Delamination Detection of Composite Cantilever Beam Coupled With Piezoelectric Transducer Using Natural Frequency Deviation

I. V. Tate, Sajal Roy, K. R. Jagtap

Abstract

Delaminations are common damages in laminated composites, which can significantly reduce the structural stiffness, which changes the dynamic response of the structures and therefore changes modal parameters such as natural frequency mode shape and damping. This paper presents effect of delamination lengths and locations on the natural frequencies of an electromechanical system consisting of a piezoelectric transducer glued on part of the upper surface of a laminated composite cantilever beam. To obtain dynamic responses of the structure transient analysis is carried out using ANSYS 13 software. From the dynamic responses frequency response functions (FRF) are obtained by using MATLAB code. Effectiveness of the proposed methodology is evaluated by numerical simulation of three delamination scenarios, i.e. delamination near fixed end, at center and at free end. It is observed that natural frequency decrease when length of delamination increases.

Keywords: Composite Cantilever Beam, Delamination, Dynamic Response, FRF, Natural Frequency, Piezoelectric Transducer.

1. Introduction

The importance of use of composite materials has been increasing consistently in different industries like civil engineering, mechanical engineering, aerospace engineering etc. due to their advantageous characteristics. One of the most remarkable properties that structures made of composites possess is their very large stiffness to weight ratio. Composites have an excellent combination of high strength and stiffness with low weight. This property
among many others has flexibility to adapt different shapes, protection against corrosion, gives the possibility to look for new engineering challenges and replace the traditional materials with composites. Composite laminates are widely used in different areas due to their easy fabrication and effectiveness and because of their versatility in the orientation of the fibers. Despite of their benefits composite materials have their disadvantages. Due to their layered nature and the interaction between both materials, fibers and matrix, composite materials are prone to different failures modes, the most common of them being delamination, which can cause irreversible damage. This failure causes the separation of the layers and induces significant loss of mechanical strength. Delamination is probably the most dangerous defect in composite materials because it can appear suddenly without any notice and it keeps developing to collapse the whole structural member. Composite materials with the defect of delamination can lose up to 60% of their stiffness and still remain visibly unchanged. The development of robust techniques for detection delamination is essential to avoid such a failure. Since delamination damage leads to stiffness loss of the structure, the modal properties like natural frequencies, damping ratio and mode shapes may vary also.

Piezoelectric materials possess intrinsic electromechanical coupling effects, by virtue of which they have found extensive applications in smart devices such as electromechanical actuators and transducers. They are largely used in active vibration control and noise suppression of sensors in structures of different scale: rockets, weapon systems, smart skin systems of submarines, and so on.

Zou et al. [1] presented different type of vibration based methods to detect delamination in structures. Dalessandro [2] provided method for modeling the dynamic response of an electromechanical system consisting of a piezoelectric transducer glued on part of the upper surface of a metallic cantilever. Sadilek et al. [3] performed frequency response analysis of hybrid aluminum beam with piezoelectric actuators. They implemented finite element model in MATLAB software. The piezoelectric actuators were driven by harmonic signals around the first eigen frequency and the beam oscillations were also investigated. Lavate et al. [4] investigated dynamic response of the composite beam. Inspection of the dynamic behavior of the composite beam for various end conditions is made by both FEM and theoretical analysis. Garcia et al. [5] developed a method for analysis of the vibration response of structures made of composites. It was also used to develop a vibration-based health monitoring procedure for such structures. Inada et al. [6] proposed two-step delamination identification method using resonant and anti-resonant frequency changes. Xu et al. [7] presented a general purpose design scheme of actively controlled smart structures with piezoelectric sensors and actuators. This scheme can make use of any finite element code with piezoelectric elements, and control design is carried out in state space form established on finite element modal analysis. Huang and Kim [8] proposed a stress function-based approach to analyze the free-edge interlaminar stresses of piezo-bonded symmetric laminates. Parashar et al. [9] presented finite element modeling of nonlinear vibration behavior of piezo-integrated structures subjected to weak electric field. Liang et al. [10] investigated interfacial debonding behavior of composite beams which include piezoelectric materials, adhesive and host beam.

