \frac{\partial J^e}{\partial T_i} + \sum_b \frac{\partial J^b}{\partial T_i} = 0 \qquad (i = 1, 2, \cdots\cdots n) \tag{9}$$

Where, the first term of left stands for summation of variation temperature Ti of internal nodes; the second term of left stands for summation of variation temperature Ti of the second type boundary and the third type boundary.

3. Application example

There is a right angle square cylinder whose section is width 2m and height 3m. Its original temperature is 100℃. Put the cylinder into liquor with temperature 10℃ to make the cylinder cooling. The coefficient of heat conductivity of cylinderλ is 50W/(m (℃). Thermal diffusivity of cylinder a is also 4m2/s. The coefficient of heat convection between surfaces of square cylinder and liquor h is 80W/(m^2·℃). Request to know the temperature distribution at 0.02 second, 0.1 second, 0.2 second and 0.4 second.

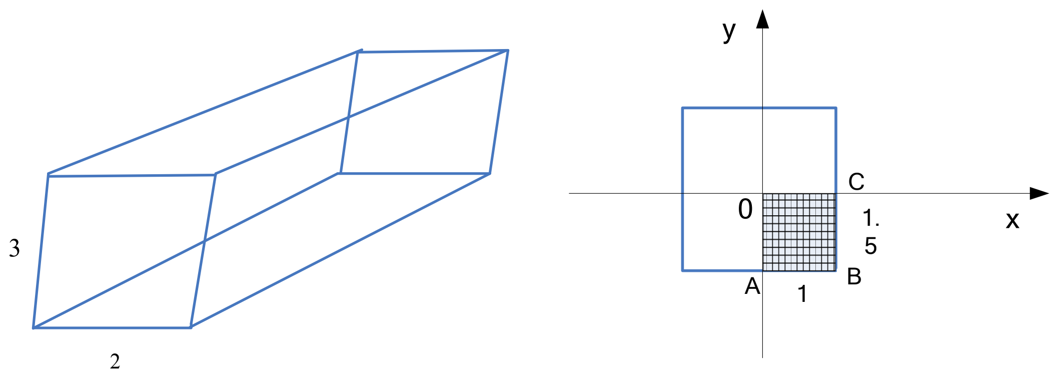

Fig.2 (a) Square cylinder with infinite length ; (b) Gridding method

This paper grids the resolving area according the method of finite element. Because of the symmetry of the graphic and simplify of the process, it can choose the bottom right of the section to analyze along. As it shows in Fig.2 (b), side AB and BC belong to the third type of boundary condition, side 0A and 0C should deal with as internal nodes. Using the method of boundary treatment introduced above, this paper work out a program to solve this problem. The result of stimulation of temperature field shows in Fig.3 (a), (b), (c) and (d).

The result of the numerical simulation has little error compared with theory solution which analyzed in reference [4]. So using the unified boundary conditions expression not only can get relatively precise result, but also can simplify the process of programming.

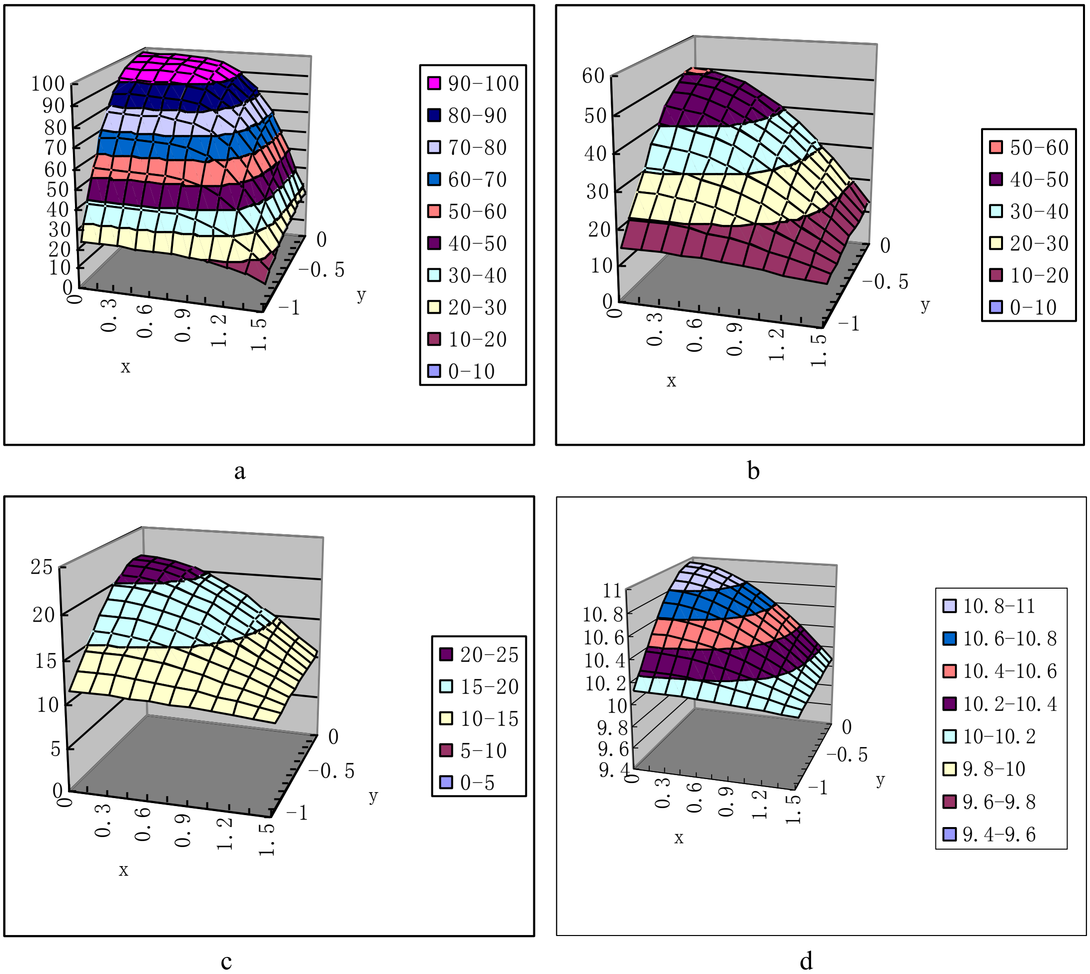

Fig. 3 (a) distribution of temperature field of time 0.02 second; (b) distribution of temperature field of time 0.1 second; (c) distribution of temperature field of time 0. 2 second; (d) distribution of temperature field of time 0.4 second.

4. Conclusions

This paper analyzes the process of boundary treatment of the method of finite element and put forward improved new boundary treatment. Then prove it was right through application example. The main findings of this paper have three points.

- Firstly, this paper puts forward the new unified expression of boundary treatment and gets the parameter values of three different boundary conditions.
- Secondly, it resolves the combining process of finite element method and the recycle design of different boundary conditions.
- Thirdly, it works out the example simulation program and got the unsteady state distribution of temperature field when infinite square cylinder putting into the cold liquor.

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