Dielectric Parameters and Wave Propagation Characteristics in Multilayer Media

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Abstract

In order to understand the plane wave propagation characteristics in multilayer media, selecting the adjacent 3-layer dielectrics, two boundaries surfaces for example, models of plane wave propagation were established within each layer. Simulation results show that both the incident and reflected waves were in the ordinary dielectric, their synthesis wave wasn’t complete stationary wave; Imaginary part of complex permittivity was attenuation factor. Other results also show when the wave struck a dielectric layer normally, the dielectric wave impedance and the thickness of the layer met the condition of a quarter wavelength matching layer, there was no reflected wave. This conclusion is of great value in many engineering and practical applications.

Keywords: Multilayer media, wave impedance, matching layer, stationary wave, complex permittivity

1. Introduction

Electromagnetic parameters such as medium permittivity, permeability, electromagnetic wave impedance are the main factors which influence radio propagation, determine the electromagnetic wave reflectivity and transmission, thereby determining the medium filtering, absorbing performance. Some special materials, their layered structure can also change periodically, therefore able to control some of the waves through or closed artificially.

Media differences are embodied not only in conductor, good conductor, ideal conductor [1], as well as perfect dielectric and non-ideal medium [2], but also similar layers in multilayer dielectric media, the wave propagation characteristics are also significant differences. It’s important to study wave propagation differences in multilayer dielectric by quantitative research, the results can provide an important basis for selecting the very material with special functions.

2. Wave equations in layered media

For arbitrary m-layered medium, its thickness is l, set the medium layered along the z direction, the direction of wave propagation, as shown in Fig. 1 (a) below. Vertically polarized plane wave as an example, set the incident surface is surface-xoz, the wave in m-layer of the uniform medium can be expressed as

\[ E_m(z) = (A_m e^{-j k_m z_m} + B_m e^{j k_m z_m}) e^{-j k_m x} \]  

\[ -H_z(z) = \frac{k_m}{\omega \mu_m} \cdot (A_m e^{-j k_m z_m} - B_m e^{j k_m z_m}) e^{-j k_m x} \]
Where, $z_m$ is the interface of the $m$ layered medium, as shown in Fig. 1 (b) below; $A_m$, $B_m$ is respectively the wave amplitude along the positive $z$ direction and the negative $z$ direction. At the $(z_m + l)$ interface, the electric field and magnetic field can be expressed as

$$E_y(z_m + l) = (A_m e^{-jk_m z_m} \cdot e^{-jk_m l} + B_m e^{jk_m z_m} \cdot e^{jk_m l}) e^{-jk_m x},$$

$$-H_z(z_m + l) = \frac{k_m}{\omega \mu_m} (A_m e^{-jk_m z_m} \cdot e^{-jk_m l} - B_m e^{jk_m z_m} \cdot e^{jk_m l}) e^{-jk_m x}.$$  

Combined with (1) and (2), we can derive the relation, expressed by matrix as

$$\begin{bmatrix} E_y(z_m) \\ H_z(z_m) \end{bmatrix} = \begin{bmatrix} A_m & B_m \\ C_m & D_m \end{bmatrix} \begin{bmatrix} E_y(z_m + l) \\ H_z(z_m + l) \end{bmatrix}.$$  

(5)

Where,

$$A_m = \cos(k_m z_m), \quad C_m = -j \frac{\sin(k_m z_m)}{\eta_m}, \quad \eta_m = \frac{\omega \mu_m}{k_m z_m},$$

$$B_m = -j \eta_m \sin(k_m z_m), \quad D_m = \cos(k_m z_m).$$  

(6)

Fig. 1. Wave propagation in layered media

Suppose the electric field and magnetic field is respectively $E_1$, $H_1$, $E_N$, $H_N$ at the first interface and the $n$-interface. According to (3), (4), by all $N-1$ layers layered dielectric media, we have

$$\begin{bmatrix} E_1 \\ H_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} E_N \\ H_N \end{bmatrix}.$$  

(7)

Here[^6]
\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
= \begin{bmatrix}
A_1 & B_1 \\
C_1 & D_1
\end{bmatrix}
\begin{bmatrix}
A_2 & B_2 \\
C_2 & D_2
\end{bmatrix}
\begin{bmatrix}
A_3 & B_3 \\
C_3 & D_3
\end{bmatrix}
\cdots
\begin{bmatrix}
A_{N-1} & B_{N-1} \\
C_{N-1} & D_{N-1}
\end{bmatrix}.
\]  

(8)

