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A full-cooperative diversity beamforming scheme in two-way amplify-and-forward relay systems ☆

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Abstract
Consider a simple two-way relaying channel in which two single-antenna sources exchange information via a multiple-antenna relay. For such a scenario, all the existing approaches that can achieve full cooperative diversity order are based on antenna/relay selection, for which the difficulty in designing the beamforming lies in the fact that a single beamformer needs to serve two destinations. In this paper, a new full-cooperative diversity beamforming scheme that ensures that the relay signals are coherently combined at both destinations is proposed, and analytical results are provided to demonstrate the performance gains. Moreover, the impact of channel estimation error is also evaluated. Finally, numerical results are provided to verify the accuracy of the provided analytical results, and also to show that this proposed scheme can outperform existing schemes based on antenna selection.

1. Introduction

Relay-assisted cooperative transmission is an efficient method to extend the coverage and improve the throughput of wireless systems. To characterize this improvement in transmission reliability, cooperative diversity for wireless relay systems is defined [1], where one relay is applied to create a virtual array for distributed transmission and signal processing. Moreover, the definition of full cooperative diversity order, i.e., the maximum achieved diversity gain for wireless relay systems, is also provided in [1]. In [2,3], more general scenarios with multiple distributed relays are studied, and it is shown that the full diversity order achieved by relay cooperation is equivalent to the number of relays without considering the direct link. In addition, centralized relay cooperative transmissions can be implemented by using multiple antennas. By careful transceiver design, multiple-input and multiple-output (MIMO) techniques can further improve relay transmission performance [4,5].

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Although relay transmissions can improve robustness of performance, they can detract from spectrum efficiency since extra radio resources are required for relaying. Network coding has been introduced as a promising solution to this problem [6,7]. Unlike the complete renewal of interference as in conventional relay transmissions, network coding encourages the use of controllable interference by mixing different messages together at the relay nodes, and thus can support the transmissions of multiple data streams simultaneously. For example, in a two-way relay scenario, the spectral efficiency can be doubled by broadcasting the network coded message at the relay, and decoding the desired message by subtracting self information at each source [11-13]. To improve the efficiency without compromising reliability, the joint design of MIMO and network coding has drawn considerable attention [8-10], especially in MIMO two-way relay systems. In [14], the sum rate for two-way relay systems is optimized through collaborative beamforming, and [15] designs the precoding matrices at the sources and the relay jointly to minimize mean-square error (MSE) for maximum multiplexing in MIMO two-way relay systems.

Despite these studies of MIMO two-way relaying channels, some challenging issues are still left as open problems. For example, consider a simple two-way relaying channel in which two single-antenna sources exchange information via a multiple-antenna relay. To such a scenario, the existing precoding design aims to maximize the sum rate, or minimize transmit power, such as proposed in [16]. However, such precoding schemes cannot achieve full-cooperative diversity, since two different beams are formed at the relay and directed to the two sources, and to the best of the authors' knowledge, how to design a full cooperative diversity relay beamformer is still an open problem. The difficulty in designing the beamforming lies in the fact that a single relay beamformer needs to serve both destinations. Moreover, the study of such a scenario is necessary since it can be commonly found in practical systems, and some typical cases are listed as follows: in cellular systems, users are exchanging information via a base station, where the base station is equipped with multiple antennas and users have only a single antenna due to the limited size and battery capacity of handsets. In wireless sensor networks, single-antenna low-cost sensors communicate with each other via a data fusion center with multiple antennas. In addition, in many emerging data networks, such as smart home and healthcare networks, it can be commonly found that low-cost single-antenna terminals/devices communicate with each other via a network hub with more capability. In this paper, we propose a solution to the addressed problem and the main contribution can be summarized as follows.

Firstly, a full-cooperative diversity relay beamformer is designed for the scenario under consideration, which is more challenging than the scenario with more antennas at the sources. Particularly, when the sources have multiple antennas, the cooperative diversity gain can be achieved by design the precoding at the sources and the relay jointly, and there exist some techniques which can achieve superior outage performance, such as signal alignment in [17], and channel parallelization based precoding design [15]. However, when each source has only a single antenna, these existing schemes cannot work since the signal space is flattened into one dimension at the sources. To the best knowledge of the authors, how to design a full-cooperative diversity beamformer for such a scenario is still an open issue. In this paper, we propose an efficient solution, and the key idea is to use the symmetry of the observation phases at both destinations, and the relay signals can be coherently combined.

Secondly, to evaluate the performance of the proposed transmission scheme, the outage probability is analyzed in this paper. Particularly an upper bound on the signal-to-noise ratio (SNR) is first developed, which facilitates the development of a closed-form expression for the outage probability. The asymptotic analysis of such an bound is also provided to demonstrate the full-cooperative diversity gain achieved by the proposed scheme. *Finally*, the impact of channel estimation error is evaluated, and both analytical and numerical results show that our proposed scheme can still achieve full cooperative diversity gains when the MSE is equivalent infinitesimal to the reciprocal of average SNR.

The rest of this paper is organized as follows. Section 2 describes the system model, and introduces the proposed full-cooperative diversity transmission scheme. In Section 3, the performance analysis of the proposed scheme is studied. The impact of channel estimation error is evaluated in Section 4. In Section 5, the simulation results are shown, followed by the conclusions in Section 5.

Notation: Vectors are denoted as boldface small letters, i.e., $\mathbf{a}$, and a_m denotes the mth element of $\mathbf{a}$ and $\mathbf{a}^T$ is the transpose of $\mathbf{a}$. $|c|$ is the Frobenius norm of c, and c can be either a vector or a number. $\mathbb{E}\{x\}$ is the expectation of x, and $\Pr\{x<p\}$ denotes the probability that the value of a random variable x is less than p. $\Gamma(d)$ is the Gamma function, $\gamma(a,x)$ denotes the lower incomplete Gamma function, $K_\nu(z)$ is the modified Bessel function with imaginary argument, and all the referred special functions follow the form given in [19]. σ^2 is the variance of additive white Gaussian noise at each antenna.

2. System model and protocol description

Consider a two-way relay system, in which two sources S_1 and S_2 exchange messages via a relay R. As illustrated in Fig. 1, each source node is equipped with one antenna, while the relay is equipped with N antennas. For simplicity, all nodes are assumed to employ time division duplexing, where the incoming channel and the corresponding outgoing channel are symmetric. All the channels are modeled as quasi-static Rayleigh fading channels, and each node has access to full perfect source-relay channel state information (CSI).

The transmission can be accomplished in two phases by applying network coding. During the first phase, both sources transmit their own messages to the relay simultaneously, and a network coded observation at the relay can

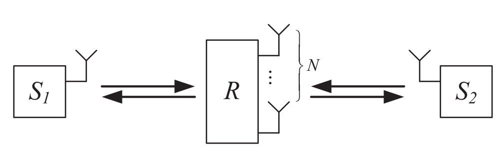

Fig. 1 System model.

be expressed as

$$\mathbf{y}_R = \mathbf{h}x_1 + \mathbf{g}x_2 + \mathbf{n}_R, \tag{1}$$

where x_1 is a message transmitted by S_1, and its transmit power is limited as $\mathbb{E}\{x_1^T x_1\} = P_1$, $\mathbf{h}$ denotes an $N \times 1$ channel vector for the link from S_1 to R, x_2 and $\mathbf{g}$ are defined similarly for S_2, the corresponding transmit power is P_2, and $\mathbf{n}_R$ is the additive Gaussian noise at the relay. Assuming that an amplify-and-forward (AF) strategy is applied, the relay broadcasts its network coding message after beamforming in the second phase. In particular, the forward message at the relay can be given as $\mathbf{t} = \beta\mathbf{Q}\mathbf{y}_R$, where $\mathbf{Q}$ is a $N \times N$ beamforming matrix at the relay, $\beta = \sqrt{\frac{P_R}{|\mathbf{Qh}|^2 P_1 + |\mathbf{Qg}|^2 P_2 + |\mathbf{Q}|^2 \sigma^2}}$ is the power normalization factor to satisfy instantaneous power constraint at the relay as introduced in [9,13,15], and P_R is the relay transmit power. Based on the channel asymmetry assumption, $\mathbf{h}^T$ and $\mathbf{g}^T$ are two $1 \times N$ channel vectors for the links from the relay to S_1 and S_2, respectively, and thus the observations at S_1 and S_2 can be expressed respectively, as follows:

$$y_1 = \mathbf{h}^T\mathbf{t} + n_1 = \beta\mathbf{h}^T\mathbf{Qh}x_1 + \beta\mathbf{h}^T\mathbf{Qg}x_2 + \beta\mathbf{h}^T\mathbf{Q}\mathbf{n}_R + n_1, \tag{2}$$

$$y_2 = \mathbf{g}^T\mathbf{t} + n_2 = \beta\mathbf{g}^T\mathbf{Qg}x_2 + \beta\mathbf{g}^T\mathbf{Qh}x_1 + \beta\mathbf{g}^T\mathbf{Q}\mathbf{n}_R + n_2, \tag{3}$$

where n_i is the additive Gaussian noise at S_i, $i = 1, 2$. Then S_i can subtract its self-interference from its observed network coded message, and obtains the desired message from the other source.

