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# Higgs inflation at the critical point

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ABSTRACT

Higgs inflation can occur if the Standard Model (SM) is a self-consistent effective field theory up to inflationary scale. This leads to an upper bound on the top Yukawa coupling, $y_t^{\text{phys}} < y_t^{\text{crit}}$ and thus on the mass of the top quark $m_t$. If $m_t$ is more than a few hundred of MeV below the critical value, the Higgs inflation predicts the universal values of inflationary indexes, $r \simeq 0.003$ and $n_s \simeq 0.97$, independently on the SM parameters. We show that in the vicinity of the critical point $y_t^{\text{crit}}$ the inflationary indexes acquire an essential dependence on $m_t$ and on the mass of the Higgs boson $M_h$. In particular, the amplitude of the gravitational waves can exceed considerably the universal value.



## 1. Introduction

The most economic inflationary scenario is based on the identification of the inflaton with the SM Higgs boson [1] and the use of the idea of chaotic initial conditions [2]. The theory is nothing but the SM with the non-minimal coupling of the Higgs field to gravity with the gravitational part of the action

$$S_G = \int d^4x\sqrt{-g}\left\{-\frac{M_P^2}{2}R - \frac{\xi h^2}{2}R\right\}. \tag{1}$$

Here $R$ is the scalar curvature, the first term is the standard Hilbert–Einstein action, $h$ is the Higgs field, and $\xi$ is a new coupling constant, fixing the strength of the "non-minimal" interaction. The presence of non-minimal coupling is required for consistency of the SM in curved space–time (see, e.g. [3]). The value of $\xi$ cannot be fixed theoretically within the SM.

The presence of the non-minimal coupling insures the flatness of the scalar potential in the Einstein frame at large values of the Higgs field. If radiative corrections are ignored, the successful inflation occurs for any values of the SM parameters provided $\xi \simeq 47000\sqrt{\lambda}$, where $\lambda$ is the Higgs boson self-coupling. This condition comes from the requirement to have the amplitude of the scalar perturbations measured by the COBE satellite. After fixing the unknown constant $\xi$ the theory is completely determined. It predicts the tilt of the scalar perturbations given by $n_s \simeq 0.97$ and the tensor-to-scalar ratio $r \simeq 0.003$. After inflationary period, the Higgs field oscillates, creates particles of the SM, and produces the Hot Big-Bang with initial temperature in the region of $10^{13-14}$ GeV [4,5].

The quantum radiative corrections can change the form of the effective potential and thus modify the predictions of the Higgs inflation. The most significant conclusion coming from the analysis of the quantum effects is that the Higgs inflation can only take place if the top quark Yukawa coupling is smaller than some critical number $y_t^{\text{crit}}$ [6–10],

$$y_t^{\text{phys}} < y_t^{\text{crit}}. \tag{2}$$

To make it exactly defined, $y_t^{\text{phys}}$ is the top quark Yukawa coupling in $\overline{\text{MS}}$ renormalisation scheme taken at $\mu_t = 173.2$ GeV, $y_t^{\text{phys}} \equiv y_t^{\text{phys}}(\mu_t)$. Roughly speaking, the Higgs self-coupling constant $\lambda$ must be positive at the energies up to the inflationary scale, leading to this constraint. In numbers [11–13],

$$y_t^{\text{crit}} = 0.9268 + 0.0057 \times \left[\frac{M_h - 125.9}{0.4} \times 0.2 + \frac{\alpha_s(M_Z) - 0.1184}{0.0007} \times 0.28\right],$$

where $\alpha_s$ is the QCD coupling at the $Z$-boson mass. Thanks to complete two-loop computations of [13] and three-loop beta functions for the SM couplings found in [14–19] this formula may have a very small theoretical error, $2 \times 10^{-4}$, with the latter number coming from an "educated guess" estimates of even higher order terms (see the discussion in [11] and more recently in [20]).

