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Logarithmic corrected $F(R)$ gravity in the light of Planck 2015

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ABSTRACT

In this Letter, we consider the theory of $F(R)$ gravity with the Lagrangian density $\pounds = R + \alpha R^2 + \beta R^2 \ln \beta R$. We obtain the constant curvature solutions and find the scalar potential of the gravitational field. We also obtain the mass squared of a scalaron in the Einstein's frame. We find cosmological parameters corresponding to the recent Plank 2015 results. Finally, we analyze the critical points and stability of the new modified theory of gravity and find that logarithmic correction is necessary to have successful model.

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1. Introduction

Recent astrophysical observations clarify the accelerated expansion of universe [1–3], which may be described by dark energy scenario. In that case, there are several dark energy models, the simplest one is the cosmological constant, however it is not a dynamical model, so there are another alternative theories such as quintessence [4–9], phantom [10–16], and quintom [17–19] models, or holographic dark energy proposal [20–23]. Moreover, there are interesting models to describe the dark energy such as Chaplygin gas [24–42].

Modification of the Einstein–Hilbert (EH) action through the Ricci scalar can describe inflation and also present accelerated expansion of universe. This called $F(R)$ gravity model, so there are several ways to construct a $F(R)$ gravity models [43–46]. In this paper we consider the particular case of the $F(R)$ gravity model where the Ricci scalar replaced by a new function,

$$F(R) = R + \alpha R^2 + \beta R^2 \ln \beta R, \tag{1.1}$$

where $\beta > 0$ is the parameter with the squared length dimension and also $\alpha > 0$. This model can describe the universe evolution without introducing the dark energy [47], where the cosmic acceleration exists due to the modified gravity. So, $F(R)$ gravity models can be replaced to the cosmological constant model.

The function given by (1.1) without logarithmic correction ($\beta = 0$) has been studied by [48,49] which is applicable to a neutron star with a strong magnetic field [50]. In order to consider effect of gluons in curved space–time, the logarithmic correction in (1.1) proposed by [51]. In the Ref. [51] a phenomenological model based on the equation (1.1) proposed. Motivated by this model, we would like to use relation (1.1) to study some cosmological parameters in the light of new data of Planck 2015.

Initial idea of the $F(R)$ gravity models successfully examined by Refs. [52–54]. Then, several models of $F(R)$ gravity introduced in the literature [55–61]. These are indeed phenomenological models which describe evolution of universe. The Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ and $c = \hbar = 1$ are used in the initial $F(R)$ gravity model [62]. Now, we would like to use logarithmic corrected $F(R)$ model given by the equation (1.1) and exam cosmological consequences of the model using recent data of Planck [63].

This paper is organized as follows. In Section 2, we introduce the model, then we study constant curvature condition in Section 3. In Section 4 we obtain the form of the scalar tensor. Cosmological parameters like tensor to scalar ratio are obtained in Section 5. Critical points and stability analyzed in Section 6. Finally, in Section 7 we give conclusion.

2. The model

We begin with the equation (1.1) to modify the Ricci scalar R in the EH action. The function $F(R)$ satisfies the conditions $F(0) = 0$, corresponding to the flat space–time without cosmological constant. Thus, the action in the Jordan frame becomes

$$S = \int d^4x\sqrt{-g}\pounds = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}F(R) + \pounds_m\right], \tag{2.1}$$

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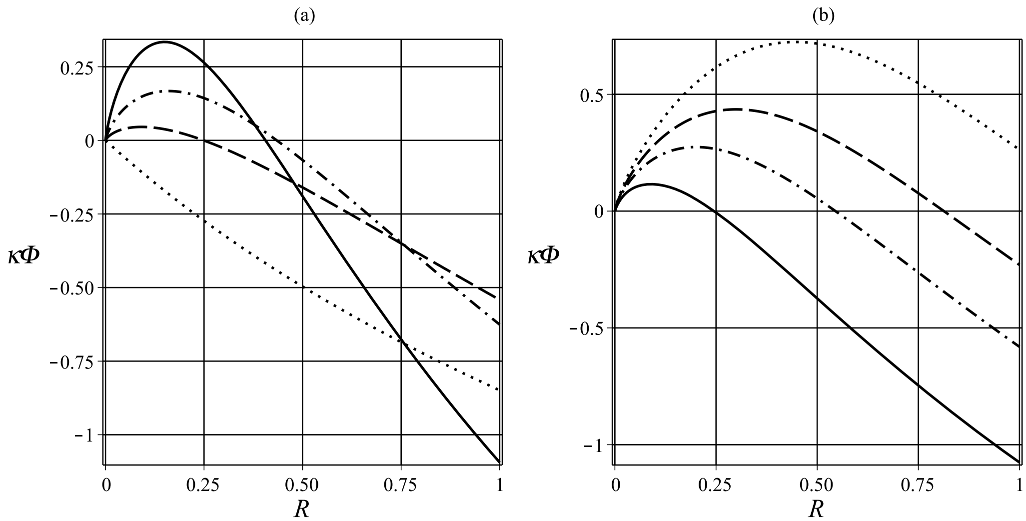

Fig. 1. The function $\kappa\Phi$ versus R. (a) $\alpha = 0.5$, $\beta = 0$ (dot), $\beta = 0.2$ (dash), $\beta = 0.4$ (dot dash), $\beta = 0.8$ (solid). (b) $\beta = 0.5$, $\alpha = 0$ (dot), $\alpha = 0.2$ (dash), $\alpha = 0.4$ (dot dash), $\alpha = 0.8$ (solid).

