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Effects of volumetric heat source and temperature dependent viscosity on natural convection flow along a wavy surface

M. A. Alim[a]*, Shahidul Alam[b], M. Miraj[c]

[a]*Department of Mathematics, Bangladesh University of Engineering and Technology, Dhaka 1000, Bangladesh*

[b]*Department of Mathematics, Northern College Bangladesh, Dhaka, Bangladesh*

[c]*Department of Mathematics, Dhaka Commerce College, Dhaka-1216, Bangladesh*

Abstract

The conjugate effects of volumetric heat source and temperature dependent viscosity on natural convection flow along a wavy surface have been investigated. The governing boundary layer equations of the present physical problem are first transformed into non-dimensional form using suitable set of dimensionless variables. The resulting nonlinear system of partial differential equations are mapped into the domain of a vertical flat plate and then solved numerically employing the implicit finite difference method, known as Keller-box scheme. The numerical results of the surface shear stress in terms of skin friction coefficient as well as the rate of heat transfer in terms of local Nusselt number are shown in tabular form and the stream lines as well as the isotherms are shown graphically for a selection of parameters set consisting of viscosity variation parameter ε, internal heat generation parameter Q of volumetric heat source and Prandtl number Pr.

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Keywords: Natural convection; volumetric heat source; internal heat generation; wavy surface; variable viscosity; finite difference method.

* Corresponding author. Tel.: +8801552345618; fax: +8802-861-3046.
E-mail address: maalim@math.buet.ac.bd

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1. Introduction

In this study, the effects of temperature dependent viscosity on natural convection flow along a wavy surface with internal heat generation due to volumetric heat source have been focused. The sinusoidal wavy surface can be viewed as an approximation to match practical geometries like cooling fin or roughened surface in heat transfer. Roughened surfaces are better heat transfer devices than a plain surface and are encountered in several heat transfer devices such as flat plate solar collectors, condensers in refrigerators etc. The effects of such non-uniformities on the vertical convective boundary layer flow of a Newtonian fluid are first studied by Yao [1] and using an extended Prantdl's transposition theorem and a finite-difference scheme. Alam et al. [2] have studied the problem of free convection from a wavy vertical surface in presence of a transverse magnetic field. Combined effects of thermal and mass diffusion on the natural convection flow of a viscous incompressible fluid along a vertical wavy surface have been investigated by Hossain and Rees [3]. Hossain et al. [4] have studied the problem of natural convection of fluid with temperature dependent viscosity along a heated vertical wavy surface. Natural convection heat and mass transfer along a vertical wavy surface have been investigated by Jang et al. [5]. Molla et al. [6] have studied natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation/absorption. Tashtoush and Al-Odat [7] investigated magnetic field effect on heat and fluid flow over a wavy surface with a variable heat flux. Yao [8] studied natural convection along a vertical complex wavy surface. Molla and Gorla [9] studied natural convection laminar flow with temperature dependent viscosity and thermal conductivity along a vertical wavy surface. Parveen and Alim [10] investigated Joule heating effect on Magnetohydrodynamic natural convection flow along a vertical wavy surface with viscosity dependent on temperature. Hossain et al. [11] investigated the natural convection flow past a permeable wedge for the fluid having temperature dependent viscosity and thermal conductivity. The present study is to incorporate the idea of the effects of volumetric heat source and temperature dependent viscosity on natural convection flow along a uniformly heated vertical wavy surface.

2. Formulation of the problem

Steady, two dimensional natural convection flow of a viscous and incompressible fluid with variable viscosity along a vertical wavy surface is considered. The surface temperature of the vertical wavy surface T_w is uniform, where $T_w > T_\infty$. The boundary layer analysis outlined below allows $\bar{\sigma}(\bar{x})$ being arbitrary, but detailed numerical work assumed that the surface exhibits sinusoidal deformations. The wavy surface may be described by

$$\bar{y}_w = \bar{\sigma}(\bar{x}) = \alpha \sin\left(n\pi\bar{x}/L\right) \tag{1}$$