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The main systematic uncertainties in required precise determination of $y_t^{\rm phys}$ from FNAL and LHC top-quark experiments lie in a poor knowledge of higher order perturbative and non-perturbative QCD effects (see e.g. [21]). Accounting for those, the value of $y_t^{\rm crit}$ is about 2 standard deviations [11,20] from the top Yukawa coupling extracted from Tevatron and LHC [22].

The determination of the inflationary indexes accounting for radiative corrections is somewhat more subtle and depends on the way the quantum computations are done (the SM with gravity is non-renormalizable, what introduces the uncertainty). In [6,9] we formulated the natural subtraction procedure (called "prescription I") which uses the field independent subtraction point in the Einstein frame (leading to scale-invariant quantum theory in the Jordan frame for large Higgs backgrounds) and computed $n_s$ and $r$ for the Higgs masses that exceeded $M_{\rm crit}$ by just a small amount of few hundreds of MeV.[1] We have shown that the values of $n_s$ and $r$ are remarkably stable in this domain and coincide with the tree estimates. However, we did not analyse what happens in the close vicinity of the critical point. Partially, this has been studied in [23], but the peculiar inflationary behaviour found in the present work was not discussed in [23].

The aim of the present paper is to study the behaviour of the inflationary indexes close to the critical point. In what follows we will use the prescription I. We expect to have qualitatively the same results in the prescription II, though the numerical values will be somewhat different. We will see that $n_s$ and $r$ acquire a strong dependence on the mass of the Higgs boson and the mass of the top quark. Thus, if the cosmological observations will show that one or both indexes do not coincide with those given by the tree analysis, they will indicate that in instead of inequality (2) we should have an equality between the top Yukawa coupling and its critical value, $y_t^{\rm phys} = y_t^{\rm crit}$.

## 2. The critical point

The behaviour of the scalar self-coupling constant $\lambda$ as a function of the $\overline{\rm MS}$ parameter $\mu$ (energy) in the SM is very peculiar. If the mass of the top quark and of the Higgs boson are varied within their experimentally allowed intervals, it can be approximated in the region of Planck energies ($M_P = 2.44 \times 10^{18}$ GeV) with a good accuracy as follows:

$$\lambda(z) = \lambda_0 + b(\log z)^2, \tag{3}$$

where

$$z = \frac{\mu}{qM_P}, \tag{4}$$

$\lambda_0$, $q$ and $b$ are some functions of the top quark (pole) mass, Higgs mass, and the strong coupling constant $\alpha_s$, see Section 4. It happens that $\lambda_0$ is small, $\lambda_0 \ll 1$ and $q$ is of the order of one.[2] To put it in words, both the value of $\lambda$ and of its beta-function, $\beta_\lambda = \mu\partial\lambda/\partial\mu$, are close to zero near the Planck scale. It is this fact that changes the behaviour of the inflationary indexes, as is demonstrated below.

The renormalisation group improved effective potential in the Einstein frame with an accuracy sufficient for the present discussion can be written as follows [9]:

[1] We also performed the computation with the use of another subtraction procedure (called "prescription II"), which has a field-independent subtraction point in the Jordan frame [8,7,10].

[2] A possible explanation of these facts may lie in the asymptotic safety of the SM [24]. This is also close to the "multiple point principle" of [25].

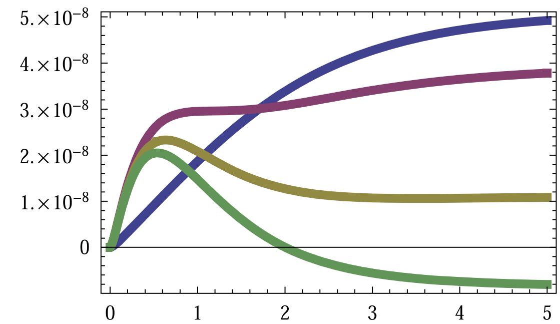


**Fig. 1.** The schematic change of the form of the effective potential depending on $\lambda_0$. For better visibility the values of $\xi$ are different for different lines. The horisontal axis corresponds to the canonically normalised field $\chi$, the vertical axis to the effective potential, all in Planck units. (For interpretation of the references to colour in this figure, the reader is referred to the web version of this article.)