In this paper effect of delamination length and location on the natural frequency of an electromechanical system consisting of a piezoelectric transducer glued on part of the upper surface of a laminated composite cantilever beam is presented. To obtain dynamic responses transient analysis of composite cantilever beam is carried out by using ANSYS 13 software. From the dynamic responses frequency response functions (FRF) are obtained by using MATLAB code. Effectiveness of the proposed methodology is evaluated by numerical simulation of three delamination scenarios, i.e. delamination near fixed end, at center and at free end. It is observed that natural frequency decrease when length of delamination increases.

2. Mathematical Model & Constitutive Equations

Electromechanical coupling is characterized by three variables i.e. the mechanical compliance, the dielectric permittivity and the piezoelectric strain coefficient. The mechanical compliance and electrical permittivity are the functions of the electrical and mechanical boundary condition, respectively. The pure mechanical stress-strain law for each piezoelectric element is extended with piezoelectric coupling. This can be than written as

\[
\sigma = C \varepsilon - e^T E
\]

\[
D = e \varepsilon - \varepsilon E
\]
Where mechanical variable $\sigma$ and $\varepsilon$ denote the stress and the strain, respectively; electrical variable $D$ and $E$ denote the electric displacement and the electric field, respectively. Note that electric displacement ($D$) and electric field ($E$) are fundamental variables in the theory of electrostatics. $C$ is the elasticity matrix, $\varepsilon$ is the piezoelectric matrix, $\varepsilon$ is the dielectric matrix. Piezoelectric matrix $\varepsilon$ accounts for the piezoelectric effect, i.e. the intrinsic coupling between mechanical and electric fields. Eq. (1) characterizes the converse piezoelectric effect that enables piezoelectric materials to function as actuators, while Eq. (2) characterizes the direct piezoelectric effect that enables piezoelectric materials to function as sensors.

The permittivity matrix $\varepsilon$ is defined as

$$
\varepsilon = \begin{bmatrix}
\varepsilon_{11} & 0 & 0 \\
0 & \varepsilon_{22} & 0 \\
0 & 0 & \varepsilon_{33}
\end{bmatrix}
$$

(3)

The piezoelectric matrix $\varepsilon$ is defined as

$$
\varepsilon = \begin{bmatrix}
0 & 0 & 0 & 0 & \varepsilon_{15} & 0 \\
0 & 0 & 0 & 0 & \varepsilon_{24} & 0 \\
\varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} & 0 & 0 & 0
\end{bmatrix}
$$

(4)

Elasticity matrix $C$ is defined as

$$
C = \begin{bmatrix}
c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\
c_{21} & c_{22} & c_{23} & 0 & 0 & 0 \\
c_{31} & c_{32} & c_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & c_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & c_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & c_{66}
\end{bmatrix}
$$

(5)

Variational principles can be used to establish the finite element equations for piezoelectric structures. The global equation of motion governing a structure system with $n$ degrees of freedom can be written as

$$
\begin{bmatrix}
M_{uu} & 0 \\
0 & 0
\end{bmatrix}\begin{bmatrix}
\dot{u} \\
\dot{\phi}
\end{bmatrix} + \begin{bmatrix}
K_{uu} & K_{u\phi} \\
K_{u\phi}^T & K_{\phi\phi}
\end{bmatrix}\begin{bmatrix}
u \\
\phi
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
$$

(6)

Where $\dot{u}$ denotes structural displacement, $\dot{\phi}$ denotes electric potential, and a dot above a variable denotes a time derivative; $M_{uu}$ is the structural mass matrix, $K_{uu}$ is the structural stiffness matrix, $K_{u\phi}$ is the piezoelectric coupling matrix, $K_{\phi\phi}$ is the dielectric stiffness matrix, and a superscript T denotes transpose of a matrix.

