Random Fixed Points for Several Classes of 1-Ball-Contractive and 1-Set-Contractive Random Maps

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A general random fixed point theorem for continuous random operators is proved. As applications, a number of random fixed points theorems for various classes of 1-set and 1-ball contractive random operators (e.g., operators of contractive type with compact or completely continuous perturbations, operators of semicontractive type, etc.) are derived. Our results unify and extend most of the known random fixed points theorems. 1991 AMS Subject Classification: 47H10, 60H25.

Key Words: random operator; 1-set-contractive map; 1-ball-contractive map, random fixed point

1. INTRODUCTION

Probabilistic functional analysis has come out as one of the momentous mathematical disciplines in view of its requirements in dealing with probabilistic models in applied problems. The study of random fixed points forms a central topic in this area and was initiated by the Prague school of probabilists in the 1950s. The theory of random fixed points received further attention after the appearance of the survey article by Bharucha-Reid [3] in 1976. Since then there has been a lot of activity in this area and several interesting results have appeared (see, for example, Beg and Shahzad [1], Itoh [4], Lin [8], Papageorgiou [12, 13], Seghal and Singh [17], Seghal and Waters [18], Xu [22], etc.). Recently, Beg and Shahzad [2], Lin [7], Liu [9, 10], Shahzad [19], and Tan and Yuan [20, 21] independently studied random fixed points of 1-set-contractive random maps satisfying different conditions. In particular Tan and Yuan [20, 21] and Liu [9] considered continuous hemicompact 1-set-contractive random operators. In this paper we consider random operators satisfying condition (A) which...
include continuous hemi-compact maps. We first prove a general random fixed point theorem for continuous random operators which gives us stochastic analogues of all deterministic fixed point theorems involving continuous mappings. We then indicate the generality of our main results (Theorems 3.1 and 3.3) and their unifying aspect by using them to deduce a number of random fixed point theorems for various classes of 1-set and 1-ball-contractive random operators (for example, operators of contractive type with compact or completely continuous perturbations, operators of semicontractive type, etc.). Present work leads to the discovery of new random fixed point theorems for more general 1-set and 1-ball-contractive random maps. For corresponding fixed point theorems, we refer to Petryshyn [14] and Reich [15, 16].

2. PRELIMINARIES

Let \((\Omega, \Sigma)\) be a measurable space with \(\Sigma\) a sigma algebra of subsets of \(\Omega\) and \(S\) a non-empty subset of a metric space \(X\). Let \(2^S\) be the family of all subsets of \(S\) and \(C(S)\) the family of all non-empty closed subsets of \(S\). A mapping \(G : \Omega \to 2^S\) is called measurable if, for each open subset \(U\) of \(S\), \(G^{-1}(U) \in \Sigma\), where \(G^{-1}(U) = \{ \omega \in \Omega : G(\omega) \cap U \neq \emptyset \}\). A mapping \(\xi : \Omega \to S\) is said to be a measurable selector of a measurable mapping \(G : \Omega \to 2^S\) if \(\xi\) is measurable and, for any \(\omega \in \Omega\), \(\xi(\omega) \in G(\omega)\). A mapping \(T : \Omega \times S \to X\) is called a random operator if, for each fixed \(x \in S\), the map \(T(\cdot, x) : \Omega \to X\) is measurable. A measurable map \(\xi : \Omega \to S\) is a random fixed point of a random operator \(T : \Omega \times S \to X\) if \(\xi(\omega) = T(\omega, \xi(\omega))\) for each \(\omega \in \Omega\).

