The Optical Flow Method Research of Particle Image Velocimetry

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Abstract

As a new experimental technique of aerodynamics, optical flow test technique gradually attracts more and more attention for the advantages of vector field measurement of pixel scale resolution and strong smoothness ability. By means of scalar constraint equation combined with smoothness constraint condition, optical flow test technique can measure global velocity vector field of high space resolution. In this paper, the theory and algorithm of integration minimization optical flow velocimetry were studied, an verification experiment, in which tracer particle images was acquired by high speed camera and velocity vector field was calculated by optical flow algorithm, was completed. The result calculated by optical flow algorithm was compared with the result calculated by PIV algorithm, and both results were significantly correlated. The research shows that optical flow test technique possesses the advantages of space resolution and velocity field smoothness.

Keywords: Optical flow; Velocity measurement; Particle image; Motion estimation; Integration minimization

1. Introduction

The quantitative measurement of flow velocity field possesses fundamental importance for understanding the physical nature of complex flow in the research of fluid dynamics and aerodynamics [1, 2]. In recent decades, along with the improvement of laser diagnostics, high-speed data acquisition and image processing, the Particle Image Velocimetry coming from Speckle method of Solid Mechanics had achieved considerable development[3, 4].

Particle Image Velocimetry algorithm(PIV) can provide two-dimensional or three-dimensional velocity vector field within two-dimensional cross-section by using cross-correlation algorithm, multi-scale iteration and sub-pixel
fitting between two adjacent tracer particle images, and its latest development can provide three-dimensional velocity vector field within a flat three-dimensional space. The velocity vector obtained by PIV is the average velocity of interpretation window, so the variation of velocity field in the interpretation window is ignored. During the exposure interval, the displacement of particle, which follows the Nyquist criterion, should be less than half of the size of interpretation window so that the spatial resolution of PIV is limited by certain restrictions. The accuracy of velocity vector field obtained by PIV is also restricted in the region with large velocity gradient.

Improving the spatial resolution of measurement, overcoming the effects of velocity gradient, understanding the space structure of fluid motion and obtaining dynamic information are very important for resolving turbulent and complex unsteady flow problem. In this paper, from the viewpoint of improving the spatial resolution of the measurement, the optical flow measurement technology was introduced into the experimental field of aerodynamics and the optical flow global flow field measurement technique based on particle image was developed and expected to enrich experimental aerodynamics technology and promote the development of aerodynamics.

2. Definition of Optical Flow Velocity Field

The velocity field obtained by cross-correlation algorithm of PIV is the velocity field of particle swarm, in fact, it is also the optical flow velocity field or the image flow velocity field. Whether that the optical flow velocity field can representing the true velocity field depend on that whether tracers can adequately follow the fluid motion. In addition, the velocity field extracted from gradient method and image correlation velocimetry is also optical flow velocity field or image flow velocity field.

Under the condition of illumination, the surface grayscale of an object will presents certain spatial distribution, which is called grayscale pattern, and the so-called “optical flow” refers to the motion of the grayscale pattern. When human’s eyes observe moving object, the motion scene of the object will show a series of continuous varying images passing through the retina (i.e. the image plane), just like a stream of light, which is called “optical flow”, passing through the retina. The optical flow image represents the spatial distribution of some kind of physical quantity, such as temperature, concentration, density or the grayscale space distribution. The optical flow coming from the movement between observer and observed object can reveal the motion information of the observed object [6].

Horn and Schunck creatively studied optical flow in 1980s. Horn et al thought that the optical flow field caused by moving object should be continuous and smooth, namely the grayscale variation projected on the image should be smooth because velocities of adjacent points on the same object should be similar. Based on this idea, additional velocity smoothness constraint, i.e. global smoothness constraint can be added to optical flow field, so the calculation problem of optical flow field could be translated into the calculus problem of variation and the motion estimation of pixel scale would be obtained [5].