2.1. Full-cooperative diversity beamforming design for studied two-way AF relay systems

The concept of diversity gains in the context of cooperative networks was first proposed for one-way relaying systems [1], where the cooperative diversity is obtained by creating a virtual antenna array and the relays are used to create independent paths between the source and the destination. It can be applied in two-way relay systems straightforwardly to evaluate the transmission reliability. For the addressed scenario with two single-antenna sources and a relay with N antennas, the full diversity gain is N, since the N relay antennas only can create at most N independent paths between two sources. To achieve full cooperative diversity gain, antenna selection is a straightforward method. Specifically, an antenna that maximizes the minimum received SNR of two sources should be selected, which was proposed in [13]. Since our studied scenario is a special case of multi-pair two-way relaying systems, it can be easily verified that antenna selection can achieve full cooperative diversity gain by following the proof of Corollary 4 in [13]. However, antenna selection does not make full use of the multiple-antenna setting at the relay, and the array gain cannot be achieved. To provide a full cooperative diversity gain scheme with better performance, the beamforming design is proposed in this subsection. Particularly, the diversity gain in the context of cooperative networks has been formally defined in [1] as follows.

Definition 2.1. The diversity gain in the context of cooperative networks, defined as α, can be obtained if the outage probability P^{out} of the transmission scheme has the high SNR asymptotic behaviorial as follows:

$$\lim_{\rho \to \infty} -\frac{\log(P^{\text{out}})}{\log(1/\rho)} = \alpha, \tag{4}$$

where ρ denotes the average SNR.

To have a closer look at cooperative diversity gains, we further derive the definition of outage probability based on the results given in [1]

$$P^{\text{out}} = \Pr\{R < R_{\text{th}}\} = \Pr\{\text{SNR} < \gamma_{\text{th}}\}, \tag{5}$$

where R is the transmission data rate, R_{th} is set as a threshold for R, SNR is the receive SNR and γ_{th} is its threshold. To evaluate the outage performance for our proposed scheme, we first derive the receive SNRs for S_1 and S_2 based on the observations given in (2) and (3), respectively,

$$\begin{aligned}\text{SNR}_1 &= \frac{\beta^2|\mathbf{h}^T\mathbf{Qg}|^2 P}{(\beta^2|\mathbf{h}^T\mathbf{Q}|^2 + 1)\sigma^2} \\ &= \frac{\lambda|\mathbf{h}^T\mathbf{Qg}|^2}{(\lambda+1)|\mathbf{Qh}|^2 + |\mathbf{Qg}|^2 + |\mathbf{Q}|^2/\rho}\rho,\end{aligned} \tag{6}$$

$$\begin{aligned}\text{SNR}_2 &= \frac{\beta^2|\mathbf{g}^T\mathbf{Qh}|^2 P}{(\beta^2|\mathbf{g}^T\mathbf{Q}|^2 + 1)\sigma^2} \\ &= \frac{\lambda|\mathbf{g}^T\mathbf{Qh}|^2}{(\lambda+1)|\mathbf{Qg}|^2 + |\mathbf{Qh}|^2 + |\mathbf{Q}|^2/\rho}\rho,\end{aligned} \tag{7}$$

where the transmit power is set as $P_1 = P_2 = P$ at S_1 and S_2, $P_R = \lambda P$ is the transmit power at the relay, and average SNR is defined as $\rho = P/\sigma^2$.

The key step to achieve full cooperative diversity gain is to design the relay beamforming matrix $\mathbf{Q}$. In particular, we focus on the signal parts of SNR_1 and SNR_2

$$\begin{aligned}\mathbf{h}^T\mathbf{Qg} &= \sum_{m=1}^{N}\sum_{n=1}^{N} q_{mn}h_m g_n \\ &= \sum_{m=1}^{N}\sum_{n=1}^{N} 0q_{mn}|h_m||g_n|e^{j(\phi_m+\theta_n)},\end{aligned} \tag{8}$$

$$\begin{aligned}\mathbf{g}^T\mathbf{Qh} &= \sum_{m=1}^{N}\sum_{n=1}^{N} q_{mn}g_m h_n \\ &= \sum_{m=1}^{N}\sum_{n=1}^{N} q_{mn}|g_m||h_n|e^{j(\theta_m+\phi_n)},\end{aligned} \tag{9}$$

where ϕ_p denotes the argument of h_p and θ_q is defined similarly for g_q. As shown in (8) and (9), when $m \neq n$, it is hard to find an appropriate choice for q_{mn} that can be helpful to $\mathbf{g}^T\mathbf{Qh}$ and $\mathbf{h}^T\mathbf{Qg}$ simultaneously. By letting $\mathbf{Q}$ as a diagonal matrix with its diagonal elements as $e^{-j(\theta_m+\phi_n)}$, we can ensure that coherent combination at both sources. Then the beamforming matrix can be defined as follows:

$$\mathbf{Q} = (q_{mn})_{N\times N} = \begin{cases} e^{-j(\phi_m+\theta_n)}, & m = n \\ 0, & m \neq n \end{cases} \tag{10}$$

Substituting (10) into (6) and (7), the SNRs at S_1 and S_2 can be further derived as

$$\text{SNR}_1 = \frac{\lambda\left(\sum_{m=1}^{N}|h_m||g_m|\right)^2}{(\lambda+1)|\mathbf{h}|^2 + |\mathbf{g}|^2 + N/\rho}\rho, \tag{11}$$

$$\mathrm{SNR}_2 = \frac{\lambda\left(\sum_{m=1}^{N}|h_m||g_m|\right)^2}{(\lambda+1)|\mathbf{g}|^2+|\mathbf{h}|^2+N/\rho}\rho. \tag{12}$$

By combining the signal from both paths coherently, our proposed beamforming scheme can achieve full cooperative diversity gain, which is demonstrated in the next section.

3. Performance analysis for proposed beamforming design

In this section, the performance of the proposed full-cooperative diversity beamforming design is evaluated. Firstly, a tractable upper bound on outage probability is derived, and then a closed-form expression for it can be obtained. Next an asymptotic high-SNR expression for the derived upper bound is analyzed, which demonstrates the cooperative diversity gain of our proposed scheme.

3.1. Upper bound of outage probability for proposed scheme

To find a feasible method to evaluate the outage probability of proposed transmission scheme with perfect CSI in this section and the impact of channel estimation error in the next section, we first derive a tractable upper bound for the SNRs, and the following lemma is presented.

Lemma 1. *Denoting that* $\mathbf{a}=[a_1\ldots a_N]^T$ *and* $\mathbf{b}=[b_1\ldots b_N]^T$, *where* a_m *and* b_n *follow independent and identical zero-mean Gaussian distributions with variances* ν, *and* $\nu\le 1$, *the random variable* $w=\frac{\left(\sum_{m=1}^{N}|a_m||b_m|\right)^2}{\frac{1}{N}\left(\sum_{m=1}^{N}|a_m|^2\right)\left(\sum_{n=1}^{N}|b_n|^2\right)}$ *can be bounded as follows with probability 1. Specifically when the number of relay antennas N is large enough*

$$\lim_{N\to\infty}\Pr\{w\ge 1\}=\lim_{N\to\infty}\Pr\left\{w=1+\frac{1}{2}(N-1)\pi^2\right\}=1. \tag{13}$$

Proof. Due to the definition of convergence with probability 1, it is equivalent to prove the following equation for any given positive number ϵ:

$$\lim_{N\to\infty}\Pr\left\{\underbrace{\left|w-\left(1+\frac{1}{2}(N-1)\pi^2\right)\right|}_{\mathcal{A}}<\epsilon\right\}=1. \tag{14}$$

Denoting $u=\frac{1}{N\nu^2}\left(\sum_{m=1}^{N}|a_m||b_m|\right)^2$, $\Pr\left\{\left|\mathcal{A}\right|<\epsilon\right\}$ can be further developed as

$$\Pr\left\{\left|\mathcal{A}\right|<\epsilon\right\} = \Pr\left\{\left|\left(\underbrace{w-u}_{\mathcal{B}_1}\right)+\left[\underbrace{u-\left(1+\frac{1}{2}(N-1)\pi^2\right)}_{\mathcal{B}_2}\right]\right|<\epsilon\right\}. \tag{15}$$

Since the event that $\{|\mathcal{B}_1|<\frac{\epsilon}{2}\}\cap\{|\mathcal{B}_2|<\frac{\epsilon}{2}\}$ is a special case of $|\mathcal{B}_1+\mathcal{B}_2|<\epsilon$, $\Pr\left\{\left|\mathcal{A}\right|<\epsilon\right\}$ can be bounded as

$$\Pr\left\{\left|\mathcal{A}\right|<\epsilon\right\}\ge\Pr\left\{\left|\mathcal{B}_1\right|<\frac{\epsilon}{2},\left|\mathcal{B}_2\right|<\frac{\epsilon}{2}\right\} \overset{(a)}{\ge}\Pr\left\{\left|\mathcal{B}_1\right|<\frac{\epsilon}{2}\right\}+\Pr\left\{\left|\mathcal{B}_2\right|<\frac{\epsilon}{2}\right\}-1, \tag{16}$$

where inequality (a) in (16) can be obtained directly by using Boole's inequality [20]. Therefore, to prove (14), we need to demonstrate that $\Pr\{|\mathcal{B}_1|<\frac{\epsilon}{2}\}$ and $\Pr\{|\mathcal{B}_2|<\frac{\epsilon}{2}\}$ are both convergent with probability 1, respectively, which are given as follows.