Let \(A\) be a non-empty bounded subset of a metric space \(X\). Let \(\alpha(\cdot)\) be the set measure of noncompactness, that is, \(\alpha(A)\) is the infimum of the numbers \(r > 0\) such that \(A\) can be covered by a finite number of subsets of \(X\) of diameter less than or equal to \(r\), and \(\chi(\cdot)\) the ball measure of noncompactness, that is \(\chi(A)\) is the infimum of the numbers \(r > 0\) such that \(A\) can be covered by a finite number of balls with centers in \(X\) and radius \(r\). It is well known that the measures \(\alpha\) and \(\chi\) are different although they have a good deal in common. Let \(T : S \to X\) be a mapping. If for every non-empty bounded subset \(A\) of \(S\) with \(\alpha(A) > 0\) we have \(\alpha(T(A)) < \alpha(A)\), then \(T\) is called set-condensing. If there exists \(k, 0 \leq k \leq 1\), such that for each non-empty bounded subset \(A\) of \(S\) we have \(\alpha(T(A)) \leq k\alpha(A)\), then \(T\) is called \(k\)-set contractive. It follows that every \(k\)-set-contractive map with \(k < 1\) is set-condensing and that every set-condensing map is \(1\)-set-contractive. As in the case of \(\alpha\), corresponding to \(\chi\) we have \(k\)-ball-contractive and ball-condensing maps. Let \(S\) be a non-empty subset of a normed space \(X\) and \(T : S \to X\). Then \(T\) is called \((1)\)
contraction if there exists $k, k < 1$, such that $\| T(x) - T(y) \| \leq k \| x - y \|$, for each $x, y \in S$; (2) nonexpansive if $\| T(x) - T(y) \| \leq \| x - y \|$ for each $x, y \in S$; (3) generalized contraction if, for each $x \in S$, there exists a number $k(x) < 1$ with $\| T(x) - T(y) \| \leq k(x) \| x - y \|$ for each $y \in S$; (4) completely continuous if it maps weakly convergent sequences into strongly convergent sequences; (5) compact if $T(A)$ is compact when $A \subset S$ is bounded; (6) uniformly strictly contractive on $S$ relative to $X$ if the map $T: X \to X$ has the property that, for each $x \in X$, there exists a number $k(x) < 1$ such that $\| T(x) - T(y) \| \leq k(x) \| x - y \|$ for each $x \in S$; (7) LANE (locally almost nonexpansive) if, for each $x \in S$ and $\varepsilon > 0$, there exists a weak neighborhood $N_x$ of $x$ in $S$ (depending also on $\varepsilon$) such that $u, v \in N_x, \| T(u) - T(v) \| \leq \| u - v \| + \varepsilon$.

Let $S$ be a non-empty closed bounded subset of a Banach space $X$ and $T: S \to X$ a continuous map.

(a) Suppose there exists a continuous mapping $V: X \times X \to X$ such that $T(x) = V(x, x)$ for $x \in S$. Then

(i) $T$ is strictly semicontractive if, for each fixed $x \in X$, $V(\cdot, x)$ is a contraction and $V(x, \cdot)$ is compact.

(ii) $T$ is weakly semicontractive if, for each $x \in X$, $V(\cdot, x)$ is nonexpansive and $V(x, \cdot)$ is compact.

(b) Suppose there exists a continuous map $V: S \times S \to X$ such that $T(x) = V(x, x)$ for $x \in S$. Then

(iii) $T$ is of strictly semicontractive type if, for each $x \in S$, $V(\cdot, x)$ is a contraction and the map $x \to V(x, \cdot)$ of $S$ into the space of continuous mappings of $S$ into $X$ with the uniform metric is compact.

(iv) $T$ is of weakly semicontractive type if, for each $x \in S$, $V(\cdot, x)$ is a nonexpansive map of $S$ into $X$ and $x \to V(x, \cdot)$ of $S$ into the space of continuous mappings of $S$ into $X$ is compact.

(c) Suppose there exists a mapping $V: X \times S \to X$ such that $T(x) = V(x, x)$ for $x \in S$. Then $T$ is of strongly semicontractive type relative to $X$ if, for each $x \in S$, the mapping $V(\cdot, x)$ is uniformly strictly contractive on $S$ relative to $X$ and $V(x, \cdot)$ is completely continuous from $S$ to $X$, uniformly for $x \in S$. For details we refer to Petryshyn [14].