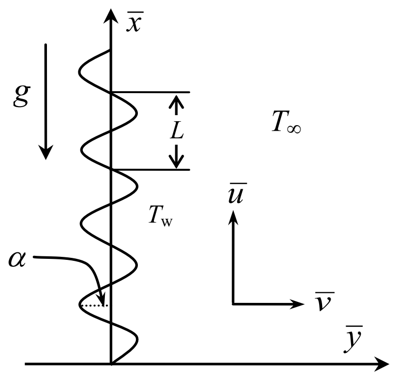

Fig. 1. Physical model and coordinate system

The geometry of the wavy surface and the two-dimensional Cartesian coordinate system are shown in Fig. 1. The conservation equations on the flow field, the continuity, momentum and energy equations can be written as:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{2}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{x}} + \frac{1}{\rho}\nabla.\left(\mu\nabla\bar{u}\right) + g\beta(T - T_\infty) \quad (3)$$

$$\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{y}} + \frac{1}{\rho}\nabla.\left(\mu\nabla\bar{v}\right) \quad (4)$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \frac{k}{\rho C_p}\nabla^2 T + \frac{Q_0(T - T_\infty)}{\rho C_p} \quad (5)$$

where $(\bar{x},\bar{y})$ are the dimensional coordinates along and normal to the tangent of the surface and $(\bar{u},\bar{v})$ are the velocity components parallel to $(\bar{x},\bar{y})$, g is the acceleration due to gravity, $\bar{p}$ is the dimensional pressure of the fluid, ρ is the density, β is the coefficient of thermal expansion, μ is the viscosity in the boundary layer, k is the thermal conductivity and C_p is the specific heat due to constant pressure. The boundary conditions relevant to the above problem are:

$$\bar{u} = 0,\ \bar{v} = 0,\ T = T_w \ \text{ at } \ \bar{y} = \bar{y}_w = \bar{\sigma}(\bar{x}) \quad \text{and } \bar{u} = 0,\ T = T_\infty,\ \bar{p} = p_\infty \text{ as } \ \bar{y} \to \infty \quad (6)$$

where T_w is the surface temperature, T_∞ is the ambient temperature of the fluid and p_∞ is the pressure of fluid outside the boundary layer. The variable viscosity chosen in this investigation that is introduced by Charraudeau [12] and used by Hossain et al. [11] as follows:

$$\mu = \mu_\infty[1 + \varepsilon^*(T - T_\infty)] \quad (7)$$

where μ_∞ is the viscosity of the ambient fluid and $\varepsilon^* = 1/\mu_f\left(\partial\mu/\partial T\right)_f$ is a constant evaluated at the film temperature of the flow $T_f = 1/2(T_w + T_\infty)$. Using Prandtl's transposition theorem to transform the irregular wavy surface into a flat surface as extended by Yao [2] and boundary-layer approximation, the following dimensionless variables were introduced for non-dimensionalizing the governing equations:

$$x = \frac{\bar{x}}{L},\ y = \frac{\bar{y} - \bar{\sigma}}{L}Gr^{\frac{1}{4}},\ p = \frac{L^2}{\rho v^2}Gr^{-1}\bar{p},\ u = \frac{\rho L}{\mu}Gr^{-1/2}\bar{u},\ v = \frac{\rho L}{\mu}Gr^{-1/4}(\bar{v} - \sigma_x\bar{u}),$$
$$\sigma_x = \frac{d\bar{\sigma}}{d\bar{x}} = \frac{d\sigma}{dx},\ Gr = \frac{g\beta(T_w - T_\infty)}{v^2}L^3,\ \theta = \frac{(T - T_\infty)}{(T_w - T_\infty)} \quad (8)$$

where θ is the non-dimensional temperature function and $(u,\ v)$ are the dimensionless velocity components parallel to $(x,\ y)$. Introducing the above dimensionless dependent and independent variables, the transformed momentum and energy equations can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (9)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + Gr^{\frac{1}{4}}\sigma_x\frac{\partial p}{\partial y} + (1 + \sigma_x^2)(1 + \varepsilon\theta)\frac{\partial^2 u}{\partial y^2} + \varepsilon(1 + \sigma_x^2)\frac{\partial\theta}{\partial y}\frac{\partial u}{\partial y} + \theta \quad (10)$$