1. *The proof of* $\lim_{N\to\infty}\Pr\{|\mathcal{B}_1|<\frac{\epsilon}{2}\}=1$: We first rewritten $\Pr\{|\mathcal{B}_1|<\frac{\epsilon}{2}\}$ as (17)

$$\Pr\left\{\left|\mathcal{B}_1\right|<\frac{\epsilon}{2}\right\} =\Pr\left\{\frac{\frac{1}{N\nu^2}\left(\sum_{m=1}^{N}|a_m||b_m|\right)^2\left|\frac{1}{N^2}\left(\sum_{m=1}^{N}||a_m|^2\right)\left(\sum_{n=1}^{N}|b_n|^2\right)-\nu^2\right|<\frac{\epsilon}{2}}{\frac{1}{N^2}\left(\sum_{m=1}^{N}|a_m|^2\right)\left(\sum_{n=1}^{N}|b_n|^2\right)}\right\} \overset{(a)}{\ge}\Pr\left\{\frac{N}{\nu^2}\left|\frac{1}{N^2}\left(\sum_{m=1}^{N}|a_m|^2\right)\left(\sum_{n=1}^{N}|b_n|^2\right)-\nu^2\right|<\frac{\epsilon}{2}\right\}, \tag{17}$$

where inequality (a) can be obtained by using the Cauchy-Schwarz inequality $(\sum_{m=1}^{N}|a_m||b_m|)^2\le(\sum_{m=1}^{N}|a_m|^2)(\sum_{n=1}^{N}|b_n|^2)$. As introduced previously, a_m and b_n are independent Gaussian distributed variables, and thus $|a_m|^2$ and $|b_n|^2$ follow exponential distributions with a common expectation ν. Based on the law of large numbers [20], we can obtain that

$$\lim_{N\to\infty}\Pr\left\{\left|\mathcal{B}_1\right|<\frac{\epsilon}{2}\right\}\ge \lim_{N\to\infty}\Pr\left\{\left|\frac{1}{N^2}\sum_{m=1}^{N}\sum_{n=1}^{N}|a_m|^2|b_n|^2-\nu^2\right|<\frac{\nu^2}{2N}\epsilon\right\}=1. \tag{18}$$

2. *The proof of* $\lim_{N\to\infty}\Pr\{|\mathcal{B}_2|<\frac{\epsilon}{2}\}=1$. u can be expanded as follows:

$$u=\frac{1}{N\nu^2}\sum_{m=1}^{N}|a_m|^2|b_m|^2+\frac{1}{N\nu^2}\sum_{m\ne n}2|a_m||b_m||a_n||b_n|. \tag{19}$$

Due to the law of large numbers, both expanded parts of u in (19) also approach constants as given in (20) and (21) when N is large

$$\lim_{N\to\infty}\Pr\left\{\underbrace{\left|\frac{1}{N\nu^2}\sum_{m=1}^{N}|a_m|^2|b_m|^2-1\right|}_{\mathcal{C}_1}<\frac{\epsilon}{4}\right\} =\lim_{N\to\infty}\Pr\left\{\left|\frac{1}{N}\sum_{m=1}^{N}|a_m|^2|b_m|^2-\nu^2\right|<\frac{\nu^2}{4}\epsilon\right\}=1, \tag{20}$$

$$\lim_{N\to\infty}\Pr\left\{\underbrace{\left|\frac{1}{N\nu^2}\sum_{m\ne n}2|a_m||b_m||a_n||b_n|-\frac{1}{2}(N-1)\pi^2\right|}_{\mathcal{C}_2}<\frac{\epsilon}{4}\right\}=1, \tag{21}$$

and (21) follows the fact that $|a_p|$ and $|b_q|$ are independent Rayleigh distributed variables. Then the convergence of

$\Pr\{|\mathcal{B}_2|<\frac{\epsilon}{2}\}$ can be proved as

$$\lim_{N\to\infty} \Pr\left\{\left|\mathcal{B}_2\right|<\frac{\epsilon}{2}\right\} = \lim_{N\to\infty} \Pr\left\{\left|\mathcal{C}_1+\mathcal{C}_2\right|<\frac{\epsilon}{2}\right\} \overset{(a)}{\geq} \lim_{N\to\infty} \Pr\left\{\left|\mathcal{C}_1\right|<\frac{\epsilon}{4},\left|\mathcal{C}_2\right|<\frac{\epsilon}{4}\right\} \overset{(b)}{\geq} \lim_{N\to\infty} \Pr\left\{\left|\mathcal{C}_1\right|<\frac{\epsilon}{4}\right\}+\lim_{N\to\infty} \Pr\left\{\left|\mathcal{C}_2\right|<\frac{\epsilon}{4}\right\}-1=1, \quad (22)$$

where inequalities (a) and (b) in (22) can be obtained by following (16).

Based on (16), (18) and (22), we can prove that the inequality $w \geq 1$ can be almost surely (a.s.) established

$$\lim_{N\to\infty} \Pr\{w\geq 1\} = \lim_{N\to\infty} \Pr\left\{w=1+\frac{1}{2}(N-1)\pi^2\right\}=1. \quad (23)$$

The lemma has been proved. □

Based on Lemma 1, the lower bound of the receive SNRs can be given as follows. When the number of relay antennas N is large, the channel vectors can be decoupled in the signal parts for both sources

$$\mathrm{SNR}_1 \overset{\text{a.s.}}{\geq} \frac{\lambda|\mathbf{h}|^2|\mathbf{g}|^2}{N(\mu|\mathbf{h}|^2+|\mathbf{g}|^2+N/\rho)}\rho, \quad (24)$$

$$\mathrm{SNR}_2 \overset{\text{a.s.}}{\geq} \frac{\lambda|\mathbf{h}|^2|\mathbf{g}|^2}{N(\mu|\mathbf{g}|^2+|\mathbf{h}|^2+N/\rho)}\rho, \quad (25)$$

where $\mu=\lambda+1$. Then an upper bound of outage probability can be derived in closed-form, which is presented in the next theorem.

Theorem 2. *An upper bound on outage probability for proposed scheme in Section 2 is given by*

$$P^{\text{out-up}} = \frac{1}{\Gamma(N)}\gamma\left(N,\frac{\mu Na}{\lambda}\right) + \sum_{j=0}^{N-1}\binom{N-1}{j}\frac{\Gamma(N-j-1)(\mu N\gamma_{\text{th}})^j\lambda^{N-j}}{\Gamma(N)}\frac{e^{-\mu Na/\lambda}}{\rho^j} - \sum_{i=0}^{N-1}\sum_{k=0}^{i}\sum_{l=0}^{N-1}2\binom{i}{k}\binom{N-1}{l} \times\frac{\mu^l N^{i+l-k}b^{(N+k-l)/2}\gamma_{\text{th}}^{i+l-k}}{\Gamma(N)\Gamma(i)\lambda^{N+i}} \times K_{k+l-N}\left(\frac{2}{\lambda\rho}\sqrt{b}\right)\frac{e^{-((\mu+1)Na)/\lambda}}{\rho^{N+i}}, \quad (26)$$

where $a=\frac{\gamma_{\text{th}}}{\rho}$ *and* $b=N^2\gamma_{\text{th}}(\mu\gamma_{\text{th}}+\lambda)$.

Proof. Without loss of the generality, the upper bound on outage probability for S_1 is derived. Denoting that $x_1=|\mathbf{h}|^2$ and $x_2=|\mathbf{g}|^2$, it can be written as

$$P^{\text{out-up}} = \Pr\left\{\frac{\lambda x_1 x_2}{N(\mu x_1+x_2+N/\rho)}\rho<\gamma_{\text{th}}\right\} = \Pr\left\{x_2<\frac{\mu Na}{\lambda}\right\} + \Pr\left\{0<x_1<\frac{N\gamma_{\text{th}}x_2+N^2a}{\lambda\rho x_2-\mu N\gamma_{\text{th}}}, x_2>\frac{\mu Na}{\lambda}\right\} = \mathcal{K}_1+\mathcal{K}_2. \quad (27)$$

As introduced previously, x_1 and x_2 are two independent Chi-squared random variables, whose probability density functions (PDFs) are given by

$$f_{x_1} = \frac{x_1^{N-1}e^{-x_1}}{\Gamma(N)}, \quad f_{x_2} = \frac{x_2^{N-1}e^{-x_2}}{\Gamma(N)}. \quad (28)$$

Then substituting (28) into (27), $\mathcal{K}_1$ and $\mathcal{K}_2$ can be obtained

$$\mathcal{K}_1 = \int_0^{\mu Na/\lambda} f_{x_2}dx_2 = \int_0^{\mu Na/\lambda}\frac{x_2^{N-1}e^{-x_2}}{\Gamma(N)}dx_2 = \frac{1}{\Gamma(N)}\gamma\left(N,\frac{\mu Na}{\lambda}\right), \quad (29)$$

$$\mathcal{K}_2 = \int_{\mu Na/\lambda}^{\infty} f_{x_2}\left(\int_0^{(N\gamma_{\text{th}}x_2+N^2a)/\lambda\rho x_2-\mu N\gamma_{\text{th}}} f_{x_1}\,dx_1\right)dx_2 = \frac{1}{[\Gamma(N)]^2}\int_{\frac{\mu Na}{\lambda}}^{\infty}\gamma\left(N,\frac{N\gamma_{\text{th}}x_2+N^2a}{\lambda\rho x_2-\mu N\gamma_{\text{th}}}\right)x_2^{N-1}e^{-x_2}\,dx_2. \quad (30)$$