Let $S$ be a non-empty subset of a metric space $(X, d)$. A mapping $T: S \to X$ is hemicompact if each sequence $(x_n)$ in $S$ has a convergent subsequence whenever $d(x_n, T(x_n)) \to 0$ as $n \to \infty$. A mapping $T: S \to X$ is said to satisfy condition (A) if for any sequence $(x_n)$ in $S$, $D \subset C(S)$ such that $d(x_n, D) \to 0$ and $d(x_n, T(x_n)) \to 0$ as $n \to \infty$; then there exists $x' \in D$ with $x' = T(x')$. It is clear that every continuous hemicompact map satisfies condition (A). We also observe that condition (A) is always true.
for continuous set-condensing or continuous ball-condensing mappings
(and in particular, continuous $k$-set or $k$-ball contractions with $0 \leq k < 1$).
A random operator $T: \Omega \times S \to X$ is continuous (set-condensing, $k$-set-
contractive, $k$-ball-contractive, LANE, nonexpansive, completely contin-
uous, compact, generalized contraction, of semicontractive type, etc.) if, for
each $\omega \in \Omega$, the map $T(\omega, \cdot): S \to X$ is so, and is said to satisfy condition
($A$) if, for any $\omega \in \Omega$, $T(\omega, \cdot)$ satisfies condition ($A$). A random operator
$T: \Omega \times S \to X$ is called weakly inward if, for any $\omega \in \Omega$, $T(\omega, x) \in \overline{I(x)}$
for all $x \in S$, where $I(x) = \{z \in X: z = x + a(y - x) \text{ for some } y \in S \text{ and } a \geq 0\}$.
When $S$ has a non-empty interior, a random operator $T: \Omega \times S \to X$ is
defined to satisfy the Leray–Schauder condition if there is a point $z$ in int$(S)$,
the interior of $S$, (depending on $\omega$) such that $T(\omega, y) = m(y - x)$, for
all $y \in \partial S$, the boundary of $S$, and $m > 1$. Throughout this paper, we shall
assume that the interior of $S$ is non-empty whenever $T$ satisfies the
Leray–Schauder condition. If $T$ is weakly inward, then it satisfies the
Leray–Schauder condition.

3. MAIN RESULTS

Using the arguments analogous to those in Itoh [4], we first establish the
following theorem and then deduce, as special cases, the random fixed
point theorems involving various types of 1-ball and 1-set-contractive
random operators.

**Theorem 3.1.** Let $S$ be a non-empty separable closed subset of a com-
plete metric space $X$ and $T: \Omega \times S \to X$ a continuous random operator for
which condition ($A$) holds. If the set $G(\omega) = \{x \in S: x = T(\omega, x)\}$ is
non-empty for each $\omega \in \Omega$, then $T$ has a random fixed point.

**Proof.** Define a mapping $G: \Omega \to 2^S \setminus \emptyset$ by $G(\omega) = \{x \in S: x =
T(\omega, x)\}$. For any non-empty closed subset $D$ of $S$, let

$$L(D) = \bigcap_{n=1}^{\infty} \bigcup_{x \in D_n} \{\omega \in \Omega: d(x, T(\omega, x)) < 2/n\},$$

where $D_n = \{x \in S: d(x, D) < 1/n\}$ and $d(x, D) = \inf\{d(x, y): y \in D\}$. We
show that $G^{-1}(D) = L(D)$. Indeed, one can easily check that $G^{-1}(D) \subset L(D)$. On the other hand, if $\omega \in L(D)$, then for each $n$, there exists $x_n \in D_n$ with $d(x_n, D) < 1/n$ and $d(x_n, T(\omega, x_n)) < 2/n$. It follows that
d$\omega \in G^{-1}(D)$ if and only if $d(x_n, D) \to 0$ and $d(x_n, T(\omega, x_n)) \to 0$ as $n \to \infty$. By condition ($A$), there
exists $x' \in D$ with $x' = T(\omega, x')$. Hence $\omega \in G^{-1}(D)$. Therefore $G^{-1}(D) = L(D)$ and $G$ is measurable. Since $G$ has closed values, by the Kura-
towski and Ryll-Nardzewski selection theorem [6], there exists a measurable mapping \( \xi: \Omega \to S \) such that \( \xi(\omega) \in G(\omega) \) for each \( \omega \in \Omega \). This \( \xi \) is the desired random fixed point of \( T \).

**Remark 3.2.** Under the hypotheses of Theorem 3.1 “deterministic solvability” of the fixed point problem implies “stochastic solvability” of it.

An immediate consequence of Theorem 3.1 is the following result.

**Theorem 3.3.** Let \( S \) be a non-empty separable closed bounded subset of a Banach space \( X \) and \( T: \Omega \times S \to X \) either a continuous \( 1 \)-set-contractive or a continuous \( 1 \)-ball-contractive random operator for which condition (A) holds. If \( T \) satisfies the Leray–Schauder condition, then \( T \) has a random fixed point.

**Proof.** By Petryshyn [14, Theorem 1], the set \( G(\omega) = \{ x \in S: x = T(\omega, x) \} \) is non-empty for each \( \omega \in \Omega \). Hence \( T \) has a random fixed point by Theorem 3.1.