$$\sigma_x(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}) = -Gr^{\frac{1}{4}}\frac{\partial p}{\partial y} + \sigma_x(1 + \sigma_x^2)(1 + \varepsilon\theta)\frac{\partial^2 u}{\partial y^2} + \varepsilon\sigma_x(1 + \sigma_x^2)\frac{\partial\theta}{\partial y}\frac{\partial u}{\partial y} - \sigma_{xx}u^2 \quad (11)$$

$$u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} = \frac{1}{\Pr}(1 + \sigma_x^2)\frac{\partial^2\theta}{\partial y^2} + Q\theta \quad (12)$$

where $\Pr = C_p\mu/k_\infty$ is the Prandtl number, $Q = Q_0L^2\Big/\left(\mu C_p Gr^{1/2}\right)$ is the heat generation parameter and

$\varepsilon = \varepsilon^*(T_w - T_\infty)$ is the viscosity variation parameter. It can easily be seen that the convection induced by the wavy surface is described by Eqs. (9)–(12). We further notice that, Eq. (11) indicates that the pressure gradient along the y-direction is $O(Gr^{-1/4})$, which implies that lowest order pressure gradient along x -direction can be determined from the inviscid flow solution. For the present problem this pressure gradient ($\partial p/\partial x = 0$) is zero. Equation (11) further shows that $Gr^{1/4}\partial p/\partial y$ is $O(1)$ and is determined by the left-hand side of this equation. Thus the eliminating of $\partial p/\partial y$ from the Eqs. (10) and (11) leads to the following equation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (1+\sigma_x^2)(1+\varepsilon\theta)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma_x\sigma_{xx}}{1+\sigma_x^2}u^2 + \varepsilon(1+\sigma_x^2)\frac{\partial\theta}{\partial y}\frac{\partial u}{\partial y} + \frac{1}{1+\sigma_x^2}\theta \tag{13}$$

The corresponding boundary conditions for the present problem then turn into

$$u = v = 0, \theta = 1 \text{ at } y = 0 \quad \text{and} \quad u = \theta = p = 0 \text{ as } y \to \infty \tag{14}$$

Now we introduce the following transformations to reduce the governing equations to a convenient form:

$$\psi = x^{3/4} f(x,\eta),\ \eta = yx^{-1/4},\ \theta = \theta(x,\eta) \tag{15}$$

where $f(\eta)$ is the dimensionless stream function, η is the pseudo similarity variable and ψ is the stream function that satisfies the equation (9) and is defined by $u = \partial\psi/\partial y,\ v = -\partial\psi/\partial x$. Using the transformation Eq. (15) into Eqs. (13) and (12), the following system of non linear equations are obtained:

$$(1+\sigma_x^2)(1+\varepsilon\theta)f''' + \frac{3}{4}ff'' + \varepsilon(1+\sigma_x^2)f''\theta' - \left(\frac{1}{2} + \frac{x\sigma_x\sigma_{xx}}{1+\sigma_x^2}\right)f'^2 + \frac{1}{1+\sigma_x^2}\theta = x\left(f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\right) \tag{16}$$

$$\frac{1}{\Pr}(1+\sigma_x^2)\theta'' + \frac{3}{4}f\theta' + Q\theta x^{\frac{1}{2}} = x\left(f'\frac{\partial\theta}{\partial x} - \theta'\frac{\partial f}{\partial x}\right) \tag{17}$$

The boundary conditions (14) now take the following form:

$$f(x,o) = f'(x,o) = 0, \theta(x,o) = 1, f'(x,\infty) = 0, \theta(x,\infty) = 0 \tag{18}$$

The physical quantities of principle interest are the skin-friction coefficients C_{fx} and the rate of heat transfer in terms of Nusselt number Nu_x which can be obtained from:

$$\tfrac{1}{2}(Gr/x)^{1/4}C_{fx} = (1+\varepsilon)\sqrt{1+\sigma_x^2}\,f''(x,0) \quad \text{and} \quad (Gr/x)^{-1/4}Nu_x = -\sqrt{1+\sigma_x^2}\,\theta'(x,0) \tag{19}$$

3. Results and discussion

Solutions are obtained in terms of the skin-friction coefficient C_{fx} and rate of heat transfer Nu_x respectively for different values of the relevant physical parameters. such as viscosity variation parameter ε, heat generation parameter Q and Prandtl number Pr and these are shown in tabular form in Table 1.