To obtain the closed-form expression, (30) can be expanded as follows by denoting $z=\frac{N\gamma_{\text{th}}x_2+N^2a}{\lambda\rho x_2-\mu N\gamma_{\text{th}}}$

$$\mathcal{K}_2 = \frac{b}{\Gamma(N)}\int_{Na/\lambda}^{\infty}\frac{\left(\mu N\gamma_{\text{th}}z+N^2a\right)^{N-1}}{(\lambda\rho z-N\gamma_{\text{th}})^{N+1}}e^{-(\mu N\gamma_{\text{th}}z+N^2a)/\lambda\rho z-N\gamma_{\text{th}}}\,dz - \frac{b}{\Gamma(N)}\sum_{i=0}^{N-1}\int_{Na/\lambda}^{\infty}\frac{z^i\left(\mu N\gamma_{\text{th}}z+N^2a\right)^{N-1}}{\Gamma(i)\left(\lambda\rho z-N\gamma_{\text{th}}\right)^{N+1}} e^{-(((\mu N\gamma_{\text{th}}z+N^2a)/\lambda\rho z-N\gamma_{\text{th}})+z)}\,dz = \mathcal{L}_1-\mathcal{L}_2, \quad (31)$$

where $\mathcal{K}_2$ is simplified by applying $\gamma(N,z)=\Gamma(N)[1-e^{-z}\left(\sum_{i=0}^{N-1}\frac{z^i}{\Gamma(i)}\right)]$ in [19]. Then we focus on deriving the closed-form expressions for $\mathcal{L}_1$ and $\mathcal{L}_2$. Particularly denoting $t=\lambda\rho z-N\gamma_{\text{th}}$, $\mathcal{L}_1$ can be derived as

$$\mathcal{L}_1 = \frac{be^{-\mu Na/\lambda}}{\Gamma(N)\lambda^N\rho^N}\int_0^{\infty}\frac{(\mu N\gamma_{\text{th}}t+b)^{N-1}}{t^{N+1}}e^{-b/\lambda\rho t}\,dt = \sum_{j=0}^{N-1}\binom{N-1}{j}(\mu N\gamma_{\text{th}})^j b^{N-j}\frac{e^{-\mu Na/\lambda}}{\Gamma(N)\rho^N}\int_0^{\infty}\frac{e^{-b/\lambda\rho t}}{t^{N-j+1}}\,dt = \sum_{j=0}^{N-1}\binom{N-1}{j}\frac{\Gamma(N-j-1)(\mu N\gamma_{\text{th}})^j\lambda^{N-j}}{\Gamma(N)}\frac{e^{-\mu Na/\lambda}}{\rho^j}, \quad (32)$$

where $\mathcal{L}_1$ is simplified by using the binomial theorem. Following the notation in (32), a closed-form expression for $\mathcal{L}_2$ can be obtained in a similar way

$$\mathcal{L}_2 = \sum_{i=0}^{N-1}\sum_{k=0}^{i}\sum_{l=0}^{N-1}\binom{i}{k}\binom{N-1}{l}\frac{\mu^l N^{i+l-k}b^{N-l}\gamma_{\text{th}}^{i+l-k}}{\Gamma(N)\Gamma(i)\lambda^{N+i}}\frac{e^{-((\mu+1)Na)/\lambda}}{\rho^{N+i}}\int_0^{\infty}t^{k+l-N-1}e^{-(b/\lambda\rho t+t/\lambda\rho)}\,dt = \sum_{i=0}^{N-1}\sum_{k=0}^{i}\sum_{l=0}^{N-1}2\binom{i}{k}\binom{N-1}{l}\frac{\mu^l N^{i+l-k}b^{(N+k-l)/2}\gamma_{\text{th}}^{i+l-k}}{\Gamma(N)\Gamma(i)\lambda^{N+i}}K_{k+l-N}\left(\frac{2}{\lambda\rho}\sqrt{b}\right)\frac{e^{-((\mu+1)Na)/\lambda}}{\rho^{N+i}}. \quad (33)$$

Then the proof is complete. □

Although the derivation of $P^{\text{out-up}}$ is based on the assumption that the number of relay antennas is large, the simulation results in the next section show that the derived upper bound is a general expression, i.e., $P^{\text{out-up}}$ is also applicable when $N=2$.

3.2. Analysis of cooperative diversity gain for the proposed scheme

To derive the achievable cooperative diversity gain of the proposed scheme, an asymptotic analysis of $P^{\text{out}-\text{up}}$ is given. Specifically it can be obtained by applying a high SNR approximation to the original expression for $P^{\text{out}-\text{up}}$ given in (27), and the derivation is given in detail as follows.

High SNR Approximation of $\mathcal{K}_1$ *in* (27): In the high SNR region, $\frac{\mu N a}{\lambda}$ approaches 0, and thus the lower incomplete gamma function in $\mathcal{K}_1$ achieves the following asymptotic form:

$$\gamma\left(N, \frac{\mu N a}{\lambda}\right) \rightarrow \frac{(\mu N \gamma_{\text{th}})^N}{N} \frac{1}{\rho^N}, \tag{34}$$

and (34) can be obtained directly by using the L'Hôpital's rule on the definition of lower incomplete gamma function given by Eq. (8.350.1) in [19] when its argument approaches zero. Then a high SNR approximation to $\mathcal{K}_1$ can be derived as

$$\mathcal{K}_1^{\infty} = \frac{(\mu N \gamma_{\text{th}})^N}{N \Gamma(N)} \frac{1}{\rho^N}. \tag{35}$$

High SNR approximation of $\mathcal{K}_2$ *in* (27): To derive an accurate approximation, we begin with the expression for $\mathcal{K}_2$ given in (30). When ρ is in high SNR region, the argument of the lower incomplete gamma function in (30) approaches

$$\frac{N\gamma_{\text{th}} x_2 + N^2 a}{\lambda \rho x_2 - \mu N \gamma_{\text{th}}} = \frac{N\gamma_{\text{th}} x_2 + (N^2 \gamma_{\text{th}}/\rho)}{\rho(\lambda x_2 - (\mu N \gamma_{\text{th}}/\rho))} \rightarrow \frac{N\gamma_{\text{th}}}{\lambda \rho}. \tag{36}$$

Then the approximation of $\mathcal{K}_2$ can be derived as follows by substituting (36) into (30):

$$\begin{aligned}\mathcal{K}_2^{\infty} &= \frac{1}{[\Gamma(N)]^2} \gamma\left(N, \frac{N\gamma_{\text{th}}}{\lambda\rho}\right) \int_{\frac{\mu N a}{\lambda}}^{\infty} x_2^{N-1} e^{-x_2}\, dx_2 \\ &= \frac{1}{[\Gamma(N)]^2} \gamma\left(N, \frac{N\gamma_{\text{th}}}{\lambda\rho}\right) \left[1 - \gamma\left(N, \frac{\mu N a}{\lambda}\right)\right].\end{aligned} \tag{37}$$

Similar to the approximation of $\mathcal{K}_1$, the asymptotic form of $\mathcal{K}_2$ can be given as follows:

$$\begin{aligned}\mathcal{K}_2^{\infty} &= \frac{N^{N-1}\gamma_{\text{th}}^N}{\lambda^N [\Gamma(N)]^2} \left[1 - \frac{(\mu N \gamma_{\text{th}})^N}{N \lambda^N} \frac{1}{\rho^N}\right] \frac{1}{\rho^N} \\ &= \frac{N^{N-1}\gamma_{\text{th}}^N}{\lambda^N [\Gamma(N)]^2} \frac{1}{\rho^N} + o\left(\frac{1}{\rho^N}\right).\end{aligned} \tag{38}$$

Based on the high SNR approximation of $\mathcal{K}_1$ and $\mathcal{K}_2$, an asymptotic form of $P^{\text{out}-\text{up}}$ is given in the following corollary, which demonstrates the cooperative diversity gain of the proposed scheme.

Corollary 3. *In the high SNR region, the derived upper bound on the outage probability for the proposed scheme, which is provided in Theorem 2, can be approximated as*

$$P_{\infty}^{\text{out-up}} = \frac{(\mu N \gamma_{\text{th}})^N}{N \Gamma(N)} \frac{1}{\rho^N} + \frac{N^{N-1}\gamma_{\text{th}}^N}{\lambda^N [\Gamma(N)]^2} \frac{1}{\rho^N} + o\left(\frac{1}{\rho^N}\right). \tag{39}$$

Substituting (39) into (4), it is easy to show that the cooperative diversity gain of the proposed scheme is N, which is the full-cooperative diversity for the studied scenario.