**Remark 3.4.** In case \( S \) is bounded, the assumption that \( (I - T(\omega, \cdot))(S) \) is a closed subset of \( X \) for each \( \omega \in \Omega \) in Tan and Yuan [21, Theorem 2.4] is redundant. We also remark that Theorem 3.3 is new in the sense that, unlike Tan and Yuan [21], we do not need \( S \) to be convex.

**Corollary 3.5** [22, Theorem 2]. Let \( S \) be a non-empty separable closed convex subset of a Banach space \( X \) and \( T: \Omega \times S \to X \) a continuous set-condensing operator that is either (i) weakly inward or (ii) satisfies the Leray–Schauder condition. If, for each \( \omega \in \Omega \), \( T(\omega, S) \) is bounded, then \( T \) has a random fixed point.

**Proof.** By results of Reich [15, 16], in both cases, the set \( G(\omega) = \{ x \in S: x = T(\omega, x) \} \) is non-empty for each \( \omega \in \Omega \). Since every continuous set-condensing map satisfies condition (A), by Theorem 3.1 \( T \) has a random fixed point.

**Remark 3.6.** (1) A careful reading of the proof of Xu [22, Theorem 2] reveals that Xu’s theorem also holds when \( S \) is separable (instead of \( X \) being separable).

(2) Theorem 2.1 of Itoh [4], Theorems 3.1–3.4 of Liu [9], Theorems 4, 5 of Lin [8], Corollary 4.1 of Lin [7], Theorem 2.4, Theorems 4.1–4.3 of Tan and Yuan [21], Theorem 3.2, Theorems 4.8, 4.9, 4.11 and Corollary 4.12 of Tan and Yuan [20] can be viewed as special cases of Theorem 3.1.

Now we indicate the generality of Theorem 3.1 and its unifying aspect by employing it to obtain a number of random fixed point theorems for random operators of contractive type and their perturbations by continuous compact or completely continuous random maps.
**Corollary 3.7.** Let $S$ be a non-empty separable closed bounded subset of a Banach space $X$, $B: \Omega \times S \to X$ a random contraction, and $C: \Omega \times S \to X$ a continuous compact random operator. If the random operator $T = B + C: \Omega \times S \to X$ satisfies the Leray–Schauder condition, then $T$ has a random fixed point.

**Proof.** Since $B: \Omega \times S \to X$ is a random contraction and $C: \Omega \times S \to X$ is a continuous compact random operator, $T = B + C: \Omega \times S \to X$ is a continuous set-condensing random operator and hence satisfies the condition on $T$. Therefore, by Theorem 3.3, $T$ has a random fixed point.

We observe that if $T(\omega, S) \subseteq S$ for each $\omega \in \Omega$, then $T$ satisfies the Leray–Schauder condition. Thus we get the following interesting result.

**Theorem 3.8.** Let $S$ be a separable closed bounded convex subset of a reflexive Banach space $X$, $B: \Omega \times S \to X$ a generalized random contraction, and $C: \Omega \times S \to X$ a completely continuous random operator. If, for any $\omega \in \Omega$, $B(\omega, x) + C(\omega, y) \subseteq S$ whenever $x, y \in S$, then $T = B + C: \Omega \times S \to X$ has a random fixed point.

**Proof.** Since $C: \Omega \times S \to X$ is a completely continuous random operator, $\alpha(C(\omega, A)) = 0$ for each subset $A$ of $S$ and each $\omega \in \Omega$. Hence $T = B + C$ is a 1-set-contractive random operator. Furthermore, $T$ satisfies the Leray–Schauder condition, since $T(\omega, S) \subseteq S$ for each $\omega \in \Omega$. To verify condition (A), fix $\omega \in \Omega$ arbitrarily. Let $(x_n)$ be any sequence in $S$, $D \subseteq C(S)$ such that $d(x_n, D) \to 0$ and $\|x_n - T(\omega, x_n)\| \to 0$ (in other words $x_n - T(\omega, x_n) \to 0$) as $n \to \infty$. Since $S$ is a closed bounded convex subset of a reflexive Banach space $X$ and $(x_n) \subseteq S$, we may assume that $x_n \to x_0$ weakly in $S$. Then using the complete continuity of $C$ we see that $C(\omega, x_n) \to C(\omega, x_0)$ as $n \to \infty$ and, therefore, $x_n - B(\omega, x_n) = x_n - T(\omega, x_n) + C(\omega, x_n) \to C(\omega, x_0)$ as $n \to \infty$. Since $B(\omega, x) + C(\omega, x_0) \subseteq S$ for each $x \in S$, Petryshyn [14, Remark 2.6] further implies that $(x_n)$ converges strongly to $x_0$. Hence $x_0 \in D$ and $x_0 - T(\omega, x_0) = 0$, and therefore, by Theorem 3.3, $T$ has a random fixed point.