$$U(\chi) \simeq \frac{\lambda(z)}{4\xi^2}\bar{\mu}^4. \tag{5}$$

Here

$$z = \frac{\bar{\mu}}{\kappa M_P} \tag{6}$$

and

$$\bar{\mu}^2 = M_P^2\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right), \tag{7}$$

where $\chi$ is the canonically normalised scalar field related to the original Higgs field by a known transformation [1] (see Eq. (11)). The $\overline{\rm MS}$ parameter $\mu$ in (4) that optimises the convergence of the perturbation theory is related to $\bar{\mu}$ as

$$\mu^2 = \alpha^2 \frac{y_t(\mu)^2}{2}\frac{\bar{\mu}^2}{\xi(\mu)} \tag{8}$$

with $\alpha \simeq 0.6$. This numerical value follows from the minimisation of the one-loop Coleman–Weinberg effective potential [6,9] (two-loop contributions do not further change $\alpha$ significantly). The expression (5) is valid for $\xi > 1$, and the scale dependence of $\xi$ can be neglected. Eqs. (4), (6), and (8) provide the connection between parameter $\kappa$, convenient for the inflationary analysis, and parameter $q$ following from the RG evolution of the $\overline{\rm MS}$ coupling constants.

Let us now consider the change of the form of the potential if $\lambda_0$, $\kappa$ and $\xi$ are varying. For $\lambda_0 \gg b/16$ the potential is a rising function of the field $\chi$, realising the "tree" Higgs inflation (see Fig. 1, blue curve). If $\lambda_0 = b/16$, a new feature appears: the first and the second derivatives of the potential are equal to zero at some point (see Fig. 1, red curve). For $\lambda_0 < b/16$ but still close to $b/16$ we get a wiggle on the potential, which is converted into a maximum for somewhat smaller $\lambda_0$ (see Fig. 1, brown line). Decreasing $\lambda_0$ even further leads to the unstable electroweak vacuum, Fig. 1 (green line). Clearly, the necessary condition for inflation to happen in the slow-roll regime is to have $dV(\chi)/d\chi > 0$ for all $\chi$, i.e. the absence of a wiggle. For $\lambda_0 \gg b/16$ all the potentials are very much similar, leading to the independence of inflationary indexes on the parameters, while if $\lambda_0$ is close to $b/16$, the form of the potential changes, and the dependence of $r$ and $n_s$ on $\lambda_0$ and $\kappa$ (and, therefore on $M_h$ and $m_t$) shows up.

The parameter $\kappa$ controls the value of $\chi$ where the wiggle would appear for $\lambda_0 = b/16$, the parameter $0 < \lambda_0 - b/16 \ll b/16$ tells how close we are to the appearance of the feature, while a combination of $\lambda_0$ and $\xi$ determines the asymptotic of the potential at large $\chi$. Let us note that inflation with large $r$ in the potentials with near vanishing $U'$ at some value of the field along the inflationary slow-roll evolution was considered in [26,27].

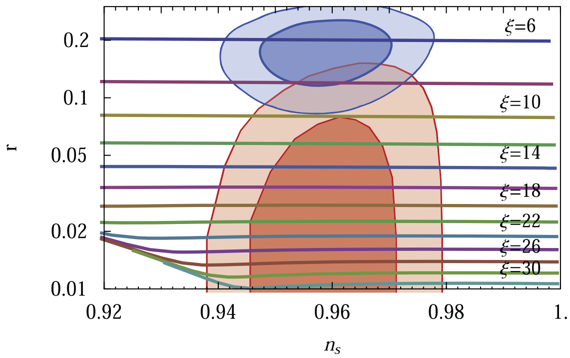


**Fig. 2.** The dependence of the inflationary indexes $n_s$ and $r$ on $\xi$ and $\kappa$, the parameter $\lambda_0$ is fixed by the COBE normalisation. Along the nearly horisontal lines $\xi$ is fixed and $\kappa$ is varying within the interval $\{0.9, 1.1\}$. We also show 1 and 2 $\sigma$ contours coming from the results of Planck [28] (red, fit with zero running of the spectral index) and the suggested region from BICEP2 [29] (blue, no foreground contributions subtracted). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