3. Integration Minimization Optical Flow Velocimetry

The grayscale pattern of optical flow is the distribution of grayscale $\xi$ in the image. In a very short time interval $\Delta t$, the distribution of $\xi$ varies very little with time and space, so $\xi$ satisfies the following equation:

$$\xi(x, y, t) = \xi(x + \Delta x, y + \Delta y, t + \Delta t)$$  \hspace{1cm} (1)

Here $\xi(x, y, t)$ is the image grayscale on time $t$ and location $(x, y)$, and $\xi(x + \Delta x, y + \Delta y, t + \Delta t)$ is the image grayscale on time $t + \Delta t$ and location $(x + \Delta x, y + \Delta y)$. Let $u = \Delta x/\Delta t$, $v = \Delta y/\Delta t$ and ignoring higher order terms, optical flow constraint equation can be obtained by Taylor expansion.

$$\xi_x u + \xi_y v + \xi_t = 0$$ \hspace{1cm} (2)

$\xi_x$, $\xi_y$, $\xi_t$ are two-dimensional space and time gradients of grayscale. Obviously, this equation, which is an indeterminate equation with two unknowns $u$, $v$, can only solve the projection of velocity vector $\vec{u}$ on the direction...
of the grayscale gradient \((\xi_x, \xi_y)\). So the additional constraint equation needs to be introduced here for solving two unknowns \(u, v\).

Each point of the image grayscale pattern could not move independently but correlatively, and velocities on adjacent positions should be continuous, i.e. the velocity smoothness constraint of image grayscale pattern. One of expressions of additional velocity smoothness constraint is introducing the regularization operator, i.e. minimizing the quadratic sum of the optical flow velocity gradients, which could eliminate singular dimension. This is very common in the inverse problem related to discrete data [9].

\[
\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial \nu}{\partial y}\right)^2
\]

As the effect of experimental noise and discrete error, equation (2) is difficult to be constantly equal to zero, so equation (2) should be combined with formula (3) by integration minimization method to seek the solution of velocity vector field \(\vec{v}(u, v)\), which has the minimum difference with the real velocity field. The mathematical expression of integration minimization optical flow velocimetry is shown here:

\[
\min_{u(x) \in \mathbb{R}^2} \int_{\mathbb{R}^2} \left[ E(u_1, u_2; x_1, x_2) \right]^2 d^2x
\]

(4)

Here \(u_i\) is dependent variable and \(x_i\) is independent variable. \(E\) is the quadratic sum of residual value of optical flow constraint equation and regularization operator, and \(E\) also represents the deviation between the velocity field \(\vec{u}\) extracted from the grayscale field and the real velocity field \(\hat{U}\), i.e. the extent of approximation. The regularization operator, which can ensure the mathematical stability of velocity vector \(\vec{u}\) in the presence of small image noise and discrete error condition, is an explicit expression of velocity field smoothness conditions in integration minimization optical flow velocimetry. Then, let \(E = E_1 + E_2\), and here:

\[
E_1 = \left[ \left( \frac{\partial}{\partial t} + \vec{u} \cdot \nabla \right) \xi \right]^2
\]

(5)

\[
E_2 = \beta^2 \nabla \vec{u} : \nabla \vec{u}
\]

(6)

\(\beta^2\) in equation (6) is a weighted value and must be chosen appropriately to reflect the experimental and numerical discrete noise.

4. Euler Characteristic Equation and Iterative Algorithm

After \(E(u_1, u_2; x_1, x_2)\) has been defined by \(E_1, E_2\) and the weighted value \(\beta^2\), the Euler characteristic equation of each dependent variable could be established by variational method. Variational method is the basic method for solving integration minimization optical flow velocimetry and with symmetric boundary condition. The Euler characteristic equation is shown as the follow equation [10, 11]:

\[
\frac{\partial E}{\partial u_i} - \sum_{j=1}^{n} \frac{\partial}{\partial x_j} \left[ \frac{\partial E}{\partial (\partial u_i / \partial x_j)} \right] = 0
\]

(7)

Here \(n\) is the index of independent variable. The following two equations can be obtained by introducing \(E_1\) and \(E_2\) into equation (7):
If the grayscale field data (two dimensional on space and one dimensional on time) has been collected in experiment, the time derivative and space gradient of each discrete point’s grayscale in the measured region could be obtained, and the equations (8) and (9) would be closed for the velocity vector \( \vec{u} \), so the velocity vector \( \vec{u} \) could be solved from these two equations.