where $\kappa = M_{pl}^{-1}$, and M_{pl} is the reduced Planck mass, and $\pounds_m$ is the matter Lagrangian density. Our main goal is to study the cosmological parameters describing inflation and the evolution of the early universe. However, we can discuss about consequences in the later time. From the equation (1.1) we obtain

$$F'(R) = 1 + \gamma R + 2\beta R \ln \beta R,$$
$$F''(R) = \lambda + 2\beta \ln \beta R, \tag{2.2}$$

where $\gamma = 2\alpha + \beta$ and $\lambda = 2\alpha + 3\beta$. The function $F(R)$ obeys the quantum stability condition $F''(R) > 0$ for $\alpha > 0$ and $\beta > 0$. This ensures the stability of the solution at high curvature. It follows from the equation (2.2) that the condition of classical stability $F'(R) > 0$ leads to

$$1 + (\gamma + 2\beta \ln \beta R)\, R > 0, \tag{2.3}$$

3. Constant curvature condition

We consider constant curvature solutions of the equations of motion that follow from the action given by (2.1) without matter. The governing equation is given by [64]

$$2F(R) - RF'(R) = 0, \tag{3.1}$$

and hence,

$$R = \frac{1}{\beta}, \tag{3.2}$$

which satisfy $0 < \beta R < 1$. Here, the condition $\frac{F'(R)}{F''(R)} > R$, is satisfied, and therefore the model can describe primordial and present dark energy, which are future stable. From the equation (2.2), we obtain

$$\frac{F'(R)}{F''(R)} = \frac{1 + \gamma R + 2\beta R \ln \beta R}{\lambda + 2\beta \ln \beta R} > R, \tag{3.3}$$

which simplifies to $\beta R < \frac{1}{2}$. Thus, the solution $R_0 = 0$ satisfy the equation (1.1) which then imply that the flat space–time is stable. The second constant curvature solution $\beta R_0 \approx 1$ does not satisfy the equation (3.3), and this leads to unstable de Sitter space–time, so describes inflation.

4. The scalar tensor form

In the Einstein frame corresponding to the scalar tensor theory of gravity, we have the following conformal transformation of the metric [65]:

$$\tilde{g}_{\mu\nu} = F'(R) g_{\mu\nu} = (1 + \gamma R + 2\beta R \ln \beta R) g_{\mu\nu}. \tag{4.1}$$

In that case the action given by the equation (2.1) with $\pounds_m = 0$ written as

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} \nabla_\mu \Phi \nabla_\nu \Phi - V(\phi) \right], \tag{4.2}$$

where ∇_μ is the covariant derivative, and $\tilde{R}$ is determined using the conformal metric in the equation (4.1). The scalar field Φ was found to be

$$\Phi = -\sqrt{\frac{3}{2}} \frac{\ln(1 + \gamma R + 2\beta R \ln \beta R)}{\kappa}. \tag{4.3}$$

In Fig. 1 we plotted the function $\kappa\Phi(R)$ for different values of β and α. From Fig. 1 (b) we can see that increasing α decreases value of the scalar field, while there is no regular behavior with variation of β. However the condition $0 < \beta R < 1$ satisfied in the plots. We can see that the scalar field (multiple by κ) is positive for small R and $\beta > 0$.

The potential V was found to be

$$V = \frac{(\gamma - \alpha) R^2 + \beta R^2 \ln \beta R}{2\kappa^2 (1 + \gamma R + 2\beta R \ln \beta R)^2}. \tag{4.4}$$

In Fig. 2 we plotted the function $\kappa^2 V$ versus R for different values of the parameters. We can see that there is at least an extremum (minimum) obtained via $V' = 0$ which means

$$\frac{R\,(\lambda + 2\beta \ln(\beta R))}{\kappa^2\,(1 + \gamma R + 2\beta R \ln(\beta R))^2} = 0, \tag{4.5}$$

therefore,

$$2\alpha + 3\beta + 2\beta \ln(\beta R) = 0. \tag{4.6}$$

Thus, using the equation (2.3) and (3.3) in the equation (4.6) with the condition $\beta R < 0.5$, we found that the flat space–time is stable with $R = 0$ and the curvature $R_0 = \frac{1}{\beta} e^{-\frac{3}{2} - \frac{\alpha}{\beta}}$ is unstable.