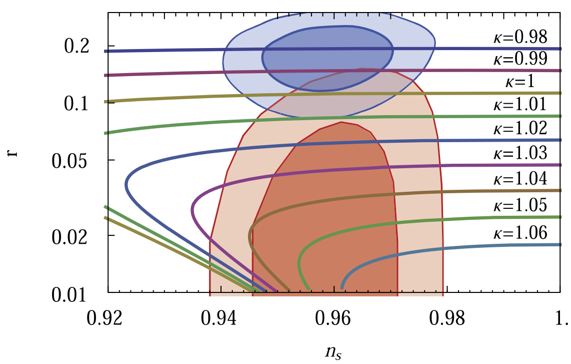


**Fig. 3.** The same as in Fig. 2, but with the grid of constant $\kappa$ lines. The parameter $\xi$ is varying within the interval $\{5, 30\}$.

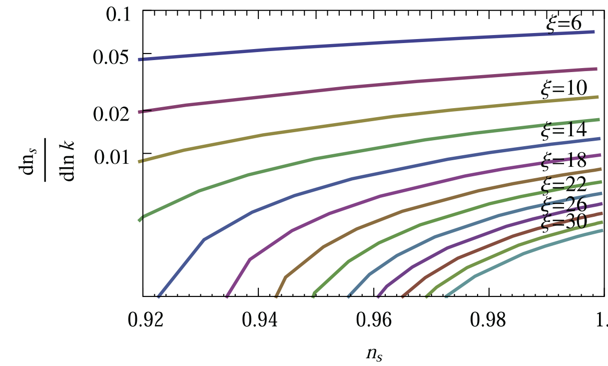


**Fig. 4.** The same as in Fig. 2, but now on the plane $(n_s, dn_s/d\ln k)$, which includes the running of the scalar spectral index.

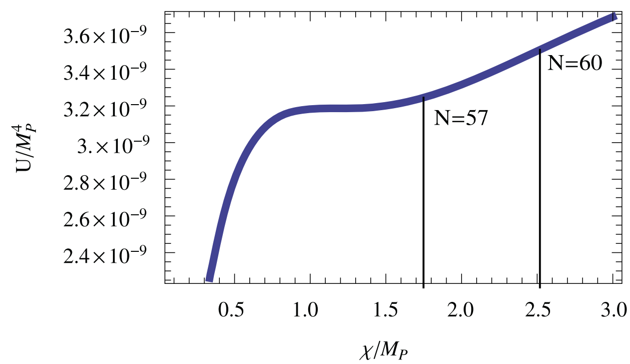


**Fig. 5.** The form of the effective potential which leads to $r = 0.1$, $n_s = 0.96$. The field values corresponding to the $N = 57$ and $N = 60$ e-foldings are marked by vertical lines, roughly indicating the observable window for inflation.

## 3. The inflationary indexes

Once the potential is known it is straightforward to determine inflationary indexes. For this end it is more convenient to use the three parameters $\kappa$, $\lambda_0$ and $\xi$ as independent variables, characterising the physics at the inflation scale (the relation to low energy observables will be discussed in Section 4). Note that the value of $b$ is stable against the change of the SM parameters in the vicinity of the critical point and can be fixed as $b \simeq 2.3 \times 10^{-5}$.

In the Higgs inflation far from the critical point the parameter $\kappa$ is irrelevant, whereas $\lambda_0$ and $\xi$ always appear in the combination $\lambda_0/\xi^2$, meaning that the potential depends on one parameter only. Fixing it from the COBE normalisation then leads to prediction of $n_s$ and $r$. Close to the critical point the situation is changed – all the three parameters are now essential. The parameter counting leads to the conclusion that any values of $n_s$ and $r$ are now possible. This expectation is confirmed by a detailed analysis, see Figs. 2 and 3.

The parameters $\xi$ and $\kappa$ define the position and the energy scale of the critical point. The small parameter $\lambda_0 - b/16$ controls the amount of e-foldings the inflation spends in the vicinity of the critical point, without significantly modifying the potential in other regions.