Table 1: Skin friction coefficient C_{fx} and the local rate of heat transfer Nu_x when $x = 4.0$ for the variation of Prandtl number Pr, heat generation parameter Q and viscosity variation parameter ε with α = 0.3.

	$Q = 0.0$, ε = 0.0		$Q = 0.0$, ε = 4.0		$Q = 0.5$, ε = 0.0		$Q = 0.5$, ε = 4.0	
Pr	C_{fx}	Nu_x	C_{fx}	Nu_x	C_{fx}	Nu_x	C_{fx}	Nu_x
0.72	0.60179	0. 32089	0.97280	0.25179	0. 91820	-0.82938	2.65921	-1.55206
1.5	0.53204	0.41205	0.83854	0.31347	0. 92217	-1.49117	3.37852	-3.51050
3.0	0.46701	0.51324	0.71961	0.38211	0. 94024	-2.72740	4.94883	-8.85316
4.5	0.43067	0.57982	0.65593	0.42736	0. 96674	-3.98924	6.91074	-16.91811
7.0	0.39300	0.65939	0.59190	0.48160	1.01435	-6.17146	11.25164	-38.64706

It is observed from Table that as the Prandtl number Pr increases, skin friction coefficient C_{fx} decreases and the rate of heat transfer Nu_x increases but the skin friction (C_{fx}) rise up and the rate of heat transfer Nu_x reduces for higher values of viscosity variation parameter ε. It is also found that the for the internal heat generation due to volumetric heat source wall friction becomes higher and the rate of heat transfer falls down significantly.

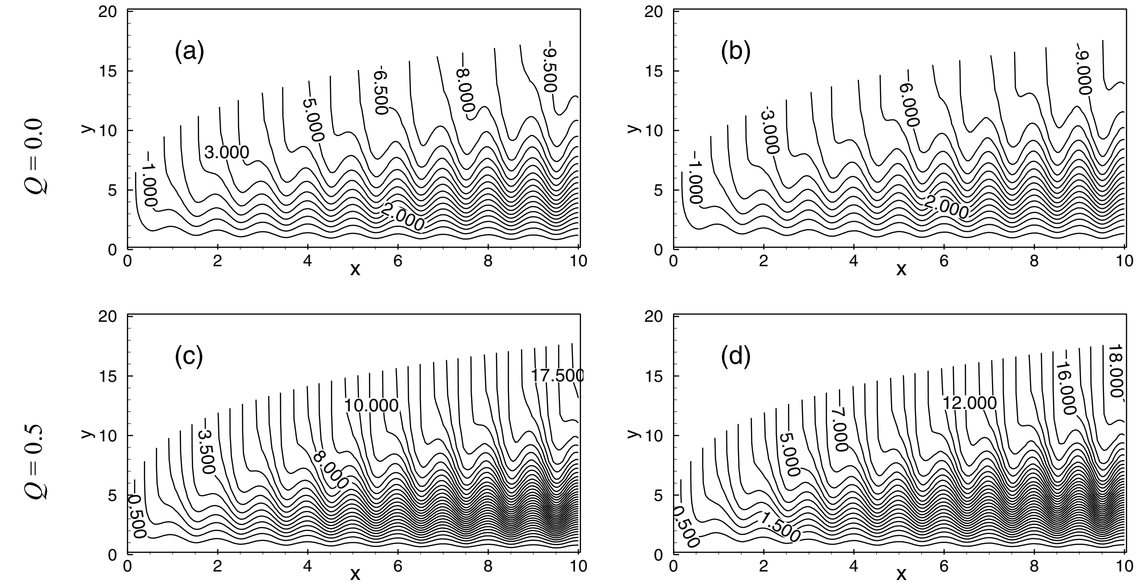

Fig. 2. Streamlines for (a) ε = 0.0, Q = 0.0; (b) ε = 4.0, Q = 0.0; (c) ε = 0.0, Q = 0.5 and (d) ε = 4.0, Q = 0.5 while Pr = 0.72 and α = 0.3