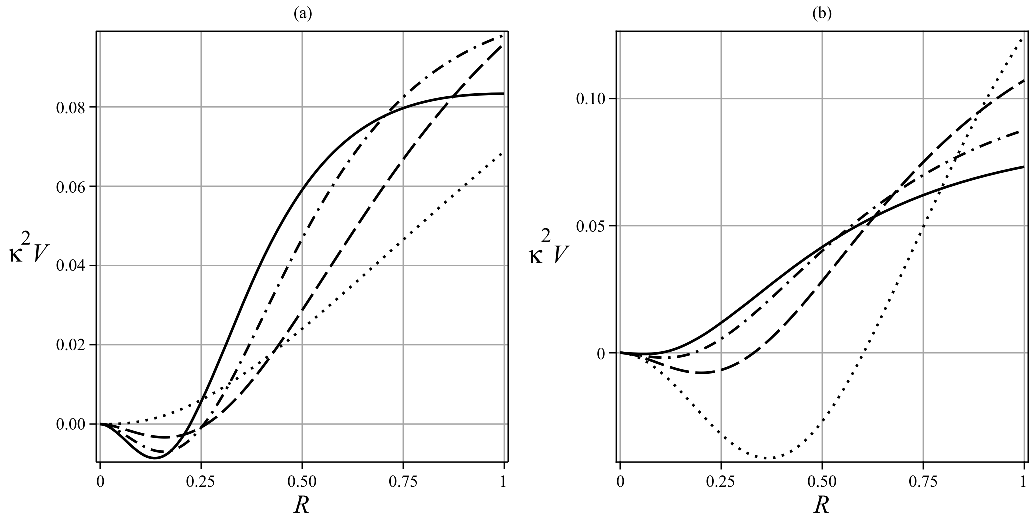

Fig. 2. The function $\kappa^2 V$ versus R. (a) $\alpha = 0.5$, $\beta = 0.1$ (dot), $\beta = 0.4$ (dash), $\beta = 0.7$ (dot dash), $\beta = 1$ (solid). (b) $\beta = 0.5$, $\alpha = 0.1$ (dot), $\alpha = 0.4$ (dash), $\alpha = 0.7$ (dot dash), $\alpha = 1$ (solid).

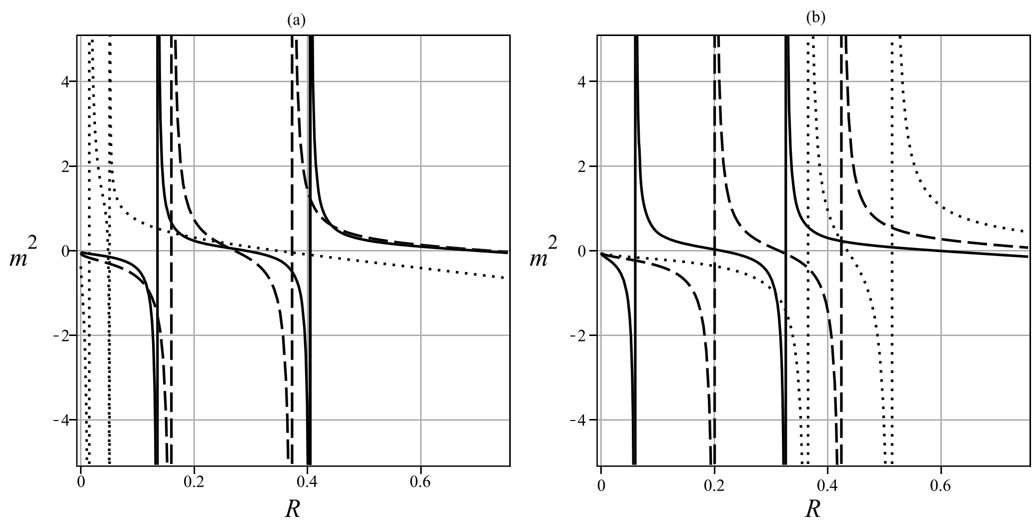

Fig. 3. The function m^2 versus R. (a) $\alpha = 0.5$, $\beta = 0.1$ (dot), $\beta = 0.4$ (dash), $\beta = 1$ (solid). (b) $\beta = 0.5$, $\alpha = 0.1$ (dot), $\alpha = 0.4$ (dash), $\alpha = 1$ (solid).

We also obtain the mass squared of a scalaron,

$$3m^2 = \frac{1}{\lambda + 2\beta\ln(\beta R)} + \frac{R}{1 + \gamma R + 2\beta R\ln(\beta R)} - \frac{4R\,(1 + \alpha R + 2\beta R\ln(\beta R))}{(1 + \gamma R + 2\beta R\ln(\beta R))^2}. \tag{4.7}$$

The plot of the function m^2 versus R is given by Fig. 3 which show periodic behavior. One can verify that $m^2 < 0$ for the constant curvature solution $R_0 = \frac{1}{\beta}e^{-\frac{3}{2}-\frac{\alpha}{\beta}}$, and therefore this solution corresponds to unstable state as it was mentioned earlier.

5. Cosmological parameters

We know that the corrections of $F(R)$ gravity model are small as compared with GR for $R \gg R_0$, where R_0 is a curvature at the present time, so we have the following conditions [66]:

$$\begin{aligned} &|F(R) - R| \ll R, \\ &|F'(R) - 1| \ll 1, \\ &\quad |RF''(R)| \ll 1. \end{aligned} \tag{5.1}$$

From the equation (1.1) we obtain

$$\begin{aligned} &\alpha R + \beta R\ln(\beta R) \ll 1, \\ &\gamma R + 2\beta R\ln(\beta R) \ll 1, \\ &\lambda R + 2\beta R\ln(\beta R) \ll 1. \end{aligned} \tag{5.2}$$

One can investigate that for $0 < \alpha < 1$ all inequalities in the equation (5.2) are satisfied. The slow-roll parameters are given by,

$$\begin{aligned} \varepsilon &= \frac{1}{2}M_{pl}^2\left(\frac{V'}{V}\right)^2, \\ \eta &= \frac{1}{2}M_{pl}^2\frac{V''}{V}. \end{aligned} \tag{5.3}$$

For the slow-roll approximation we need the conditions $\varepsilon \ll 1$, and $\eta \ll 1$. One can obtain the slow-roll parameters expressed through the curvature from the equations (4.4)–(4.7) as follows:

$$\varepsilon = \frac{1}{3}\left[\frac{\beta R - 1}{(\gamma - \alpha)R + \beta R\ln(\beta R)}\right]^2, \tag{5.4}$$