3. Numerical Study

Two composite cantilevered beams (one unidirectional and one quasi-isotropic beam) of length 250mm and width 70mm is considered for numerical study as shown in Figure 1a. Figure 1b shows the cross section of the unidirectional (all 0°) beam and the cross section of the quasi-isotropic ($[0/45/-45/90]_T$) beam is shown in Figure 1c. Beam has 8 layer of glass fiber, thickness of each layer is 0.33mm and therefore total thickness of the beam is 2.64mm.
The mechanical properties of glass composite beam are as follows:

- $E_{11} = 40.5 \text{ Gpa}$, $E_{22} = 12.5 \text{ Gpa}$, $G_{12} = 3.96 \text{ Gpa}$
- $\nu_{12} = 0.29$

Density of material is 1900 kg/m$^3$ (for 60% fiber volume fraction)

The PZT-5H is used as a piezoelectric actuator, and its material properties are given as follows:

- $C = \begin{bmatrix} 12.6 & 7.95 & 8.41 & 0 & 0 & 0 \\ 0 & 12.6 & 8.41 & 0 & 0 & 0 \\ 0 & 0 & 11.7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.35 \end{bmatrix}$ Gpa
  
- $e = \begin{bmatrix} 0 & 0 & -6.5 \\ 0 & 0 & -6.5 \\ 0 & 0 & 23.3 \\ 0 & 17 & 0 \\ 17 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ C/m$^2$
- $\varepsilon = \begin{bmatrix} 27 & 0 & 0 \\ 0 & 27 & 0 \times 10^8 F/m \\ 0 & 0 & 29 \end{bmatrix}$

3.1. Transient coupled field analysis of PZT composite beam

Dynamic response of beam is obtained by transient coupled-field finite element analysis using commercial finite element package ANSYS 13. Piezoelectric composite beam modelled for present analysis is shown in Figure 1a. Composite beam is modelled layerwise by giving different properties to each layer. The lamina thickness is 0.33mm,
which is almost one third of the PZT’s thickness (1 mm). Piezoelectric transducer having thickness 1 mm and length 10 mm is glued to the upper surface of composite beam to provide mechanical coupling between PZT and composite beam and VOLT DOF is coupled to provide electrical coupling between two materials. PZT is modelled by giving piezoelectric properties such as permittivity matrix and piezoelectric matrix. The Plane 42 element is used for composite beam and Plane 13 element for PZT. For transient coupled field analysis we need to specify time integration parameters i.e. ALPHA=0.25, DELTA=0.5 and THETA=0.5. Two electrodes are defined on piezoelectric transducer from which bottom electrode is grounded and load is applied on top electrode only. A PZT is excited by giving excitation charge to top electrode at 2 kHz. A time step of $2.5 \times 10^4$ s is used to precisely obtain dynamic response. Same time step and loads are used for transient analysis of each delamination case. Dynamic responses obtained from transient analysis are converted to FRF by using MATLAB code.

3.2. Delamination Simulation for unidirectional beam

To study the effectiveness of proposed methodology three damage scenarios has been considered.

1. Delamination near fixed end of the beam

![](image1)

Fig 2a.Delamination near fixed end of beam

2. Delamination near center of the beam

![](image2)

Fig 2b.Delamination near center of beam

3. Delamination near free end of the beam.

![](image3)

Fig 2c.Delamination near free end of beam

3.3. Delamination Simulation for quasi-isotropic beam

In quasi isotropic beam location of delamination is changed as shown in figure 3

1. Delamination near fixed end of the beam

![](image4)

Fig 3a.Delamination near fixed end of beam
2. Delamination near center of the beam

3. Delamination near free end of the beam.

4. Result and discussion

4.1. Unidirectional Beam

4.1.1. Delamination near the Fixed End

Delamination of 90mm and 120 mm are introduced between 2nd and 3rd layer from bottom layer near fixed end respectively. For each delamination case transient analysis are carried out and FRFs are obtained to observe shift in natural frequencies. Figure 4 shows FRFs for 90mm and 120mm delamination near fixed end between 2nd and 3rd layer from the bottom layer. From Table 1 it has been observed that there is no change in the first natural frequency but there is significant change in the other natural frequencies. Maximum change is observed in the 2nd natural frequency for both the delamination lengths.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Without Delamination</th>
<th>90 mm Delamination</th>
<th>% change in frequency</th>
<th>120mm Delamination</th>
<th>% change in frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>31.25</td>
<td>31.25</td>
<td>0</td>
<td>31.25</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>191.4</td>
<td>164.1</td>
<td>14.26</td>
<td>148.4</td>
<td>22.46</td>
</tr>
<tr>
<td>3</td>
<td>507.8</td>
<td>492.2</td>
<td>3.07</td>
<td>476.6</td>
<td>6.14</td>
</tr>
</tbody>
</table>