The following corollary is a special case of Theorem 3.8.

**Corollary 3.9.** Let $S$ be a non-empty separable closed bounded convex subset of a reflexive Banach space $X$ and $T: \Omega \times S \to S$ a random generalized contraction. Then $T$ has a random fixed point.

Now, if in Theorem 3.8 we strengthen the hypothesis on $S$, then the condition on $T$ in Theorem 3.8 can be somewhat relaxed.

**Theorem 3.10.** Let $S$ be a closed ball with center at origin and radius $r$, in a separable reflexive Banach space $X$ and let $B: \Omega \times S \to X$ and $C: \Omega \times S \to X$ be as in Theorem 3.8. If, for any $\omega \in \Omega$, $B(\omega, x) + C(\omega, y) \in$
S for \(x \in \partial S\) and \(y \in S\), then \(T = B + C: \Omega \times S \to X\) has a random fixed point.

Proof. For each \(\omega \in \Omega\), \(T'(\omega, x) = B'(\omega, x) + C'(\omega, x)\) for \(x \in S\), where \(B'(\omega, x) = (x + S(\omega, x))/2\) and \(C'(\omega, x) = C(\omega, x)/2\) for \(x \in S\). Then \(B': \Omega \times S \to X\) is a generalized random contraction, \(C': \Omega \times S \to X\) is a completely continuous random operator, and \(T\) and \(T'\) have the same random fixed point. As in Petryshyn [14, proof of Theorem 2.5], for any \(\omega \in \Omega\), \(B'(\omega, x) + C'(\omega, y) \in S\) whenever \(x, y \in S\). Hence, Theorem 3.10 follows from Theorem 3.8.

If we somewhat strengthen the condition on \(B\), then the assertion of Theorem 3.8 remains valid without the condition "for any \(\omega \in \Omega\), \(B(\omega, x) + C(\omega, y) \in S\) whenever \(x, y \in S\)" even for compact operator \(C\).

Theorem 3.11. Let \(S\) be a non-empty separable closed bounded convex subset of a reflexive Banach space \(X\), \(C: \Omega \times S \to X\) a continuous compact random operator, and \(B: \Omega \times S \to X\) a uniformly strictly contractive random operator on \(S\) relative to \(X\). If \(T = B + C: \Omega \times S \to X\) satisfies the Leray–Schauder condition, then \(T\) has a random fixed point.

Proof. Since \(T\) is \(1\)-set-contractive, in view of Theorem 3.3, it suffices to show that \(T\) satisfies condition (A). Fix any \(\omega \in \Omega\). Let \(\{x_n\}\) be any sequence in \(S\), \(D \in C(S)\) such that \(d(x_n, D) \to 0\) and \(\|x_n - T(\omega, x_n)\| \to 0\) (that is, \(x_n - T(\omega, x_n) \to 0\)) as \(n \to \infty\). Since \(\{x_n\}\) is bounded and \(C\) is compact we may assume that \(C(\omega, x_n) \to y\) in \(X\). Therefore

\[x_n - B(\omega, x_n) = x_n - T(\omega, x_n) + C(\omega, x_n) \to y\]

or

\[x_n - F(\omega, x_n) \to 0\]

as \(n \to \infty\),

where \(F: \Omega \times X \to X\) is a uniformly strictly contractive random operator defined by \(F(\omega, x) = B(\omega, x) + y\). Since \(X\) is reflexive, there exists a subsequence \(\{x_{m_n}\}\) of \(\{x_n\}\) such that \(x_{m_n} \to x_0\) weakly as \(m \to \infty\). By Kirk [5], \(\{x_n\}\) is a Cauchy sequence which necessarily converges strongly to \(x_0\). This and the continuity of \(T\) imply that \(x_0 \in D\) and \(x_0 - T(\omega, x_0) = 0\). Hence \(T\) has a random fixed point.