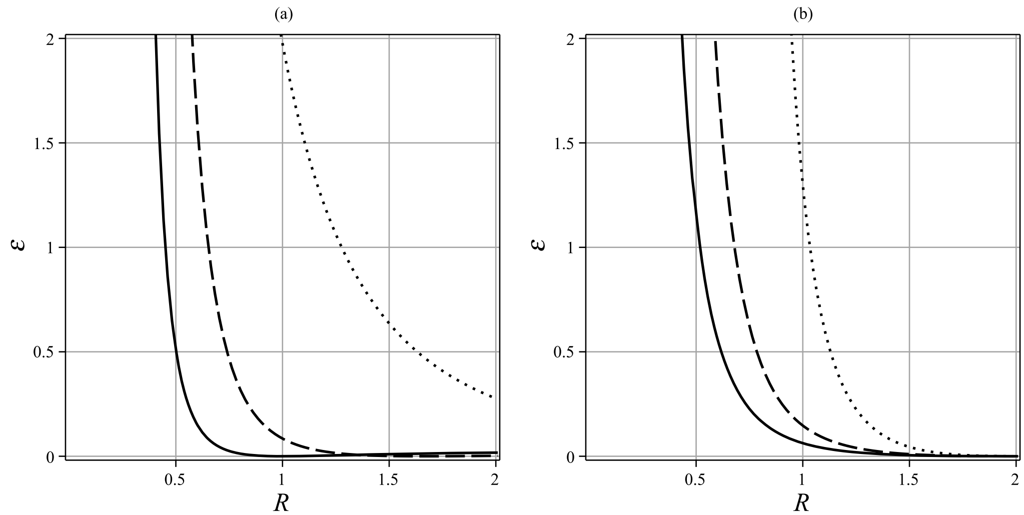

Fig. 4. The function ε versus R. (a) $\alpha = 0.5$, $\beta = 0.1$ (dot), $\beta = 0.6$ (dash), $\beta = 1$ (solid). (b) $\beta = 0.5$, $\alpha = 0.1$ (dot), $\alpha = 0.6$ (dash), $\alpha = 1$ (solid).

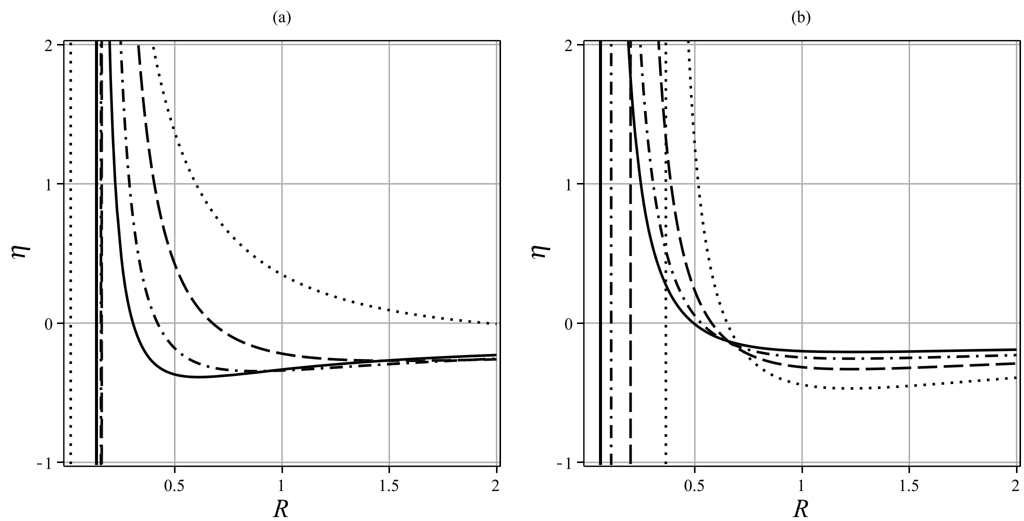

Fig. 5. The function η versus R. (a) $\alpha = 0.5$, $\beta = 0.1$ (dot), $\beta = 0.4$ (dash), $\beta = 0.7$ (dot dash), $\beta = 1$ (solid). (b) $\beta = 0.5$, $\alpha = 0.1$ (dot), $\alpha = 0.4$ (dash), $\alpha = 0.7$ (dot dash), $\alpha = 1$ (solid).

and

$$\eta = \frac{2}{3}\left[\frac{1 - 4\beta R + \beta^2 R^2 - 2\alpha\beta R^2 - 2\beta^2 R^2 \ln(\beta R)}{R(\lambda + 2\beta\ln(\beta R))(1 + \alpha R + \beta R \ln(\beta R))}\right]. \tag{5.5}$$

The plot of the function ε versus R is given by Fig. 4 for different parameters β and α.

The plot of the function η versus R is given in Fig. 5 for different parameters α and β.

The equation $\varepsilon = 1$ has the solution $\beta R = 0.420$. The equation $|\eta| = 1$ is satisfied when $\beta R = 0.1158$, $\beta R = 0.0142 + 0.0485I$ for $0 < \beta < 0.18$, and when $\beta R = 0.2429$, $\beta R = 0.0129 - 0.7917I$ for $0.18 < \beta < 1$. In the case of $0.1158 < \beta R < 1$ and $0.2429 < \beta R < 1$, we have $|\eta| < 1$. Therefore, the slow-roll approximation, $\varepsilon < 1$ and $|\eta| < 1$, takes place when $0.420 < \beta R < 1$.