Table 1. Percentage change in natural frequencies for 90mm and 120mm delamination near fixed end in unidirectional beam.
4.1.2. Delamination at the center of the beam

In this case also delamination of 90 mm and 120 mm is introduced between 2nd and 3rd layer from the bottom layer near center. For each delamination case transient analysis are carried out and FRFs are obtained to observe shift in natural frequencies. Figure 5 shows FRFs for 90 mm and 120 mm delamination near center between 2nd and 3rd layer from the bottom layer. From table 2 it has been observed that there is no change in the first natural frequency but there is significant change in the other natural frequencies. Maximum change is observed in the 3rd natural frequency for both the delamination lengths.

Table 2 shows percentage change in natural frequencies for 90 mm and 120 mm delamination at center end in unidirectional beam.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Without Delamination</th>
<th>90 mm Delamination</th>
<th>% change in frequency</th>
<th>120 mm Delamination</th>
<th>% change in frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>31.25</td>
<td>31.25</td>
<td>0</td>
<td>31.25</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>191.4</td>
<td>183.6</td>
<td>4.07</td>
<td>168</td>
<td>12.22</td>
</tr>
<tr>
<td>3</td>
<td>507.8</td>
<td>429.7</td>
<td>15.38</td>
<td>386.7</td>
<td>23.84</td>
</tr>
<tr>
<td>4</td>
<td>878.9</td>
<td>843.8</td>
<td>3.99</td>
<td>757.8</td>
<td>13.78</td>
</tr>
</tbody>
</table>

Fig. 5 FRFs for delamination near center between 2nd and 3rd lamina from bottom of the beam

4.1.3. Delamination near the Free End

The third damage case is delamination near free end, where delamination is introduced between 2nd and 3rd lamina from the bottom layer near free end. Same lengths of delamination are considered i.e. 90 mm and 120 mm. For each delamination case transient analysis are carried out and FRFs are obtained to observe shift in natural frequencies.

Table 3 Percentage change in natural frequencies for 90 mm and 120 mm delamination near free end

<table>
<thead>
<tr>
<th>Mode</th>
<th>Without Delamination</th>
<th>90 mm Delamination</th>
<th>% change in frequency</th>
<th>120 mm Delamination</th>
<th>% change in frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>31.25</td>
<td>31.25</td>
<td>0</td>
<td>27.34</td>
<td>12.51</td>
</tr>
</tbody>
</table>
Table 3 shows percentage change in natural frequencies for 90mm and 120mm delamination near free end in unidirectional beam. From table 3 it has been observed that there is no change in the first natural frequency for 90mm delamination near the free end but there is significant change in the other natural frequencies. Maximum change is observed in the 2nd natural frequency for both the delamination lengths.

Figure 6 shows FRFs for 90mm and 120mm delamination near free end between 2nd and 3rd layer from the bottom layer.

4.2. Quasi-isotropic [0/45/-45/90]s Beam

4.2.1. Delamination near Fixed End

A beam with orientation [0/45/-45/90]s is known as quasi-isotropic beam. Similar to the unidirectional beam case, delamination of 90mm and 120mm is introduced between 2nd and 3rd lamina from the top layer near fixed end. For each delamination case transient analysis are carried out and FRFs are obtained to observe shift in natural frequencies. Figure 7 shows FRFs for 90mm and 120mm delamination near center between 2nd and 3rd lamina from top. Unlike from unidirectional beam change has been observed in the first natural frequency of quasi-isotropic beam for both delamination lengths and there is significant change in the other natural frequencies also. Maximum change is observed in the 2nd natural frequency for both the delamination lengths.
Table 4 shows percentage change in natural frequencies for 90mm and 120mm delamination near fixed end in quasi-isotropic beam.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Without Delamination</th>
<th>90 mm Delamination</th>
<th>% change in frequency</th>
<th>120mm Delamination</th>
<th>% change in frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>27.34</td>
<td>23.44</td>
<td>14.26</td>
<td>23.44</td>
<td>14.26</td>
</tr>
<tr>
<td>2</td>
<td>160.2</td>
<td>125</td>
<td>21.97</td>
<td>109.4</td>
<td>31.7</td>
</tr>
<tr>
<td>3</td>
<td>433.6</td>
<td>394.5</td>
<td>9.01</td>
<td>375</td>
<td>13.51</td>
</tr>
<tr>
<td>4</td>
<td>773.4</td>
<td>699.2</td>
<td>9.59</td>
<td>628.9</td>
<td>18.68</td>
</tr>
</tbody>
</table>