In Fig. 2 (3) we show the lines of constant $\xi$ ($\kappa$) on the plane $(n_s, r)$, the parameter along the line is associated with the variation of $\kappa$ ($\xi$) within the interval $\{0.9, 1.1\}$ ($\{5, 30\}$).

In Fig. 4 we show the running of the scalar index as a function of $\kappa$ and $\xi$. One can see that it is positive. A word of caution should be said in this place, warning from the use of the available inflationary constraints from CMB measurement. With the potential (5) close to the critical point the behaviour of the spectral index in the observable inflationary region is complicated (i.e. expansion in terms of running and running of the running of the spectral index over the observable inflationary window is not a good approximation, contrary to the case of power law potentials). Thus it is necessary to make the complete fit of the perturbation spectrum, generated form the potential (5) to the CMB observations, like it is suggested in [30–32]. This analysis goes beyond the scope of the present letter.

The picture of the Higgs-inflation potential which gives $r = 0.1$, $n_s = 0.96$ is shown in Fig. 5.

A very interesting feature of the inflation near the critical point is the drastic decrease of the necessary non-minimal coupling $\xi$ down to a number of the order of ten. The large value of $\xi$, necessary for the Higgs inflation far from the critical point, effectively introduces a new strong-coupling threshold $\Lambda \sim M_P/\xi$ well below the Planck scale, if the scattering of the SM particles is considered around the EW vacuum [33,34]. Though this fact does not invalidate the self-consistency of the Higgs inflation [35,36] which occurs at large Higgs fields, it requires the UV completion of the SM or self-healing of high energy scattering [37,38] at energies much smaller than the Planck scale. The Higgs inflation at the critical point does not require any new cutoff scale, essentially different from the Planck scale.

The evolution of the Universe after the Higgs inflation at the critical point is different from that for the case $\xi \gg 1$. If $\xi \gg 1$, the Universe after inflation is "matter dominated" due to oscillations of the Higgs field. The transition to the radiation dominated Universe occurs due to particle production after some time, but not later than after $\mathcal{O}(\xi/2\pi)$ oscillations [4,5]. For $\xi \sim 10$ we have the radiation-dominated epoch right after inflation is finished. Of course, the system will come to thermal equilibrium only after a number of oscillations of the Higgs field.

## 4. Connection between low energy and high energy observables

The aim of this section is elucidating the relation between the parameters of the inflationary potential $\kappa$ and $\lambda_0$ and top quark and Higgs masses measured in low energy experiments. The main problem here is that the Standard Model in the Einstein frame is essentially non-polynomial and thus non-renormalizable, meaning that the required connection cannot be found without extra assumptions about the structure of the underlying fundamental theory [35].

The most conservative (and thus most predictive ) hypothesis of the absence of new physics between the Planck and Fermi scales still has the uncertainties in the relation between low energy and high energy parameters [35]. To discuss these uncertainties let us consider (the most important numerically) interactions of the top quark with the Higgs field, given in the Einstein frame by

$$L_t = \frac{y_t}{\sqrt{2}}\bar{t}tF(\chi). \tag{9}$$

Here

$$F(\chi) = \frac{h}{\Omega}, \tag{10}$$

$\Omega^2 = 1+\xi h^2/M_P^2$ is the conformal factor, and the canonically normalised field $\chi$ is related to the Higgs field $h$ via

$$\frac{dh}{d\chi} = \frac{\Omega^2}{\sqrt{\Omega^2+\xi(6\xi+1)h^2/M_P^2}}. \tag{11}$$

To remove the divergencies in the arbitrary background fields, the counter-terms must be added to the action. The scalar correction to the $\bar{t}th$ vertex requires the modification of $y_t$ as

$$y_t \to y_t + \frac{y_t^3}{16\pi^2}\left(\frac{9}{4\epsilon}+C_t\right)F'^2 \tag{12}$$

and the top quark loop contribution to scalar self-interaction gives

$$\lambda \to \lambda - \frac{y_t^4}{16\pi^2}\left(\frac{3}{\epsilon}-C_\lambda\right)F'^4, \tag{13}$$

where $\epsilon$ is the parameter of dimensional regularisation, $C_t$ and $C_\lambda$ are the (arbitrary) constant parts of the counter-terms, and $F' = dF/d\chi$. These constant parts cannot be fixed theoretically with the use of the SM Lagrangian and must be found from observations or from (unknown yet) UV complete theory hosting the SM at low energies.