The age of the inflation can be obtained by calculating the e-fold number,

$$N_e = \frac{3}{2}\int_{R_{end}}^{R_0} \frac{R[(\gamma - \alpha) + \beta\ln(\beta R)](\lambda + 2\beta\ln(\beta R))}{\lambda + 2\beta(1 - \beta R)\ln(\beta R) - 2(\alpha + \beta)\lambda R} dRd. \tag{5.6}$$

Here the value R_{end} corresponds to the time of the end of inflation when ε or $|\eta|$ are close to 1. We find that selected value of βR and β in previous give $50 < N_e < 60$ to solve the flatness and horizon problems.

The index of the scalar spectrum power law due to density perturbations is given by,

$$n_s = 1 - 6\varepsilon + 2\eta. \tag{5.7}$$

The tensor-to-scalar ratio is defined by,

$$r_s = 16\varepsilon. \tag{5.8}$$

The Planck experiment results [63] tell that

$$n_s = 0.968 \pm 0.006,$$
$$r_s < 0.11, \tag{5.9}$$

while adding BICEP2, Keck Array, and Planck (BKP) B-mode data yield to

$$r_s < 0.09. \tag{5.10}$$

We can use above data to fix parameters. One can check that our selected values of α and β give good result in agreement with observational data.

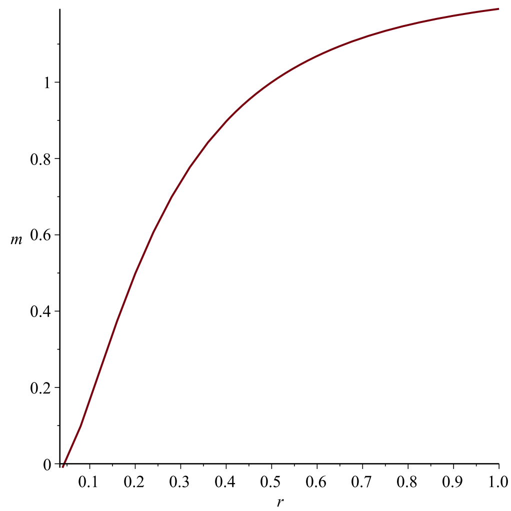

Fig. 6. m versus r.

6. Critical points and stability

There are several ways to specify viable conditions of $F(R)$ gravity theories such as positivity of the effective gravitational coupling [67], stability of cosmological perturbations [68], equivalence principle and Solar-System constraints [69], and asymptotic behavior of the Λ-CDM in the large curvature regime [53]. Here, we use autonomous equations to investigate critical points and stability of the model.

In order to investigate critical points of equations of motion, it is useful to introduce the following dimensionless parameters [43],

$$x_1 = -\frac{\dot{F}'(R)}{HF'(R)} = -\frac{(\lambda + 2\beta \ln(\beta R))\dot{R}}{H(1+\gamma R + 2\beta R \ln(\beta R))}, \tag{6.1}$$

$$x_2 = -\frac{F(R)}{6H^2F'(R)} = -\frac{R+\alpha R^2 + \beta R^2 \ln(\beta R)}{6H^2(1+\gamma R + 2\beta R \ln(\beta R))}, \tag{6.2}$$

$$x_3 = 2 - \frac{\dot{H}}{H^2}, \tag{6.3}$$

$$x_4 = -\frac{\kappa^2 \rho_{rad}}{3F(R)H^2}, \tag{6.4}$$

$$m = \frac{RF''(R)}{F'(R)} = 1 + \frac{2\beta R - 1}{1+\gamma R + 2\beta R \ln(\beta R)}, \tag{6.5}$$

$$r = -\frac{RF'(R)}{F(R)} = -2 + \frac{1-\beta R}{1+\alpha R + \beta R \ln(\beta R)}, \tag{6.6}$$

where H is Hubble parameter, and the dot denote the derivative with respect to the time. The deceleration parameter q is given by $q = 1 - x_3$. The critical points for the system of equations can be studied by the investigation of the function $m(r)$. Equations of motion in the absence of the radiation, $\rho_{rad} = 0$ ($x_4 = 0$), with the help of above equations can be written in the form of autonomous equations [43]. One can discuss the critical points of the system of equations by the study of the function $m(r)$ which shows the deviation from the ΛCDM model. The plot of the function $m(r)$ is given in by Fig. 6.

A de Sitter point in the absence of radiation, $x_4 = 0$, corresponds to the parameters $x_1 = 0$, $x_2 = -1$, and $x_3 = 2$ ($\dot{H} = 0$, $H = \frac{R}{12}$, $r = -2$). This point corresponds to the constant curvature solutions. The effective equation of state (EoS) parameter, w_{eff}, and the parameter of matter energy fraction, Ω_m, are given for this point by the following equations,

$$\omega_{eff} = -1 - \frac{2\dot{H}}{3H^2} = -1, \tag{6.7}$$

and

$$\Omega_m = 1 - x_1 - x_2 - x_3 = 0, \tag{6.8}$$

which correspond to the dark energy. This point mimics a cosmological constant and the deceleration parameter becomes $q = -1$. The constant curvature solution $\beta R \approx 1$ corresponds to unstable de Sitter space.