4. Performance analysis when channel estimation error exists

Based on the perfect CSI assumption, we have evaluated the diversity gains of the proposed beamforming scheme in the addressed scenario. However, channel estimation error is commonly found in practical wireless systems, and in this section its impact will be investigated. Particularly the channel vectors, $\mathbf{h}$ and $\mathbf{g}$, can be expressed as the sums of channel estimates $\hat{\mathbf{h}}$ and $\hat{\mathbf{g}}$ and independent channel estimation errors $\mathbf{e}_1$ and $\mathbf{e}_2$, respectively,

$$\mathbf{h} = \hat{\mathbf{h}} + \mathbf{e}_1, \quad \mathbf{g} = \hat{\mathbf{g}} + \mathbf{e}_2, \tag{40}$$

where $\mathbf{e}_1$ and $\mathbf{e}_2$ are complex Gaussian distributed with variances ϵ. Substituting (40) into (2) and (3), the observations at S_1 and S_2 can be expressed as follows when the CSI is imperfect:

$$\begin{aligned}y_1 &= \hat{\beta}\hat{\mathbf{h}}^T \mathbf{Q}\hat{\mathbf{h}} x_1 + \hat{\beta}\hat{\mathbf{h}}^T \mathbf{Q}\hat{\mathbf{g}} x_2 + \hat{\beta}\left(2\hat{\mathbf{h}}^T \mathbf{Q}\mathbf{e}_1 + \mathbf{e}_1^T \mathbf{Q}\mathbf{e}_1\right) x_1 \\ &\quad + \hat{\beta}\left(\hat{\mathbf{h}}^T \mathbf{Q}\mathbf{e}_2 + \mathbf{e}_1^T \mathbf{Q}\hat{\mathbf{g}} + \mathbf{e}_1^T \mathbf{Q}\mathbf{e}_2\right) x_2 \\ &\quad + \hat{\beta}\left(\mathbf{h}^T + \mathbf{e}_1^T\right) \mathbf{Q}\mathbf{n}_R + n_1,\end{aligned} \tag{41}$$

$$\begin{aligned}y_2 &= \hat{\beta}\hat{\mathbf{g}}^T \mathbf{Q}\hat{\mathbf{g}} x_2 + \hat{\beta}\hat{\mathbf{g}}^T \mathbf{Q}\hat{\mathbf{h}} x_1 + \hat{\beta}\left(2\hat{\mathbf{g}}^T \mathbf{Q}\mathbf{e}_2 + \mathbf{e}_2^T \mathbf{Q}\mathbf{e}_2\right) x_2 \\ &\quad + \hat{\beta}\left(\hat{\mathbf{g}}^T \mathbf{Q}\mathbf{e}_1 + \mathbf{e}_2^T \mathbf{Q}\hat{\mathbf{h}} + \mathbf{e}_2^T \mathbf{Q}\mathbf{e}_1\right) x_1 \\ &\quad + \hat{\beta}(\mathbf{g}^T + \mathbf{e}_2^T) \mathbf{Q}\mathbf{n}_R + n_2.\end{aligned} \tag{42}$$

where $\hat{\beta}$ is the power normalization factor at the relay when channel estimation error exists, given by

$$\hat{\beta} = \sqrt{\frac{P_R}{|\mathbf{Q}\hat{\mathbf{h}}|^2 P_1 + |\mathbf{Q}\hat{\mathbf{g}}|^2 P_2 + |\mathbf{Q}|^2 \epsilon P_1 + |\mathbf{Q}|^2 \epsilon P_2 + |\mathbf{Q}|^2 \sigma^2}}. \tag{43}$$

Since perfect CSI is not accessible in practice, the proposed beamforming scheme is based on channel estimates, and thus the beamforming matrix $\mathbf{Q}$ is given by

$$\mathbf{Q} = (q_{mn})_{N\times N} = \begin{cases} e^{-j(\hat{\phi}_m + \hat{\theta}_n)}, & m = n \\ 0, & m \neq n \end{cases}, \tag{44}$$

where $\hat{\phi}_m$ and $\hat{\theta}_n$ are the arguments of $\hat{h}_m$ and $\hat{g}_n$, respectively. Due to (41), (42), (43) and (44), when channel estimation errors exist, the signal-to-interference-plus-noise ratios (SINRs) at S_1 and S_2 can be (45) and (46), respectively

$$\text{SINR}_1 = \frac{\lambda\left(\sum_{m=1}^{N} |\hat{h}_m| |\hat{g}_m|\right)^2}{\lambda\left[5|\hat{\mathbf{h}}|^2\epsilon + |\hat{\mathbf{g}}|^2\epsilon + (N^2+N)\epsilon^2\right]\rho + \lambda\left(|\hat{\mathbf{h}}|^2 + N\epsilon\right) + \left(|\hat{\mathbf{h}}|^2 + |\hat{\mathbf{g}}|^2 + 2N\epsilon + N/\rho\right)}\rho, \tag{45}$$

$$\text{SINR}_2 = \frac{\lambda\left(\sum_{m=1}^{N} |\hat{h}_m| |\hat{g}_m|\right)^2}{\lambda\left[5|\hat{\mathbf{g}}|^2\epsilon + |\hat{\mathbf{h}}|^2\epsilon + (N^2+N)\epsilon^2\right]\rho + \lambda(|\hat{\mathbf{g}}|^2 + N\epsilon) + \left(|\hat{\mathbf{h}}|^2 + |\hat{\mathbf{g}}|^2 + 2N\epsilon + N/\rho\right)}\rho. \tag{46}$$

Recalling Lemma 1, SINR_1 and SINR_2 can be bounded as follows when N is large:

$$\mathrm{SINR}_1 \overset{\text{a.s.}}{\geq} \frac{\lambda\left|\hat{\mathbf{h}}\right|^2\left|\hat{\mathbf{g}}\right|^2}{\lambda N\left[5|\hat{\mathbf{h}}|^2\epsilon+|\hat{\mathbf{g}}|^2\epsilon+(N^2+N)\epsilon^2\right]\rho+\lambda N\left(|\hat{\mathbf{h}}|^2+N\epsilon\right)+N\left(|\hat{\mathbf{h}}|^2+|\hat{\mathbf{g}}|^2+2N\epsilon+N/\rho\right)}\rho, \tag{47}$$

$$\mathrm{SINR}_2 \overset{\text{a.s.}}{\geq} \frac{\lambda\left|\hat{\mathbf{h}}\right|^2\left|\hat{\mathbf{g}}\right|^2}{\lambda N\left[5|\hat{\mathbf{g}}|^2\epsilon+|\hat{\mathbf{h}}|^2\epsilon+(N^2+N)\epsilon^2\right]\rho+\lambda N(|\hat{\mathbf{g}}|^2+N\epsilon)+N\left(|\hat{\mathbf{h}}|^2+|\hat{\mathbf{g}}|^2+2N\epsilon+N/\rho\right)}\rho. \tag{48}$$

Then an upper bound of outage probability can be obtained when channel estimation error exists, which is given in the following theorem.

Theorem 4. *When channel estimation errors exist, with variance ϵ, an upper bound of outage probability of the proposed scheme can be written as*

$$\begin{aligned}\mathcal{P}^{\text{out}-\text{up}} &= \frac{1}{\Gamma(N)}\gamma\left[N,\frac{c\gamma_{\text{th}}}{\lambda\rho(1-\epsilon)}\right] \\ &+\sum_{j=0}^{N-1}\binom{N-1}{j}\frac{c^j\gamma_{\text{th}}^j\lambda^{N-j}\Gamma(N-j-1)}{(1-\epsilon)^{N+j}\Gamma(N)}\frac{e^{-c\gamma_{\text{th}}/((1-\epsilon)\lambda\rho)}}{\rho^j} \\ &-\sum_{i=0}^{N-1}\sum_{k=0}^{i}\sum_{l=0}^{N-1}2\binom{i}{k}\binom{N-1}{l} \\ &\frac{(cd\gamma_{\text{th}}^2+\lambda q\gamma_{\text{th}}\rho)^{(N+k-l)/2}c^l d^{i-k}\gamma_{\text{th}}^{i+l-k}}{\lambda^{N+i}\rho^{N+i}(1-\epsilon)^{(N+k+l)/2}\Gamma(N)\Gamma(i)} \\ &e^{-(c\gamma_{\text{th}}+(1-\epsilon)d\gamma_{\text{th}})/(1-\epsilon)\lambda\rho}K_{k+l-N}\left(\frac{2}{\lambda\rho}\sqrt{\frac{cd\gamma_{\text{th}}^2+\lambda q\gamma_{\text{th}}\rho}{1-\epsilon}}\right),\end{aligned} \tag{49}$$

where $c=(5\lambda\epsilon\rho+\lambda+1)N$, $d=(\lambda\epsilon\rho+1)N$ *and* $q=[\lambda(N^2+N)\epsilon^2\rho+N\lambda\epsilon+2N\epsilon+N/\rho]N$.

Proof. We focus on the outage probability of S_1 without loss of the generality. Denoting $\hat{x}_1=|\hat{\mathbf{h}}|^2$ and $\hat{x}_2=|\hat{\mathbf{g}}|^2$, the outage probability can be bounded as follows based on (47)

$$\mathcal{P}^{\text{out}-\text{up}}=\Pr\left\{\frac{\lambda\hat{x}_1\hat{x}_2}{c\hat{x}_1+d\hat{x}_2+q}\rho<\gamma_{\text{th}}\right\}. \tag{50}$$

Then $\mathcal{P}^{\text{out}-\text{up}}$ in (50) can be further derived as

$$\begin{aligned}\mathcal{P}^{\text{out}-\text{up}} &= \Pr\left\{\hat{x}_2<\frac{c\gamma_{\text{th}}}{\lambda\rho}\right\} \\ &+\Pr\left\{0<\hat{x}_1<\frac{d\gamma_{\text{th}}\hat{x}_2+q\gamma_{\text{th}}}{\lambda\rho\hat{x}_2-c\gamma_{\text{th}}},\hat{x}_2>\frac{c\gamma_{\text{th}}}{\lambda\rho}\right\} \\ &=\mathcal{J}_1+\mathcal{J}_2.\end{aligned} \tag{51}$$

Note that $\hat{x}_1$ and $\hat{x}_2$ follow the gamma distribution, and so their PDFs can be written as

$$f_{\hat{x}_1}=\frac{\hat{x}_1^{N-1}e^{-\hat{x}_1/(1-\epsilon)}}{(1-\epsilon)^N\Gamma(N)},\quad f_{\hat{x}_2}=\frac{\hat{x}_2^{N-1}e^{-\hat{x}_2/(1-\epsilon)}}{(1-\epsilon)^N\Gamma(N)}. \tag{52}$$