For the small Higgs backgrounds $h \ll M_P/\xi$ the derivative $F'$ is close to one, and the parameters $C_t$ and $C_\lambda$ are absorbed in the definition of low-energy top Yukawa coupling and scalar self-coupling and thus are unobservable at low energies. However, in the inflationary region, at $h > M_P/\xi$, the derivative $F'$ goes to zero, and it is the couplings $y_t$ and $\lambda$, rather than $y_t^{\text{phys}} = y_t + \frac{y_t^3}{16\pi^2}C_t$ and $\lambda^{\text{phys}} = \lambda - \frac{y_t^4}{16\pi^2}C_\lambda$, that contribute to cosmological observables. The transition between two regimes (for $\xi > 1$) occurs approximately at $h^* = \frac{1}{2\sqrt{6}}\frac{M_P}{\xi}$, corresponding to the fastest falloff of the function $F'^2$. It can be well approximated by a sudden jump of the coupling constant from $y_t^{\text{phys}}$ to $y_t$ at $h = h^*$. The obvious inflationary requirement is that in the domain $h < h^*$ the *physical, low energy* $\lambda^{\text{phys}}$ must be positive, i.e. the inequality (2) must be valid.

To determine the parameters of the inflationary potential (5) we define the “inflationary” top and Higgs masses $m_t^*$ and $M_h^*$ that lead to $y_t$ and $\lambda$ at high energies through the renormalisation group evolution (SM running up to $h^*$ and chiral SM running afterwords [9]), *without any jumps* at $h = h^*$. The inflationary masses are related to the physical masses $m_t$ and $M_h$ as follows:

$$m_t^* = m_t\left(1-\frac{y_t^2 C_t}{16\pi^2}\right), \qquad M_h^* = M_h\left(1-\frac{y_t^4 C_\lambda}{16\pi^2}\frac{h_0^2}{M_h^2}\right), \tag{14}$$

where $h_0 = 250$ GeV is the vacuum expectation value of the Higgs field, and all the constants are taken at low energy scale. The presence of the unknown coefficients $C_t$ and $C_\lambda$ results exactly from the unremovable uncertainty following from the non-renormalizable character of the SM coupled to gravity in the non-minimal way. Numerically (in the units of GeV, and for $M_h \simeq 126$ GeV),

$$m_t^* \simeq m_t - C_t, \qquad M_h^* \simeq M_h - 3C_\lambda. \tag{15}$$

Now, we can combine the discussion of inflation with the low energy parameters accounting for the uncertainties discussed above. The functions $\lambda_0$, $q$ and $b$ can be expressed through the high energy inflationary parameter and can be found from the analysis of the renormalisation group running for $\lambda$. The fitting formulas are given below[3]:

$$\begin{aligned}\lambda_0 &= 0.003297\big(\big(M_h^*-126.13\big)-2\big(m_t^*-171.5\big)\big),\\ q &= 0.3\exp\big(0.5\big(M_h^*-126.13\big)-0.03\big(m_t^*-171.5\big)\big),\\ b &= 0.00002292-1.12524\times10^{-6}\big(\big(M_h^*-126.13\big)\\ &\quad -1.75912\big(m_t^*-171.5\big)\big),\end{aligned} \tag{16}$$

where $M_h^*$ and $m_t^*$ are to be taken in GeV.