For the other critical point,

$$(x_1, x_2, x_3) = (\frac{3m}{1+m}, -\frac{1+4m}{2(1+m)^2}, \frac{1+4m}{2(1+m)}), \tag{6.9}$$

we can find $x_3 = \frac{1}{2}$, $m \approx 0$, $r = -1$, and EoS of a matter era is $\omega_{eff} = 0$ ($a = a_0 t^{\frac{2}{3}}$). Then, we have a viable matter dominated epoch prior to late-time acceleration. The equation $m = -r - 1$ has the solution $m = 0$, $r = -1$, $R = 0$, corresponding to this point. One can verify using the equation (6.6) that $m(r = -1) = 0$. As a result, the condition $m(r = -1) > -1$ holds and we have the standard matter era. Therefore, the correct description of the standard matter era occurs in the model under consideration. The equation $m(r) = -r - 1$ with the help of the equations (6.5) and (6.6) becomes

$$\frac{1-2\beta R}{1+\gamma R + 2\beta R \ln(\beta R)} = \frac{1-\beta R}{1+\alpha R + \beta R \ln(\beta R)} \tag{6.10}$$

Equation (6.10) possesses two solutions: the trivial solution $x = \beta R = 0$ ($m = 0$, $r = -1$) corresponding to the second critical point discussed above, and the nontrivial solution. The nontrivial numerical solution of the equation (6.10) for $\alpha = 0.4$ gives $x \approx 3.71$, $m \approx 1.36$, and $r \approx -2.36$.

7. Conclusion

In this Letter, we suggested a new model of modified $F(R)$ gravity representing the effective gravity model which can describe the evolution of universe. Usually, the main purpose of $F(R)$ gravity models is to solve the late-time cosmic acceleration. However, it is possible to study inflation. Our main goal is to compute some inflationary parameters and compare with observational data. The constant curvature solutions, $\beta R = 0.420$, and $\beta R = 0.919$ were obtained corresponding to the flat and the de Sitter space–time, respectively. The de Sitter space–time gives the acceleration of universe and corresponds to inflation. The flat space–time is stable but the de Sitter space–time is unstable in the model and it goes with the maximum of the effective potential. The slow-roll parameters ε, η and the e-fold number of the model were evaluated. The model gives e-fold number $50 < N_e < 60$ characterizing the age of inflation. Agreement of our results with observational data suggests that the logarithmic corrections are useful and may be necessary to construct successful model. We show by the analysis of critical points of autonomous equations that the standard matter era exists and the standard matter era conditions are satisfied. The model may be alternative to GR, and can describe early-time inflation.

There are also more comprehensive model to study inflation, such as $F(R)$ proportional to polynomial inflation [70] with logarithmic correction. We left this point as future work.

References

[1] A.G. Riess, et al., Supernova Search Team Collaboration, Observational evidence from supernovae for an accelerating universe and a cosmological constant, Astron. J. 116 (1998) 1009, arXiv:astro-ph/9805201.

[2] A.G. Riess, et al., Supernova Search Team Collaboration, Type Ia supernova discoveries at $z > 1$ from the Hubble Space Telescope: evidence for past deceleration and constraints on dark energy evolution, Astron. J. 607 (2004) 665, arXiv:astro-ph/0402512.

[3] S. Perlmutter, et al., Supernova Cosmology Project Collaboration, Measurements of Omega and Lambda from 42 high redshift supernovae, Astrophys. J. 517 (1999) 565, arXiv:astro-ph/9812133.

[4] B. Ratra, P.J.E. Peebles, Cosmological consequences of a rolling homogeneous scalar field, Phys. Rev. D 37 (1988) 3406.

[5] C. Wetterich, Cosmology and the fate of dilatation symmetry, Nucl. Phys. B 302 (1988) 668.

[6] A.R. Liddle, R.J. Scherrer, A classification of scalar field potentials with cosmological scaling solutions, Phys. Rev. D 59 (1999) 023509, arXiv:astro-ph/9809272.

[7] Z.-K. Guo, N. Ohta, Y.-Z. Zhang, Parametrizations of the dark energy density and scalar potentials, Mod. Phys. Lett. A 22 (2007) 883, arXiv:astro-ph/0603109.

[8] M. Khurshudyan, E. Chubaryan, B. Pourhassan, Interacting quintessence models of dark energy, Int. J. Theor. Phys. 53 (2014) 2370, arXiv:1402.2385 [gr-qc].

[9] S. Dutta, E.N. Saridakis, R.J. Scherrer, Dark energy from a quintessence (phantom) field rolling near potential minimum (maximum), Phys. Rev. D 79 (2009) 103005, arXiv:0903.3412.

[10] R.R. Caldwell, A phantom menace?, Phys. Lett. B 545 (2002) 23, arXiv:astro-ph/9908168.

[11] R.R. Caldwell, M. Kamionkowski, N.N. Weinberg, Phantom energy and cosmic doomsday, Phys. Rev. Lett. 91 (2003) 071301, arXiv:astro-ph/0302506.

[12] S. Nojiri, S.D. Odintsov, Quantum de Sitter cosmology and phantom matter, Phys. Lett. B 562 (2003) 147, arXiv:hep-th/0303117.

[13] V.K. Onemli, R.P. Woodard, Quantum effects can render $w < -1$ on cosmological scales, Phys. Rev. D 70 (2004) 107301, arXiv:gr-qc/0406098.

[14] E.N. Saridakis, Theoretical limits on the equation-of-state parameter of phantom cosmology, Phys. Lett. B 676 (2009) 7, arXiv:0811.1333.

[15] E.N. Saridakis, Phantom evolution in power-law potentials, Nucl. Phys. B 819 (2009) 116, arXiv:0902.3978.