Then $\mathcal{J}_1$ and $\mathcal{J}_2$ in (51) can be derived respectively, as follows:

$$\begin{aligned}\mathcal{J}_1 &= \int_0^{c\gamma_{\text{th}}/\lambda\rho}f_{\hat{x}_2}\,d\hat{x}_2=\int_0^{c\gamma_{\text{th}}/\lambda\rho}\frac{\hat{x}_2^{N-1}e^{-\hat{x}_2/(1-\epsilon)}}{(1-\epsilon)^N\Gamma(N)}\,d\hat{x}_2 \\ &=\frac{1}{\Gamma(N)}\gamma\left[N,\frac{c\gamma_{\text{th}}}{\lambda\rho(1-\epsilon)}\right],\end{aligned} \tag{53}$$

$$\begin{aligned}\mathcal{J}_2 &= \int_{c\gamma_{\text{th}}/\lambda\rho}^{\infty}\left(\int_0^{(d\gamma_{\text{th}}\hat{x}_2+q\gamma_{\text{th}})/\lambda\rho\hat{x}_2-c\gamma_{\text{th}}}f_{\hat{x}_1}\,d\hat{x}_1\right)f_{\hat{x}_2}\,d\hat{x}_2 \\ &=\frac{1}{(1-\epsilon)^{2N}[\Gamma(N)]^2}\int_{c\gamma_{\text{th}}/\lambda\rho}^{\infty}\gamma\left(N,\frac{d\gamma_{\text{th}}\hat{x}_2+q\gamma_{\text{th}}}{\lambda\rho\hat{x}_2-c\gamma_{\text{th}}}\right) \\ &\times\hat{x}_2^{N-1}e^{-\hat{x}_2/(1-\epsilon)}\,d\hat{x}_2.\end{aligned} \tag{54}$$

Similar to (31), $\mathcal{J}_2$ can be expanded as (55) by denoting $\hat{z}=\frac{d\gamma_{\text{th}}\hat{x}_2+q\gamma_{\text{th}}}{\lambda\rho\hat{x}_2-c\gamma_{\text{th}}}$.

$$\begin{aligned}\mathcal{J}_2 &= \frac{cd\gamma_{\text{th}}^2+\lambda q\gamma_{\text{th}}\rho}{(1-\epsilon)^{2N}\Gamma(N)}\int_{c\gamma_{\text{th}}/\lambda\rho}^{\infty} \\ &\times\frac{(c\gamma_{\text{th}}\hat{z}+q\gamma_{\text{th}})^{N-1}}{(\lambda\rho\hat{z}-d\gamma_{\text{th}})^{N+1}}e^{-(c\gamma_{\text{th}}\hat{z}+q\gamma_{\text{th}})/(1-\epsilon)(\lambda\rho\hat{z}-d\gamma_{\text{th}})}\,d\hat{z}- \\ &\times\frac{cd\gamma_{\text{th}}^2+\lambda q\gamma_{\text{th}}\rho}{(1-\epsilon)^{2N}\Gamma(N)\Gamma(i)}\sum_{i=0}^{N-1}\int_{c\gamma_{\text{th}}/\lambda\rho}^{\infty} \\ &\times\hat{z}^i\frac{(c\gamma_{\text{th}}\hat{z}+q\gamma_{\text{th}})^{N-1}}{(\lambda\rho\hat{z}-d\gamma_{\text{th}})^{N+1}}e^{-(c\gamma_{\text{th}}\hat{z}+q\gamma_{\text{th}})/((1-\epsilon)(\lambda\rho\hat{z}-d\gamma_{\text{th}})-\hat{z})}\,d\hat{z}=\mathcal{T}_1-\mathcal{T}_2.\end{aligned} \tag{55}$$

Denoting $\hat{t}=\lambda\rho\hat{z}-d\gamma_{\text{th}}$, explicit expressions of $\mathcal{T}_1$ and $\mathcal{T}_2$ can be given by (56) and (57), respectively, which are similar to (32) and (33),

$$\begin{aligned}\mathcal{T}_1 &= \frac{(cd\gamma_{\text{th}}^2+\lambda q\gamma_{\text{th}}\rho)e^{-c\gamma_{\text{th}}/((1-\epsilon)\lambda\rho)}}{\lambda^N\rho^N(1-\epsilon)^{2N}\Gamma(N)} \\ &\times\int_0^{\infty}\frac{(c\gamma_{\text{th}}\hat{t}+cd\gamma_{\text{th}}^2+\lambda q\gamma_{\text{th}}\rho)^{N-1}}{\hat{t}^{N+1}}e^{-(cd\gamma_{\text{th}}^2+\lambda q\gamma_{\text{th}}\rho)/(1-\epsilon)\lambda\rho\hat{t}}\,d\hat{t} \\ &=\sum_{j=0}^{N-1}\binom{N-1}{j}\frac{c^j\gamma_{\text{th}}^j\lambda^{N-j}\Gamma(N-j-1)}{(1-\epsilon)^{N+j}\Gamma(N)}\frac{e^{-c\gamma_{\text{th}}/((1-\epsilon)\lambda\rho)}}{\rho^j},\end{aligned} \tag{56}$$

$$\begin{aligned}\mathcal{T}_2 &= \sum_{i=0}^{N-1}\frac{(cd\gamma_{\text{th}}^2+\lambda q\gamma_{\text{th}}\rho)e^{-(c\gamma_{\text{th}}+(1-\epsilon)d\gamma_{\text{th}})/(1-\epsilon)\lambda\rho}}{(\lambda\rho)^{N+i}(1-\epsilon)^{2N}\Gamma(N)\Gamma(i)} \\ &\times\int_0^{\infty}(\hat{t}+d\gamma_{\text{th}})^i\frac{(c\gamma_{\text{th}}\hat{t}+cd\gamma_{\text{th}}^2+\lambda q\gamma_{\text{th}}\rho)^{N-1}}{\hat{t}^{N+1}} \\ &\times e^{-((cd\gamma_{\text{th}}^2+\lambda q\gamma_{\text{th}}\rho)/(1-\epsilon)\lambda\rho\hat{t})-(\hat{t}/\lambda\rho)}\,d\hat{t}\end{aligned}$$

$$= \sum_{i=0}^{N-1}\sum_{k=0}^{i}\sum_{l=0}^{N-1} 2\binom{i}{k}\binom{N-1}{l} \times \frac{(cd\gamma_{th}^2+\lambda q\gamma_{th}\rho)^{(N+k-l)/2} c^l d^{i-k}\gamma_{th}^{i+l-k}}{\lambda^{N+i}\rho^{N+i}(1-\epsilon)^{(N+k+l)/2}\Gamma(N)\Gamma(i)} \times e^{-(c\gamma_{th}+(1-\epsilon)d\gamma_{th})/(1-\epsilon)\lambda\rho} K_{k+l-N}\left(\frac{2}{\lambda\rho}\sqrt{\frac{cd\gamma_{th}^2+\lambda q\gamma_{th}\rho}{1-\epsilon}}\right). \quad (57)$$

Substituting (53), (56) and (57) into (51), the theorem follows. □

4.1. Impact of channel estimation errors on cooperative diversity gain

To evaluate the impact of channel estimation errors, the approximation of $\mathcal{P}^{out-up}$ is studied when the SNR is high. In particular, we focus on two typical cases that are analyzed in details as follows.

1. *When the mean square error ϵ is a constant*: For a constant ϵ, we focus on the items related to the SNR ρ in (49). In particular, the argument including ρ in $\mathcal{J}_1$ and $\mathcal{T}_1$ can be approximated as follows when the average SNR is high:

$$\frac{c\gamma_{th}}{\lambda\rho(1-\epsilon)} = \frac{(5\lambda\epsilon\rho+\lambda+1)N\gamma_{th}}{\lambda\rho(1-\epsilon)} \rightarrow \frac{5\epsilon N\gamma_{th}}{1-\epsilon}. \quad (58)$$

Substituting (58) (into 53) and (56), $\mathcal{J}_1$ and $\mathcal{T}_1$ can be written asymptotically as follows:

$$\mathcal{J}_1^{\infty} = \frac{1}{\Gamma(N)}\gamma\left(N, \frac{5\epsilon N\gamma_{th}}{1-\epsilon}\right), \quad (59)$$

$$\mathcal{T}_1^{\infty} = \sum_{j=0}^{N-1}\binom{N-1}{j}\frac{(5\epsilon N\gamma_{th})^j\Gamma(N-j-1)}{(1-\epsilon)^{N+j}\Gamma(N)} e^{-5\epsilon N\gamma_{th}/(1-\epsilon)}. \quad (60)$$

Similarly, the arguments related to ρ in $\mathcal{T}_2$ be approximated as

$$\frac{cd\gamma_{th}^2+\lambda q\gamma_{th}\rho}{\rho^2} \rightarrow (5\gamma_{th}+N+1)\lambda^2N^2\gamma_{th}\epsilon^2 = \zeta,$$
$$\frac{c}{\rho}\rightarrow 5\lambda N\epsilon, \frac{d}{\rho}\rightarrow \lambda N\epsilon. \quad (61)$$

Substituting (61) into (57), $\mathcal{T}_2$ can be approximated as

$$\mathcal{T}_2^{\infty} = \sum_{i=0}^{N-1}\sum_{k=0}^{i}\sum_{l=0}^{N-1}\frac{2\binom{i}{k}\binom{N-1}{l}5^l\zeta^{(N+k-l)/2}(\epsilon N\gamma_{th})^{i+l-k}}{\lambda^{N+k-l}(1-\epsilon)^{(N+k+l)/2}\Gamma(N)\Gamma(i)} e^{-((6-\epsilon)\epsilon N\gamma_{th})/(1-\epsilon)\lambda\rho}K_{k+l-N}\left(\frac{2}{\lambda}\sqrt{\frac{\zeta}{1-\epsilon}}\right). \quad (62)$$