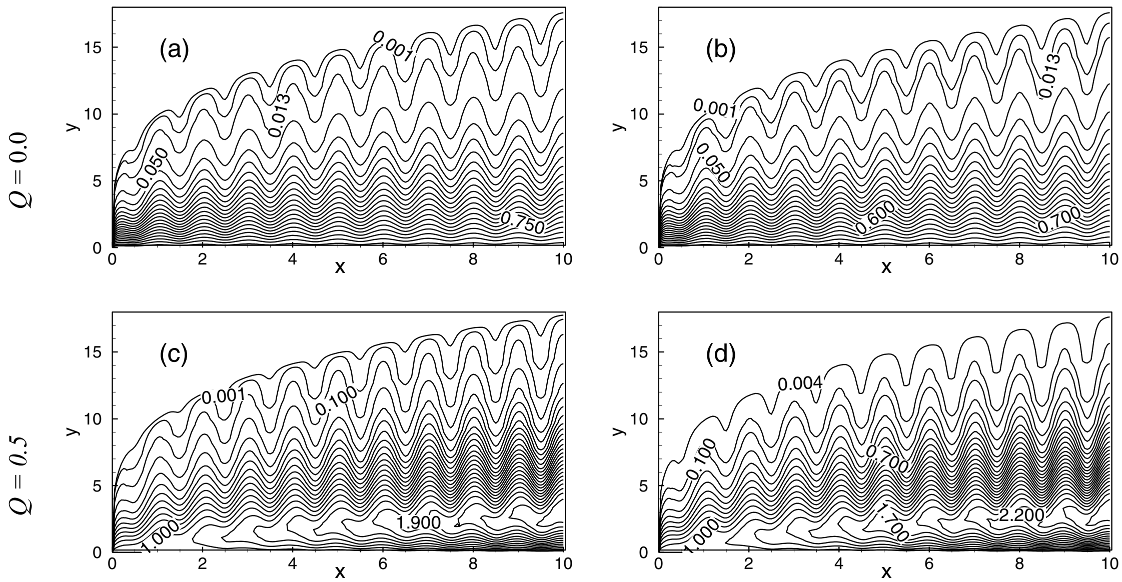

Fig. 3. Isotherms for (a) ε = 0.0, Q = 0.0; (b) ε = 4.0, Q = 0.0; (c) ε = 0.0, Q = 0.5 and (d) ε = 4.0, Q = 0.5 while Pr = 0.72 and α = 0.3

The combined effects of viscosity variation parameter ε and heat generation parameter Q on the development of streamlines which are displayed in Figs. 2(a)-2(d) with amplitude of the wavy surface $\alpha = 0.3$ and Prandtl number Pr = 0.72. It is observed from Fig. 2(a) that the maximum value of stream function ψ_{max} is 12.456 for $Q = 0.0$ and $\varepsilon = 0.0$.Figure 2(b) displays the results that an increasing values of ε, the momentum boundary layer thickness enhances. In this case the maximum value of stream function is $\psi_{max} = 11.982$.

For higher values of heat generation parameter Q the boundary layer becomes thinner and the maximum value of stream function ψ_{max} is 20.221 that is shown in Fig. 2(c). The effects of viscosity variation parameter ε(= 0.0 and 4.0) and the heat generation parameter Q (= 0.0 and 0.5) on the isotherms for $\alpha = 0.3$ and Pr = 0.72 are shown in Figs. 3(a)-3(d). From these figures it is observed that temperature of the fluid rise up significantly due to volumetric heat generation and temperature as well as the thermal boundary layer thickness increase for the combined effect of the viscosity variation parameter ε and heat generation parameter Q.

4. Conclusion

The effects of temperature dependent viscosity variation parameter ε, heat generation parameter Q and Prandtl number Pr on momentum and heat transfer have been studied numerically. From the present investigation the following conclusions may be drawn:

- The frictional force at the wall enhances for the higher values of heat generation parameter Q, the viscosity variation parameter ε and the Prandtl number Pr over the whole boundary layer but the rate of heat transfer reduces significantly for all these cases.
- The skin friction reduces and the rate of heat transfer rise up significantly for higher values of the Prandtl number without effect of viscosity variation and heat generation.

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