Now we obtain from Theorem 3.3 various random fixed point theorems for operators of semicontractive type.

Theorem 3.12. Let \(S\) be a non-empty separable closed bounded subset of a Banach space \(X\) and \(T: \Omega \times S \to X\) a continuous random operator that satisfies the Leray–Schauder condition. Then \(T\) has a random fixed point if \(T\)
satisfies one of the following conditions:

(i) $T$ is strictly semicontractive.

(ii) $T$ is weakly semicontractive and satisfies condition (A).

Proof. (i) By Petryshyn [14, Lemma 3.1] $T$ is $k$-ball-contractive with $k < 1$. To prove the theorem, it suffices to show that $T$ satisfies condition (A). Fix $\omega \in \Omega$ arbitrarily. Let $(x_n)$ be any sequence in $S$, $D \in C(S)$ such that $d(x_n, D) \to 0$ and $\|x_n - T(\omega, x_n)\| \to 0$ (equivalently, $x_n - T(\omega, x_n) \to 0$) as $n \to \infty$. Then the set $(x_n)$ is precompact by the same way as in the proof of [14, Theorem 3.1]. Without loss of generality, we may assume that $(x_n)$ itself converges to some $x_0 \in S$. Hence $x_0 \in D$ and $x_0 - T(\omega, x_0) = 0$ and so, by Theorem 3.3, $T$ has a random fixed point.

(ii) If $T$ satisfies (ii), then by Petryshyn [14, Theorem 3.1] $T$ is 1-ball-contractive. This theorem just follows from Theorem 3.3.

As another application of Theorem 3.3, we obtain the following result.

**Theorem 3.13.** Let $S$ be a non-empty separable closed bounded subset of a Banach space $X$ and $T: \Omega \times S \to X$ a continuous random operator that satisfies the Leray–Schauder condition. Then $T$ has a random fixed point if $T$ satisfies one of the following conditions:

(i) $T$ is of strictly semicontractive type.

(ii) $T$ is of weakly semicontractive type and satisfies condition (A).

Proof. (i) If $T$ satisfies (i), then by Petryshyn [14, Lemma 3.2] $T$ is $k$-set-contractive with $k < 1$. From the proof of Theorem 3.12, we can easily see that $T$ satisfies condition (A). Hence, the result follows from Theorem 3.3.

(ii) If $T$ satisfies (ii), then again by Petryshyn [14, Lemma 3.2] $T$ is 1-set-contractive. Therefore, by Theorem 3.3, $T$ has a random fixed point.

**Corollary 3.14.** Let $S$ be a non-empty separable closed bounded convex subset of a reflexive Banach space $X$ and $T: \Omega \times S \to X$ a strongly semicontractive type random operator relative to $X$ that satisfies the Leray–Schauder condition. Then $T$ has a random fixed point.

Proof. Since $T$ is of strongly semicontractive type relative to $X$, it is of semicontractive type. By Petryshyn [14, Lemma 3.2 and p. 338] $T$ is 1-set-contractive. Furthermore, $T$ satisfies condition (A). Indeed, fix any $\omega \in \Omega$. Let $(x_n) \subset S$ be a sequence, $D \in C(S)$ such that $d(x_n, D) \to 0$ and $\|x_n - T(\omega, x_n)\| \to 0$. Then from Kirk [5, Theorem 2] $x_n \to x_0 \in S$ and so $x_0 \in D$ and $x_0 - T(\omega, x_0) = 0$. Hence this corollary follows immediately from Theorem 3.3.
It is well known that if $X$ is a reflexive Banach space, $S$ a closed bounded convex subset of $X$, and $T: S \to X$ a continuous LANE mapping, then $T$ is 1-set-contractive. So using this fact together with Theorem 3.3, we get at once the following corollary.

**Corollary 3.15.** Let $S$ be a non-empty separable closed bounded convex subset of a reflexive Banach space $X$, $B: \Omega \times S \to X$ a continuous LANE random operator, and $C: \Omega \times S \to X$ a completely continuous random operator. If the random operator $T = B + C: \Omega \times S \to X$ satisfies the Leray–Schauder condition and condition (A), then $T$ has a random fixed point.

**REFERENCES**


22. H. K. Xu, Some random fixed point theorems for condensing and nonexpansive operators, *Proc. Amer. Math. Soc.* **110** (1990), 395–400.