Due to (59), (60) and (62), $\mathcal{P}^{out-up}$ can be written asymptotically as follows when ϵ is a constant:

$$\mathcal{P}_{\infty}^{out-up} = \frac{1}{\Gamma(N)}\gamma\left(N, \frac{5\epsilon N\gamma_{th}}{1-\epsilon}\right) + \sum_{j=0}^{N-1}\binom{N-1}{j} \frac{(5\epsilon N\gamma_{th})^j\Gamma(N-j-1)}{(1-\epsilon)^{N+j}\Gamma(N)} e^{-5\epsilon N\gamma_{th}/(1-\epsilon)} - \sum_{i=0}^{N-1}\sum_{k=0}^{i}\sum_{l=0}^{N-1} \frac{2\binom{i}{k}\binom{N-1}{l}5^l\zeta^{(N+k-l)/2}(\epsilon N\gamma_{th})^{i+l-k}}{\lambda^{N+k-l}(1-\epsilon)^{(N+k+l)/2}\Gamma(N)\Gamma(i)} e^{-((6-\epsilon)\epsilon N\gamma_{th})/(1-\epsilon)\lambda\rho}K_{k+l-N}\left(\frac{2}{\lambda}\sqrt{\frac{\zeta}{1-\epsilon}}\right). \quad (63)$$

Since $\mathcal{P}_{\infty}^{out-up}$ is a constant unrelated to ρ for high SNR, the diversity order is 0 when the mean square error ϵ is a constant. Therefore, in this case, the outage performance saturates at a constant instead of continuing to decrease for perfect CSI.

2. *When the mean square error $\epsilon \sim 1/\rho$*: In this part, we consider another case in which ϵ is equivalent infinitesimal to $1/\rho$, i.e., $\frac{\epsilon}{1/\rho}\rightarrow 1$ when the average SNR ρ is high. Particularly, the argument of the incomplete gamma function in $\mathcal{J}_1$ can be approximated as follows when $\rho\rightarrow 0$ and $\epsilon \sim 1/\rho$:

$$\frac{c\gamma_{th}}{\lambda\rho(1-\epsilon)} = \frac{(5\lambda\epsilon\rho+\lambda+1)N\gamma_{th}}{\lambda(\rho-\epsilon\rho)} \rightarrow \frac{(6\lambda+1)N\gamma_{th}}{\lambda(\rho-1)} \rightarrow 0. \quad (64)$$

Recalling (34), the asymptotic form of $\mathcal{J}_1$ can be expressed as

$$\mathcal{J}_1^{\infty} = \frac{1}{N\Gamma(N)}\left[\left(6+\frac{1}{\lambda}\right)N\gamma_{th}\right]^N \frac{1}{(\rho-1)^N} \sim \frac{1}{N\Gamma(N)}\left[\left(6+\frac{1}{\lambda}\right)N\gamma_{th}\right]^N \frac{1}{\rho^N}. \quad (65)$$

To simplify the approximation of $\mathcal{J}_2$, we begin the asymptotic analysis from (54), where the argument of the incomplete gamma function can be approximated as

$$\frac{d\gamma_{th}\hat{x}_2+q\gamma_{th}}{\lambda\rho\hat{x}_2-c\gamma_{th}} \rightarrow \frac{(\lambda+1)N}{\lambda\rho}. \quad (66)$$

Substituting (66) into (54), $\mathcal{J}_2$ can be further derived as

$$\mathcal{J}_2^{\infty} = \frac{1}{[\Gamma(N)]^2}\gamma\left[N, \frac{(\lambda+1)N}{\lambda\rho}\right]\int_{c\gamma_{th}/\lambda\rho}^{\infty}\hat{x}_2^{N-1}e^{-\hat{x}_2/(1-\epsilon)}\,d\hat{x}_2 = \frac{1}{[\Gamma(N)]^2}\gamma\left[N, \frac{(\lambda+1)N}{\lambda\rho}\right] \times \gamma\left\{\Gamma(N) - \gamma\left[N, \frac{c\gamma_{th}}{\lambda\rho(1-\epsilon)}\right]\right\}. \quad (67)$$

Then the asymptotic form of $\mathcal{J}_2$ can be obtained by following (65),

$$\mathcal{J}_2^{\infty} = \frac{N^{N-1}}{[\Gamma(N)]^2}\left(1+\frac{1}{\lambda}\right)^N \times \left\{\Gamma(N) - \frac{1}{N}\left[\left(6+\frac{1}{\lambda}\right)N\gamma_{th}\right]^N\frac{1}{\rho^N}\right\}\frac{1}{\rho^N}. \quad (68)$$

Based on (65) and (68), the following asymptotic result of $\mathcal{P}^{out-up}$ can be obtained when $\rho\rightarrow 0$ and $\epsilon \sim 1/\rho$

$$\mathcal{P}_{\infty}^{out-up} \sim \frac{1}{N\Gamma(N)}\left[\left(6+\frac{1}{\lambda}\right)N\gamma_{th}\right]^N\frac{1}{\rho^N} + \frac{N^{N-1}}{\Gamma(N)}\frac{1}{\rho^N}$$

$$+o\left(\frac{1}{\rho^N}\right). \tag{69}$$

The last equation demonstrates that the diversity order of the proposed scheme is N, which means that the full diversity gains can be achieved. Considering that the mean square error of the typical channel estimation methods follows the fact that $\epsilon \sim 1/\rho$, our proposed full-cooperative diversity beamforming scheme is still applicable when the channel estimation error exists.

5. Numerical results

In this section, numerical results are provided to evaluate the transmission performance and demonstrate the accuracy of theoretical analysis in the previous section. In Fig. 2, the cumulative distribution functions (CDF) of w defined in Lemma 1 is plotted when the number of relay antennas is set as $N = 3, 4, 5, 6$. As shown in the figure, $\Pr\{w > 1\}$ raises as N increases, and approaches 1 when $N \geq 3$. Such results verify the inequality established with probability 1 in Lemma 1.

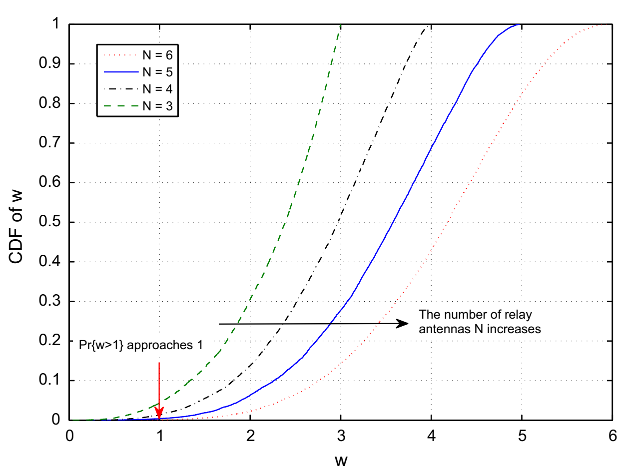

Fig. 2 The CDF of w in Lemma 1.

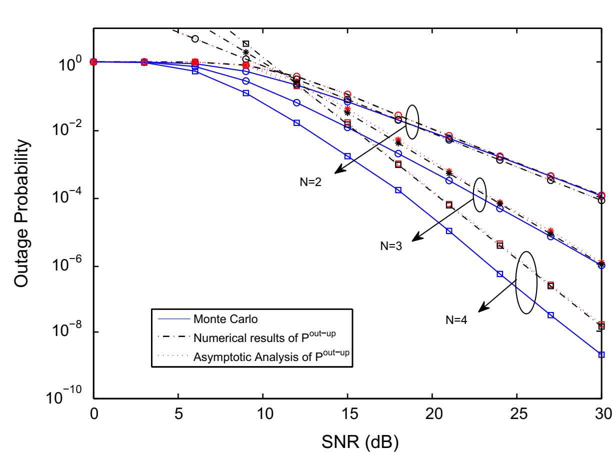

Fig. 3 The derived upper bound on the outage probability, $N = 2, 3, 4$ and $r_{th} = 1$ bit/s/Hz.

Fig. 3 gives numerical results for the derived upper bound on the outage probability and the corresponding asymptotic analysis, where N is set as 2, 3 and 4. The simulation results show that the derived upper bound is quite tight in the high SNR region, especially when N is small. Recalling (23), which shows that the signal power can be lower bounded by abandoning the component of $(2\sum_{m\neq n}|h_m||g_m||h_n||g_n|)/N$, which approaches $\frac{1}{2}(N-1)\pi^2$ when N is large. Therefore, the power of $(2\sum_{m\neq n}|h_m||g_m||h_n||g_n|)/N$ increases as N increases, which enlarges the gap between the exact outage probability and our derived upper bound. However, it is important to point out that removing such a component does not cause any changes in the achievable diversity gain, which can be confirmed by studying the slopes of the curves for the exact outage probability and the asymptotic one. In Fig. 4, the outage performance with different power settings is evaluated for our proposed scheme. Specifically the number of relay antennas is fixed at $N = 3$, and the ratio of relay power to source power λ is set as 0.5, 1, and 2, respectively. As shown in the figure, the outage probability of proposed scheme can be improved as the transmit power increases. In addition, the curves with different power settings have the same slope, which demonstrates the

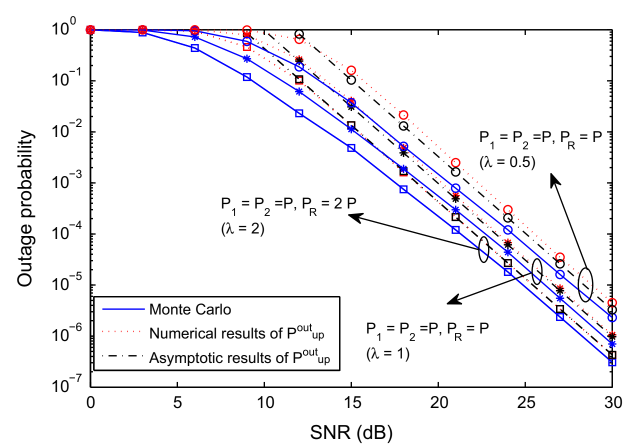

Fig. 4 The outage performance at different power settings, $N = 3$ and $r_{th} = 1$ bit/s/Hz.