[16] G. Gupta, E.N. Saridakis, A.A. Sen, Non-minimal quintessence and phantom with nearly flat potentials, Phys. Rev. D 79 (2009) 123013, arXiv:0905.2348.

[17] Z.K. Guo, Y.S. Piao, X.M. Zhang, Y.Z. Zhang, Cosmological evolution of a quintom model of dark energy, Phys. Lett. B 608 (2005) 177, arXiv:astro-ph/0410654.

[18] W. Zhao, Quintom models with an equation of state crossing -1, Phys. Rev. D 73 (2006) 123509, arXiv:astro-ph/0604460.

[19] Y.F. Cai, E.N. Saridakis, M.R. Setare, J.Q. Xia, Quintom cosmology: theoretical implications and observations, Phys. Rep. 493 (2010) 1, arXiv:0909.2776.

[20] H. Li, Z.K. Guo, Y.Z. Zhang, A tracker solution for a holographic dark energy model, Int. J. Mod. Phys. D 15 (2006) 869, arXiv:astro-ph/0602521.

[21] J. Sadeghi, B. Pourhassan, Z.A. Moghaddam, Interacting entropy-corrected holographic dark energy and IR cut-off length, Int. J. Theor. Phys. 53 (2014) 125, arXiv:1306.2055.

[22] M.R. Setare, J. Zhang, X. Zhang, Statefinder diagnosis in a non-flat universe and the holographic model of dark energy, J. Cosmol. Astropart. Phys. 0703 (2007) 007, arXiv:gr-qc/0611084.

[23] E.N. Saridakis, Holographic dark energy in braneworld models with moving branes and the $w = -1$ crossing, J. Cosmol. Astropart. Phys. 0804 (2008) 020, arXiv:0712.2672.

[24] H. Saadat, B. Pourhassan, FRW bulk viscous cosmology with modified Chaplygin gas in flat space, Astrophys. Space Sci. 343 (2013) 783.

[25] U. Debnath, A. Banerjee, S. Chakraborty, Role of modified Chaplygin gas in accelerated universe, Class. Quantum Gravity 21 (2004) 5609, arXiv:gr-qc/0411015.

[26] H. Saadat, B. Pourhassan, FRW bulk viscous cosmology with modified cosmic Chaplygin gas, Astrophys. Space Sci. 344 (2013) 237.

[27] B. Pourhassan, Viscous modified cosmic Chaplygin gas cosmology, Int. J. Mod. Phys. D 22 (2013) 1350061, arXiv:1301.2788.

[28] J. Sadeghi, B. Pourhassan, M. Khurshudyan, H. Farahani, Time-dependent density of modified cosmic Chaplygin gas with cosmological constant in non-flat universe, Int. J. Theor. Phys. 53 (2014) 911.

[29] E.V. Linder, R.J. Scherrer, Aetherizing Lambda: barotropic fluids as dark energy, Phys. Rev. D 80 (2009) 023008, arXiv:0811.2797.

[30] K.N. Ananda, M. Bruni, Cosmological dynamics and dark energy with a nonlinear equation of state: a quadratic model, Phys. Rev. D 74 (2006) 023523, arXiv:astro-ph/0512224.

[31] B. Pourhassan, E.O. Kahya, FRW cosmology with the extended Chaplygin gas, Adv. High Energy Phys. 2014 (2014) 231452, arXiv:1405.0667.

[32] B. Pourhassan, Unified universe history through phantom extended Chaplygin gas, arXiv:1504.04173 [gr-qc].

[33] B. Pourhassan, E.O. Kahya, Extended Chaplygin gas model, Results Phys. 4 (2014) 101.

[34] E.O. Kahya, B. Pourhassan, Observational constraints on the extended Chaplygin gas inflation, Astrophys. Space Sci. 353 (2014) 677.

[35] F.C. Santos, M.L. Bedran, V. Soares, On the thermodynamic stability of the generalized Chaplygin gas, Phys. Lett. B 636 (2006) 86.

[36] Y.S. Myung, Thermodynamics of Chaplygin gas, Astrophys. Space Sci. 335 (2011) 561.

[37] K. Karami, S. Ghaffari, M.M. Soltanzadeh, The generalized second law for the interacting generalized Chaplygin gas model, Astrophys. Space Sci. 331 (2011) 309.

[38] M. Salti, Thermodynamics of Chaplygin gas interacting with cold dark matter, Int. J. Theor. Phys. 52 (2013) 4583.

[39] F.C. Santos, M.L. Bedran, V. Soares, On the thermodynamic stability of the modified Chaplygin gas, Phys. Lett. B 646 (2007) 215.

[40] S. Bhattacharya, U. Debnath, Thermodynamics of modified Chaplygin gas and tachyonic field, Int. J. Theor. Phys. 51 (2012) 565.

[41] U. Debnath, M. Jamil, Correspondence between DBI-essence and modified Chaplygin gas and the generalized second law of thermodynamics, Astrophys. Space Sci. 335 (2011) 545.

[42] T. Bandyopadhyay, Thermodynamics of Gauss–Bonnet brane with modified Chaplygin gas, Astrophys. Space Sci. 341 (2012) 689.

[43] L. Amendola, R. Gannouji, D. Polarski, S. Tsujikawa, Conditions for the cosmological viability of $f(R)$ dark energy models, Phys. Rev. D 75 (2007) 083504, arXiv:gr-qc/0612180.