4.2.2. Delamination at the Center

In this case also delamination of 90mm and 120 mm is introduced between 2\textsuperscript{nd} and 3\textsuperscript{rd} lamina from top layer near center. For each delamination case transient analysis are carried out and FRFs are obtained to observe shift in natural frequencies. Figure 8 shows FRFs for 90mm and 120mm delamination near center between 2\textsuperscript{nd} and 3\textsuperscript{rd} lamina from top. As given in table 5 significant change in the natural frequencies is observed for every mode of vibration. Maximum change is observed in the 3\textsuperscript{rd} natural frequency for both the delamination lengths.
Table 5 shows percentage change in natural frequencies for 90mm and 120mm delamination near center in quasi-isotropic beam.

Table 5 Percentage change in natural frequencies for 90mm and 120mm delamination near center

<table>
<thead>
<tr>
<th>Mode</th>
<th>Without Delamination</th>
<th>90 mm Delamination</th>
<th>% change in frequency</th>
<th>120mm Delamination</th>
<th>% change in frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>27.34</td>
<td>23.44</td>
<td>14.26</td>
<td>23.44</td>
<td>14.26</td>
</tr>
<tr>
<td>2</td>
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<td>4.93</td>
<td>128.9</td>
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<tr>
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<td>24.33</td>
<td>293</td>
<td>32.43</td>
</tr>
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<td>4</td>
<td>773.4</td>
<td>699.2</td>
<td>9.59</td>
<td>593.8</td>
<td>23.22</td>
</tr>
</tbody>
</table>

4.2.3. Delamination near Free End

The third damage case is delamination near free end, where delamination is introduced between 2nd and 3rd lamina from top layer near free end. Same lengths of delamination are considered i.e. 90mm and 120mm. For each delamination case transient analysis are carried out and FRFs are obtained to observe shift in natural frequencies. Figure9 shows FRFs for 90mm and 120mm delamination near center between 2nd and 3rd layer from top.

As given in table 6 significant change in the natural frequencies is observed for every mode of vibration. Maximum change is observed in the 4th natural frequency for 90mm delamination and in 2nd natural frequency for 120mm delamination near free end.

Table 6 Percentage change in natural frequencies for 90mm and 120mm delamination near free end

<table>
<thead>
<tr>
<th>Mode</th>
<th>Without Delamination</th>
<th>90 mm Delamination</th>
<th>% change in frequency</th>
<th>120mm Delamination</th>
<th>% change in frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>27.34</td>
<td>23.44</td>
<td>14.26</td>
<td>19.63</td>
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<td>625</td>
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</table>
5. Conclusion

In this study natural frequency deviation due to delamination in composite cantilever beam coupled with PZT is investigated numerically. ANSYS FE software is used to estimate dynamic responses and MATLAB code is used to obtain FRFs. From the proposed study it is concluded that natural frequency of beam will reduce significantly if delamination is present in beam at any location. Natural frequency reduction is observed for both unidirectional and quasi-isotropic beams when length of delamination changed from 90mm to 120mm. In the unidirectional beam there is no change in first natural frequency for both the delamination lengths when the delamination is near fixed end and at the center, but the changes observed when 120mm delamination present near the free end. Unlike unidirectional beam change has been observed in the first natural frequency of quasi-isotropic beam for both the delamination lengths and there are significant changes in the other natural frequencies. Maximum changes in the natural frequency are observed usually at 2nd or 3rd mode for the beams.

References