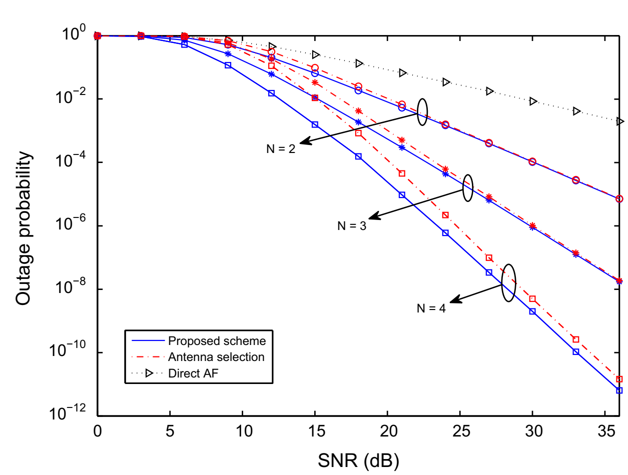

Fig. 5 The outage probability performance, $N = 2, 3, 4$ and $r_{th} = 1$ bit/s/Hz.

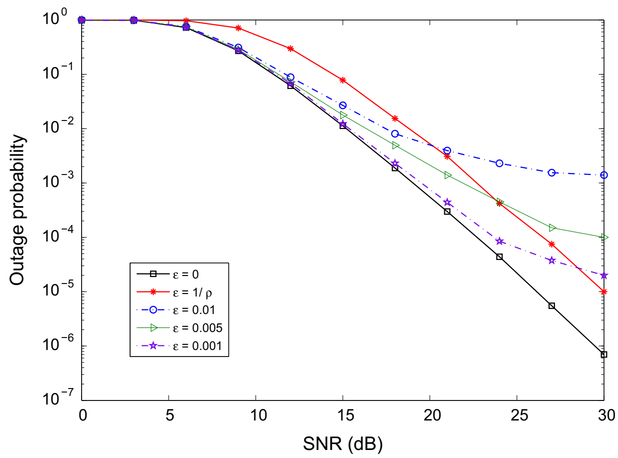

Fig. 6 The outage probability performance with channel estimation errors, $N = 3$ and $\lambda = 1$ and $r_{th} = 1$ bit/s/Hz.

accuracy of the developed analytic results. The curves for the upper bound and the asymptotic analysis in both Figs. 3 and 4 have the same slope as the Monte-Carlo curves, and thus the analytical results in Section 3 can be testified.

To further evaluate the outage performance of the proposed scheme, a comparison of the antenna selection scheme and the direct AF scheme for the studied scenario is provided in Fig. 5, with the numbers of relay antennas are given as $N = 2, 3, 4$. Since there are no additional operations at the relay, the number of relay antennas does not have much impact on the diversity gain achieved by the direct AF scheme, and thus we evaluate the performance only of the best case when N is set as 4. As shown in the figure, the proposed scheme can achieve better performance than the antenna selection scheme, especially in the low SNR region. The performance gain achieved by the proposed beamforming scheme is analog to the advantage of maximal ratio combining (MRC) over selection combining in conventional single-input multiple-output systems, where selection combining uses only a single antenna and MRC can ensure the optimal use of all antennas. It has been verified in [18] that MRC can achieve full-diversity, and has better performance than selection combining, which is consistent with our results.

In Fig. 6, the outage probability for the proposed scheme with channel estimation errors is plotted. In the figure, we provide results for different error variances ϵ. With such channel estimation errors, an error floor for the outage probability is evident in the high SNR region. As ϵ decreases, which means that the estimation accuracy is improved, the error floor can be lowered, and the outage performance approaches that with perfect CSI. Moreover, the curve steadily decreases when $\epsilon = 1/\rho$, and thus the cooperative diversity gains can be ensured.

6. Conclusion

To achieve full-cooperative diversity gain, a joint beamforming and network coding scheme has been proposed for two-way relay systems. By combining the messages from different paths at the sources, transmission reliability can be improved. A closed-form upper bound on the outage probability has been derived, and a high SNR asymptotic analysis has also been given to demonstrate the achieved cooperative diversity gain. Moreover, we have considered the case in which the channel estimation is imperfect, and studied the impact of channel estimation error on the outage probability and the diversity gains. Simulation results have been provided to verify the derived analytical results, which also show that our proposed scheme outperforms the antenna selection scheme in the studied scenario.

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References

[1] J.N. Laneman, D.N.C. Tse, G.W. Wornell, Cooperative diversity in wireless networks: Efficient protocols and outage behavior, IEEE Trans. Inf Theory 50 (December (12)) (2004) 3062-3080.
[2] S. Atapattu, N. Rajatheva, C. Tellambura, Performance analysis of TDMA relay protocols over Nakagami-m fading, IEEE Trans. Veh. Technol. 59 (January (1)) (2010) 93-104.
[3] Y. Zou, Y.D. Yao, B. Zheng, Diversity-multiplexing tradeoff in selective cooperation for cognitive radio, IEEE Trans. Commun. 60 (September (9)) (2012) 2467-2481.
[4] S. Jin, M.R. McKay, C. Zhong, K.-K. Wong, Ergodic capacity analysis of amplify-and-forward MIMO dual-hop systems, IEEE Trans. Inf. Theory 56 (May (5)) (2010) 2204-2224.
[5] W. Xu, X. Dong, W.-S. Lu, Joint precoding optimization for multiuser multi antenna relaying downlinks using quadratic programming, IEEE Trans. Signal Process. 59 (May (5)) (2011) 1228-1235.
[6] M. Peng, C. Yang, Z. Zhao, W. Wang, H. Chen, Cooperative network coding in relay-based IMT-Advanced systems, IEEE Trans. Wireless Commun. 8 (March (3)) (2009) 1247-1259.
[7] Z. Han, X. Zhang, H.V. Poor, High performance cooperative transmission protocols based on multiuser detection and network coding, IEEE Commun. Mag. 8 (May (5)) (2009) 2352-2361.
[8] S. Zhang, S. Liew, P. Lam, Physical layer network coding, in: Proceedings of the ACM MobiCom '06, 2006, p. 358-365 (September).
[9] Z. Ding, K. Leung, D.L. Goeckel, D. Towsley, On the study of network coding with cooperative diversity, IEEE Trans. Wireless Commun. 8 (March (3)) (2009) 1247-1259.
[10] Z. Chen, H. Liu, W. Wang, A novel decoding-and-forward scheme with joint modulation for two-way relay channel, IEEE Commun. Lett. 14 (December (12)) (2010) 1149-1151.
[11] G. Amarasuriya, C. Tellambura, M. Ardakani, Joint beamforming and antenna selection for two-way amplify-and-forward MIMO relay networks, in: Proceedings of the IEEE International Conference on Communications '12, June 2012, pp. 4829-4834.
[12] T. Cui, F. Gao, T. Ho, A. Nallanathan, Distributed space-time coding for two-way wireless relay networks, in: Proceedings of the IEEE International Conference on Communications '08, May 2008, pp. 3888-3892.
[13] Z. Zhao, Z. Ding, M. Peng, W. Wang, K.K. Leung, A special case of multi-way relay channel: when beamforming is not applicable, IEEE Trans. Wireless Commun. 10 (July (7)) (2011) 2046-2051.

[14] M. Zeng, R. Zhang, S. Cui, On design of collaborative beamforming for two-way relay networks, IEEE Trans. Signal Process. 59 (May (5)) (2011) 2284–2295.

[15] R. Wang, M. Tao, Joint source and relay precoding designs for MIMO two-way relaying based on MSE criterion, IEEE Trans. Signal Process. 60 (March (3)) (2012) 1352–1365.

[16] R. Zhang, Y.-C. Liang, C.C. Chai, S. Cui, Optimal beamforming for two-way multi-antenna relay channel with analogue network coding, IEEE J. Sel. Areas Commun. 27 (June (5)) (2009) 699–712.

[17] N. Lee, J. Lim, J. Chun, Degrees of freedom of the MIMO Y channel: Signal space alignment for network coding, IEEE Trans. Inf. Theory 56 (July) (2010) 3332–3342.

[18] A. Goldsmith, Wireless Communications, Cambridge University Press, Colombia, UK, 2005.

[19] I.S. Gradshteyn, I.M. Ryzhik, Table of Integrals, Series, and Products, sixth ed., Academic Press, Waltham, Massachusetts, US, 2000.

[20] G.R. Grimmett, D.R. Stirzaker, Probability and Random Processes, second ed., Clarendon Press, Oxford, UK, 1992.