These equations can be used now to determine the dependence of the cosmological parameters on $m_t^*$ and $M_h^*$. Since the value of the scalar tilt $n_s$ depends very strongly on $\kappa$, we fix it in the experimentally allowed region $n_s \in \{0.94, 0.98\}$ and present in Figs. 6 and 7 the dependence of $r$ and required $\xi$ on high energy values of the top quark and Higgs masses. Also, Fig. 8 shows the running of the spectral index. Note, that change of $n_s$ within the observable region corresponds to extremely small change of the relation between $\xi$, $M_h^*$, and $m_t^*$, which is completely within the widths of the lines on Figs. 6 and 7, and corresponds to some change of the prediction of $dn_s/d\ln k$ on Fig. 8.

The combination of particle physics and cosmological measurements allows to fix unknown parameters $C_t$ and $C_\lambda$. If we take, for instance, $r = 0.12$, then $M_h^* \simeq 122.6$ GeV and $m_t^* \simeq 169.8$ GeV, $\xi \simeq 8$. To get the physics low energy value of the Higgs mass $M_h = 125.6$ GeV we need $C_\lambda \simeq 1$. The value of $C_t \simeq 1.5$ would bring the physical top mass to $m_t \simeq 171.5$ GeV, consistent with the measured top quark mass within 2 $\sigma$ uncertainties. The relative change of the top Yukawa coupling at $h = h^*$ is $(y_t^{\text{phys}} - y_t)/y_t \simeq 0.024$, while the change in $\lambda$ is very sensitive to the physical Higgs and top masses and can be made equal to zero by tuning $m_t$ and $M_h$ within few MeV.

## 5. Conclusions

The Higgs inflation for $y_t^{\text{phys}} < y_t^{\text{crit}}$ is a predictive theory for *cosmology*, as the values of the inflationary indexes are practically

[3] We used the analysis made in [11] to produce the fitting formulas and fixed $\alpha_s = 0.1184$. The account of more precise mapping at the electroweak scale made in [13] can change the extraction of the Higgs and top masses from the cosmological data by amount of $\mathcal{O}(100)$ MeV.

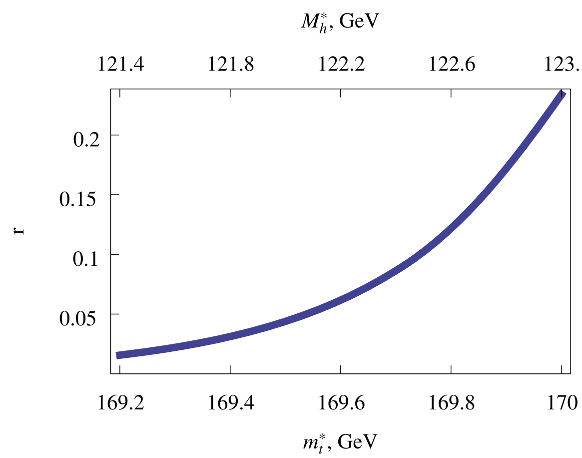


**Fig. 6.** The dependence of the tensor-to-scalar ratio $r$ on the high energy inflationary Higgs boson and top quark masses $M_h^*$ and $m_t^*$. The value of $n_s$ along the curve is within the interval $n_s \in \{0.94, 0.98\}$.

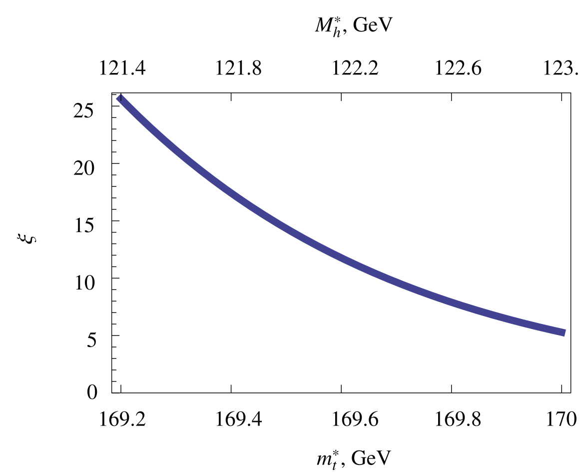


**Fig. 7.** The dependence of the required non-minimal coupling on the high energy inflationary Higgs boson and top quark masses $M_h^*$ and $m_t^*$.