The space discretization of grayscale’s time derivative and space gradient is shown at Fig.1, where \( i, j \) represent the space coordinate \( x, y \) and \( k \) represents the time coordinate \( t \). The pixel grayscale at point \( i, j \) on time \( k \) is expressed as \( \xi_{i,j,k} \) which is discretized by unit scale. The discretization schemes of grayscale’s time derivative and space gradient are shown in equations (10), (11) and (12).

\[
\xi_x = \frac{1}{4}(\xi_{i+1,j,k} - \xi_{i,j,k} + \xi_{i+1,j+1,k} - \xi_{i,j+1,k} + \xi_{i+1,j,k+1} - \xi_{i,j,k+1} - \xi_{i,j+1,k+1} + \xi_{i+1,j+1,k+1})
\]

(10)

\[
\xi_y = \frac{1}{4}(\xi_{i,j+1,k} - \xi_{i,j,k} + \xi_{i+1,j+1,k} - \xi_{i,j+1,k} + \xi_{i,j+1,k+1} - \xi_{i,j,k+1} + \xi_{i+1,j,k+1} - \xi_{i,j+1,k+1})
\]

(11)

\[
\xi_t = \frac{1}{4}(\xi_{i,j,k+1} - \xi_{i,j,k} + \xi_{i+1,j,k+1} - \xi_{i,j+1,k} + \xi_{i,j+1,k+1} - \xi_{i,j,k+1} + \xi_{i+1,j+1,k+1} - \xi_{i,j+1,k+1})
\]

(12)

Equations (10), (11) and (12) indicate that time derivative and space gradient have second order accuracy and are the one of the cube center in figure 1. The Laplace discretization schemes of velocity components are equations (13), (14), (15) and (16), which are also second order accuracy.

\[
\nabla^2 u = \kappa(\vec{u}_{i,j,k} - u_{i,j,k})
\]

(13)

\[
\nabla^2 v = \kappa(\vec{v}_{i,j,k} - v_{i,j,k})
\]

(14)

\[
- u_{i,j,k} = \frac{1}{6}(u_{i-1,j,k} + u_{i+1,j,k} + u_{i,j-1,k} + u_{i,j+1,k}) + \frac{1}{12}(u_{i-1,j-1,k} + u_{i+1,j-1,k} + u_{i-1,j+1,k} + u_{i+1,j+1,k})
\]

(15)
\[-v_{i,j,k} = \frac{1}{6} \left( v_{i-1,j,k} + v_{i+1,j,k} + v_{i,j-1,k} + v_{i,j+1,k} + v_{i-1,j+1,k} + v_{i+1,j+1,k} \right) + \frac{1}{12} \left( v_{i-1,j-1,k} + v_{i+1,j-1,k} + v_{i-1,j+1,k} + v_{i+1,j+1,k} \right) \]

(16)

Here \( \kappa = 3 \). Then equations (17), (18) could be obtained after introducing equations (13) and (14) into equations (8) and (9) and here \( \alpha^2 = \kappa \beta^2 \).

\[
\left( \varepsilon_x^2 + \alpha^2 \right) u + v \xi_x \xi_y = \alpha^2 \bar{u} - \xi_x \xi_y
\]

(17)

\[
u \xi_x \xi_y + \left( \varepsilon_y^2 + \alpha^2 \right) v = \alpha^2 \bar{v} - \xi_x \xi_y
\]

(18)

The following equations could be further deduced from equations (17), (18):

\[
u = \frac{- \xi_x \left( \varepsilon_x u + \xi_y v + \xi_y \right)}{\left( \alpha^2 + \varepsilon_x^2 + \varepsilon_y^2 \right)}
\]

(19)

\[
u = \frac{- \xi_x \left( \varepsilon_x u + \xi_y v + \xi_y \right)}{\left( \alpha^2 + \varepsilon_x^2 + \varepsilon_y^2 \right)}
\]

(20)

According to equations (19) and (20), the iterative discretization scheme could be established, and the movement information would be extracted from optical flow field.