The coefficients of reflection and transmission of the multi-layered media \(^4\) are as

\[
\Gamma = \frac{A + B/\eta_N - \eta_i (C + D/\eta_N)}{A + B/\eta_N + \eta_i (C + D/\eta_N)},
\]

\[
\tau = \frac{2}{A + B/\eta_N + \eta_i (C + D/\eta_N)},
\]

(9)

The corresponding reflectivity and reflectivity

\[
r = |\Gamma|^2 \text{ and } t = |\tau|^2.
\]

(10)

Multi-layered media equivalent wave impedance \(^5\):

\[
\eta_{ed} = \eta_2 e^{j\beta_2 z + \Gamma} e^{-j\beta_2 z}.
\]

(11)

Where, \(\Gamma\) is the coefficient of reflection at the interface between Zone 2 and 3.

3. Numerical Simulation and Analysis

3.1. For Any Dielectric Layer

We now consider the case when a wave travelling in a dielectric medium (medium 1) impinges normally upon a medium 2, as depicted in Fig. 2, simulate wave propagation in many cases, to observe the characteristics. Let electric field of the incident wave in Area 1 be\(^5\):

\[
\tilde{E}_i(z,t) = \tilde{\varepsilon}_i E_{im} e^{-j\beta_2 z} e^{j\omega t}.
\]

(12)

Electric field of the reflected wave in Zone 1 can be expressed as:

\[
\tilde{E}_r(z,t) = \tilde{\varepsilon}_r E_{im} e^{j\beta_2 z} e^{j\omega t},
\]

(13)

and synthetic wave field in zone 1 is:

\[
\tilde{E}_s(z,t) = \tilde{\varepsilon}_r E_{im} (e^{-j\beta_2 z} + \Gamma e^{j\beta_2 z}) e^{j\omega t}.
\]

(14)

We know that the transmitted wave is the incident wave of zone 2, its electric field is as:

\[
\tilde{E}_{2s}(z,t) = \tilde{E}_i(z,t) = \tilde{\varepsilon}_s \tau E_{im} e^{-j\beta_2 z} e^{j\omega t}.
\]

(15)

The relative dielectric permittivity can be expressed as

\[
\varepsilon_r = \varepsilon' - j\varepsilon''.
\]

(16)

Supposing the relative dielectric constants of media 2 and 3 as: 4.32 + 2.27\(j\) ; 2.13 + 1.61\(j\), for meeting the human visual capabilities, the frequency is reduced billions of times, electric fields of the plane waves are simulated as shown in Fig. 2.

Conclusion: both incident and reflected waves exist in zone 1 and 2. Since the intensity of the reflected wave is lower than that of incident, so their synthetic wave is not entirely stationary wave, synthesis wave moves in the same direction with the incident, but in slower pace. There is always a minimum of synthetic wave, however, the minimum
value never was 0. There is not reflected wave in the last dielectric layer, but only incident wave; for the plural form of a dielectric constant, the wave in the dielectric is attenuated, the imaginary part of the plural is attenuation factor.

Fig. 2  plane waves in layered media composed of three dielectric layers

3.2. A quarter wavelength matching layer

When the thickness $d$ of medium 2 meets $d = \frac{\lambda_2}{4}$, and the intrinsic impedance of medium 2 meets

$$\eta_2 = \sqrt{\eta_1 \eta_3},$$

and $\eta_{ed} = \eta_1$. Wave reflection coefficient of the interface between medium 1 and medium 2 is as

$$\Gamma_1 = 0.$$  \hspace{1cm} (18)

At this very point, zone 2 is called a quarter wavelength matching layer, because this coating can eliminate reflection, and therefore it have important practical significance.

As showing in Fig. 2, zone 2 is the quarter wavelength matching layer, simulation results show that, despite zone 2 have incident and reflected waves and synthetic wave, but zone 1 is not reflected wave.
In addition, by using dual principles, which transform relation is $E \rightarrow H$, $H \rightarrow -E$ and $\mu \rightarrow \varepsilon$, $\varepsilon \rightarrow \mu$, the formula for the horizontal polarization can be got from that for vertical polarization. If with normal incidence, only the component of $H_y$ and $E_x$ exists.

4. Conclusions

For multilayer media which composed of different medium, there are incident wave, reflected wave in each layer. If you need that there is no reflected wave in a certain layer ($m$), you need to set the parameters in the next layer ($m+1$), selecting the capacitivity $\varepsilon_{m+1}$ and insuring wave impedance to meet $\eta_{m+1} = \sqrt{\eta \eta_{m+2}}$, and selecting the thickness of $m+1$ layer a quarter of the wavelength. That is, let the $m+1$ layer be a quarter wavelength matching layer.

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References