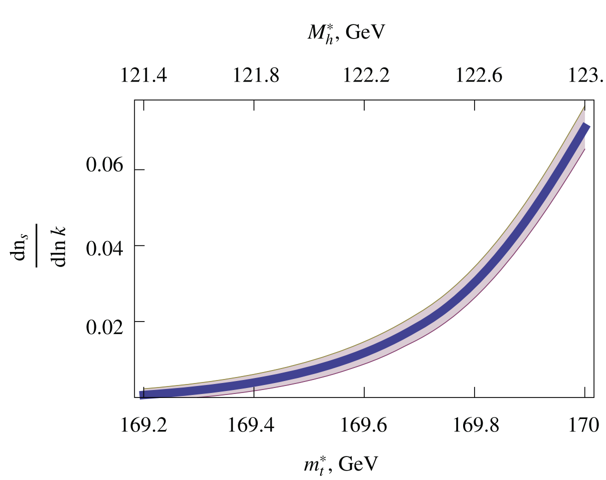


**Fig. 8.** The same as in Fig. 6, but for the running of the scalar spectral index. The shaded area corresponds to change of $\kappa$ leading to change of $n_s$ within the Planck allowed values (for Figs. 6 and 7 this area is within the thickness of the lines on the plots).

independent of the SM parameters. Near the critical point the situation completely changes, and we get a strong dependence of $n_s$ and $r$ on the precise values of the *inflationary masses* of the top quark and the Higgs boson $m_t^*$ and $M_h^*$. In this regime the Higgs inflation becomes a predictive theory for *high energy domain of particle physics*, as any deviation of inflationary indexes from the tree values tells that we are at the critical point, fixing thus the inflationary values of masses of the top quark and the Higgs boson $m_t^*$ and $M_h^*$. It is amazing that a possible detection of large tensor-to-scalar ratio $r$ in [29] gives the inflationary top quark and Higgs boson masses close to their experimental values $m_t$ and $M_h$. Though the exact relation between $m_t^*$, $M_h^*$ and the physical masses $m_t$, $M_h$ requires the knowledge of the full UV complete theory, we can see that the uncertainties related to the transition from low and high energies (constants $C_t$ and $C_\lambda$) are quite small.

Let us also note, that though this uncertainty prevents us from exact predicton of the top-quark and Higgs masses from the CMB observations at present, the bound on these masses related to the metastability of the electroweak vacuum [11–13] remains unmodified (within $\sim 100$ MeV precision), as far as it is connected with the Higgs potential becoming negative at scales below $M_P/\xi$, where any corrections become important.

Also, though prediction of the inflationary parameters $r$ and $n_s$ is impossible for the Higgs inflation near the critical point, the predictions for the higher derivatives (shape) of the spectra of the inflationary perturbations can be done, and thus provide a non-trivial way to check the model against the CMB data.

We conclude with a word of caution. All results here are based on the assumption of the validity of the SM up to the Planck scale. If this hypothesis is removed, the Higgs inflation remains a valid cosmological theory, but its predictability is lost even far from the critical point. For example, the modification of the kinetic term of the Higgs field at large values of $H$, leads to a considerable modification of $r$ [39–41] (see also [42–44] for generalised Higgs inflation with Horndenski type terms). The change of the structure of the Higgs-gravity interaction to, for instance,

$$M_P^2 R\sqrt{1+\xi|H|^2/M_P^2}, \tag{17}$$

will make the potential in the Einstein frame quadratic with respect to the field $\chi$ and thus would modify $r$ and $n_s$, making them the same as in the chaotic inflation with free massive scalar field. Another assumption is about the absence of operators suppressed by the Planck scale (or various tree level unitarity violation scales [35]), which may be justified by a special scale (or shift in the Einstein frame) symmetry of the UV complete theory. Adding them would change introduce further uncertainties in inflationary physics [45], cf. [46] for importance of such terms for the stability of electroweak vacuum.

While this paper was in preparation, the article [47] appeared, where the possibility to have large value of $r$ for the Higgs inflation close to the critical point was also pointed out.

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