[44] Z. Girones, A. Marchetti, O. Mena, C. Pena-Garay, N. Rius, Cosmological data analysis of $f(R)$ gravity models, J. Cosmol. Astropart. Phys. 1011 (2010) 004, arXiv:0912.5474.

[45] E.O. Kahya, M. Khurshudyan, B. Pourhassan, R. Myrzakulov, A. Pasqua, Higher order corrections of the extended Chaplygin gas cosmology with varying G and Λ, Eur. Phys. J. C 75 (2015) 43, arXiv:1402.2592.

[46] S. Domazet, V. Radovanovic, M. Simonovic, H. Stefancic, On analytical solutions of $f(R)$ modified gravity theories in FLRW cosmologies, Int. J. Mod. Phys. D 22 (2013) 1350006, arXiv:1203.5220.

[47] S. Nojiri, S.D. Odintsov, Unified cosmic history in modified gravity: from $F(R)$ theory to Lorentz non-invariant models, Phys. Rep. 505 (2011) 59, arXiv:1011.0544.

[48] A.vS. Arapoglu, C. Deliduman, K.Y. Eksi, Constraints on perturbative $f(R)$ gravity via neutron stars, J. Cosmol. Astropart. Phys. 1107 (2011) 020, arXiv:1003.3179.

[49] M. Orellana, F. Garcia, F.A. Teppa Pannia, G.E. Romero, Structure of neutron stars in R-squared gravity, Gen. Relativ. Gravit. 45 (2013) 771, arXiv:1301.5189.

[50] M.-K. Cheoun, et al., Neutron stars in a perturbative $f(R)$ gravity model with strong magnetic fields, J. Cosmol. Astropart. Phys. 1310 (2013) 021, arXiv:1304.1871.

[51] H. Alavirad, J.M. Weller, Modified gravity with logarithmic curvature corrections and the structure of relativistic stars, Phys. Rev. D 88 (2013) 124034, arXiv:1307.7977.

[52] W. Hu, I. Sawicki, Models of $f(R)$ cosmic acceleration that evade Solar-System tests, Phys. Rev. D 76 (2007) 064004, arXiv:0705.1158.

[53] S.A. Appleby, R.A. Battye, Do consistent $F(R)$ models mimic general relativity plus Λ?, Phys. Lett. B 654 (7) (2007), arXiv:0705.3199.

[54] A.A. Starobinsky, Disappearing cosmological constant in $f(R)$ gravity, JETP Lett. 86 (2007) 157, arXiv:0706.2041.

[55] S.I. Kruglov, Born – Infeld-like modified gravity, Int. J. Theor. Phys. 52 (2013) 2477, arXiv:1202.4807.

[56] S.I. Kruglov, On exponential modified gravity, Int. J. Mod. Phys. A 28 (2013) 13501194, arXiv:1204.6709.

[57] S.I. Kruglov, Modified arctan-gravity model mimicking a cosmological constant, Phys. Rev. D 89 (2014) 064004, arXiv:1310.6915.

[58] G. Cognola, E. Elizalde, S. Nojiri, S.D. Odintsov, L. Sebastiani, S. Zerbini, Class of viable modified $f(R)$ gravities describing inflation and the onset of accelerated expansion, Phys. Rev. D 77 (2008) 046009, arXiv:0712.4017.

[59] Eric V. Linder, Exponential gravity, Phys. Rev. D 80 (2009) 123528, arXiv:0905.2962.

[60] M. Banados, P.G. Ferreira, Eddington's theory of gravity and its progeny, Phys. Rev. Lett. 105 (2010) 011101, arXiv:1006.1769.

[61] P. Pani, V. Cardoso, T. Delsate, Compact stars in Eddington inspired gravity, Phys. Rev. Lett. 107 (2011) 031101, arXiv:1106.3569.

[62] A.A. Starobinsky, A new type of isotropic cosmological models without singularity, Phys. Lett. B 91 (1980) 99.

[63] P.A.R. Ade, et al., Planck Collaboration, Planck 2015 results. XIII. Cosmological parameters, arXiv:1502.01589.

[64] J.D. Barrow, A.C. Ottewill, The stability of general relativistic cosmological theory, J. Phys. A, Math. Gen. 16 (1983) 2757.

[65] G. Magnano, L.M. Sokolowski, On physical equivalence between nonlinear gravity theories, Phys. Rev. D 50 (1994) 5039, arXiv:gr-qc/9312008.
[66] S. Appleby, R. Battye, A. Starobinsky, Curing singularities in cosmological evolution of $F(R)$ gravity, J. Cosmol. Astropart. Phys. 1006 (2010) 005, arXiv:0909.1737.
[67] R. Brustein, G. Dvali, G. Veneziano, A bound on the effective gravitational coupling from semiclassical black holes, J. High Energy Phys. 0910 (2009) 085, arXiv:0907.5516.
[68] A. de la Cruz-Dombriz, D. Saez-Gomez, On the stability of the cosmological solutions in $f(R, G)$ gravity, Class. Quantum Gravity 29 (2012) 245014, arXiv:1112.4481.
[69] S. Capozziello, S. Tsujikawa, Solar System and equivalence principle constraints on $f(R)$ gravity by chameleon approach, Phys. Rev. D 77 (2008) 107501, arXiv:0712.2268.
[70] Q.-G. Huang, A polynomial $f(R)$ inflation model, J. Cosmol. Astropart. Phys. 1402 (2014) 035, arXiv:1309.3514.