5. Verification Experiment of Optical Flow Velocity Field Measurement

The preliminary calculation program had been compiled according to the optical flow algorithm and verified by the vortex structure observation experimental of backward facing step interval flow field.

The difficulty in this experiment is to guarantee the experiment conditions satisfying the constraint equation of optical flow velocimetry. For effective calculation of optical flow velocity field, this algorithm requires identical illumination condition for each image of continuous shooting and tiny displacement of corresponding pixels on adjacent images.

The experiment was conducted in a small low-speed wind tunnel system which was set up temporarily in the laboratory. The whole system was composed of wind tunnel and flow control sub system, tracer particles generation and seeding sub system, laser illumination and optical path sub system, image acquisition sub system and computer processing sub system. The experiment schematic diagram is shown in Fig.2.
The flow speed of wind tunnel test section is controlled by a controllable turbo fan motor and within the range of 1m/s to 20 m/s by regulating the frequency converter. The wind tunnel inhaled air flow mixed with tracer particles coming from three seeding tubes, and after passing through the cellular, the air and the tracer particles mixed uniformly. An Argon ion continuous laser, beam expander lens and cylindrical lens were applied in this experiment to achieve identical illumination condition. A high-speed camera with maximum frequency of 1500HZ was used to capture images and ensure tiny displacement at scale of pixel between adjacent images. A calibration plate should be placed in the experiment area, and CCD camera would record the image of calibration plate to determine the actual measurement area size.

6. Results and Analysis of the Verification Experiment

The image of backward facing step internal flow field captured by the high-speed camera is shown in Fig.7, and the dashed box in the image is the computational domain. A large number of continuously captured images recorded the information of continuously varying optical flow field.

The distribution of the velocity field calculated by the optical flow algorithm is pixel scale resolution and shown in Fig.8. The local velocity vector distribution in dashed box in Fig.8 is shown in Fig.9, and the velocity distribution of high resolution would be significant for the research on flow field structure. Fig.10 displays the velocity vector distribution by the interval of $8 \times 8$ pixels, which is accordance with the minimum resolution of PIV (Fig.11). Compared with Fig.11, the optical flow result is consistent with that of PIV at velocity magnitude, direction and velocity contour lines distribution, and there is no distinct difference in the vortex structure and location. In addition, the velocity contour’s edges of optical flow result are smoother than that of PIV, and it is related with introducing smoothness constraint condition into the optical flow algorithm.

By comparison, it can be concluded that the optical flow algorithm could obtain a smoother velocity field than PIV in the conditions of pixel scale and is more suitable for velocity field measurement of complex flow.
7. Analysis of Correlation

Fig.12 is the relation curve of the weighted value $\beta^2$ and the correlation coefficient between optical flow result and PIV result. From the curve, the correlation coefficient $r$ rapidly increases when $1 \leq \beta^2 \leq 8$; the correlation coefficient $r$ gently increases when $8 \leq \beta^2 \leq 16$; the correlation coefficient $r$ gradually change when $16 \leq \beta^2$; the correlation coefficient $r$ reach the maximum value when $\beta^2 = 20$.

As a weighted value, $\beta^2$ reflect the relative influence of the experimental and numerical discrete noise. If $\beta^2$ is relative large value, the noise is also relative large. Particle image is different from LIF image and its grayscale distribution has discontinuity of certain extent, so the weighted value $\beta^2$ need to be increased to reduce error. The choice of $\beta^2$ should be further researched.

8. Conclusion

Through the analysis of optical flow algorithm, the experiment requirement of optical flow velocimetry was studied and the test platform was successfully set up. By comparison between optical flow result and PIV result, it can be concluded that the optical flow algorithm could obtain a smoother velocity field than PIV in the conditions of pixel scale and is more suitable for velocity field measurement of complex flow.

References