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Construction of a robust warm inflation mechanism

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Abstract

A dissipative mechanism is presented, which emerges in generic interacting quantum field systems and which leads to robust warm inflation. An explicit example is considered, where using typical parameter values, it is shown that considerable radiation can be produced during inflation. The extension of our results to expanding spacetime also is discussed.
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1. Introduction

Inflationary dynamics inherently is a multifield problem, since the vacuum energy that drives inflation eventually must convert to radiation, which generally is comprised of a variety of particle species. Phenomenologically it has been shown that the inflation and radiation production phases can be two well separated periods in scenarios generically termed supercooled (or isentropic) inflation (for a review see [1]), or radiation production can occur concurrently with inflationary expansion in scenarios generically termed warm (or non-isentropic) inflation [2]. Warm inflation is a broader picture, since the extent of radiation production during inflation is variable, so that supercooled inflation emerges as the limiting case of zero radiation production.

Although by now considerable work has demonstrated its phenomenological significance [3], one key barrier to the warm inflation picture has been establishing plausibility of its dynamics from first principles quantum field theory. To some extent this point has been overemphasized for warm inflation, since in similar respects particle production during the far out-of-equilibrium reheating phase of supercooled inflation is not well understood, thus leaving incompleteness also to this picture. However, for supercooled inflation, since particle production is assumed not to affect large scale structure formation during inflation, thus the main observational predictions, these shortcomings are cast aside as secondary concerns. Nevertheless, without a solution here, this picture is unproven. On the other hand, the warm inflation picture makes no a priory assumption that particle production does not affect

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large scale structure formation. As such, the particle production problem appears more acute here. More basically a proper understanding of particle production should mean that theory itself can decide which or to what extent either of these two pictures is valid. Undoubtedly, no theory based on inflationary expansion will ever emerge, until particle production in quantum field theory is adequately understood.

This is a major problem, which must be tackled in steps. Fair enough is to attempt to see how well either picture of inflation can be understood from first principles and *en route* hope a clearer general picture eventually will emerge. For warm inflation, there is greater possibility to understand particle production, and eventually reach closure at a theoretical level about the viability of this picture as a description of the early universe. The reason is that recall in this picture the scalar inflaton field is required to have a slow, overdamped motion. As such, adiabatic methods of quantum field theory are applicable here, and these are the only methods for which dissipation can be unarguably analyzed.

The road toward a first principles warm inflation picture primarily has been hindered by basic gaps in the understanding of dissipative quantum field theory, which during the course of developing warm inflation are being filled [4–9]. The first attempt to understand warm inflation dynamics utilized finite temperature dissipative quantum field theory, since some formalism already existed here [10–14]. Based on this work [4], statements of a general sort have been made about the impossibility of warm inflation dynamics [6]. However, these criticisms failed to recognize that the key problems were specific to the restrictive constraints of the high-T approximation and were not reflexive of warm inflation in general.

Intrinsically, warm inflation is an out-of-equilibrium problem, in that it is not tied to any specific equilibrium statistical state, but rather simply requires radiation production concurrent with the overdamped relaxation of a global order parameter. Although the actual statistical state during warm inflation may not be very far from an equilibrium state, at present the problem is simply technical limitations in describing the scope of such states. Furthermore, as has been noted [2,7], very little radiation production during inflation, at the scale of tens of orders of magnitude below the vacuum energy density, is already sufficient to affect large scale structure formation and create an adequately high post-inflation temperature.

With these thoughts in mind, in [7] a simple attempt was made to circumvent the specific constraints of the high-temperature formalism, by examining dissipation at zero-temperature. The point there was to investigate a suggestions learned from our high-temperature analysis, that alleviation of the constraints specific to the high-T approximation would adequately allow realizing robust radiation production during warm inflation. The main purpose of [7] was to develop the necessary formalism, but in addition one suggestive mechanism was identified that could realize this point, which involved a scalar Φ field (whose zero mode can be associated, e.g., with the inflaton) exciting heavy χ-bosons which then decay into lighter ψ-fermions. This Letter reports a detailed investigation of this process and demonstrates that it is a robust mechanism for warm inflation. For this, in Section 2 a linear response derivation will be presented, which in the adiabatic regime and at leading order is equivalent to the closed time Lagrangian formalism, but is simpler and physically more transparent. Then in Section 3 an alternative derivation is presented, using canonical methods. From this approach, the origin of particle production and energy balance for this mechanism will be clarified. Next, Section 4 gives a physical picture to the mechanism and supplies an explicit numerical example to demonstrate the extent of radiation production it yields during inflation. Section 5 discusses the extension of the calculation to expanding spacetime. Finally the conclusions are given in Section 6.

2. A model for robust radiation production

We consider a multi-field model, first studied in [7], of a scalar field Φ interacting with a set of scalar fields χ_j, $j = 1, \dots, N_\chi$, which in turn interact with fermion fields ψ_k, $k = 1, \dots, N_\psi$, with Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{m_{0\phi}^2}{2}\Phi^2 - \frac{\lambda}{4!}\Phi^4 + \sum_{j=1}^{N_\chi}\left[\frac{1}{2}(\partial_\mu \chi_j)^2 - \frac{m_{0\chi_j}^2}{2}\chi_j^2 - \frac{f_j}{4!}\chi_j^4 - \frac{g_j^2}{2}\Phi^2\chi_j^2\right]$$

$$+\sum_{k=1}^{N_\psi}\bar{\psi}_k\left[i\not{\partial}-m_{0\psi_k}-\sum_{j=1}^{N_\chi}h_{kj}\chi_j\right]\psi_k. \tag{1}$$

The regime of interest for warm inflation, that is studied here is $m_{\chi_j} > 2m_{\psi_k} > m_\phi$, where these are the renormalized and, if relevant, background field dependent masses.

By decomposing Φ in terms of a homogeneous classical part, $\varphi(t)$, and its fluctuations ϕ, the effective equation of motion (EOM) for φ emerges as

$$\ddot{\varphi}(t)+m_{0\phi}^2\varphi(t)+\frac{\lambda}{6}\varphi^3(t)+\frac{\lambda}{2}\varphi(t)\langle\phi^2\rangle+\frac{\lambda}{6}\langle\phi^3\rangle+\sum_{j=1}^{N_\chi}g_j^2\left[\varphi(t)\langle\chi_j^2\rangle+\langle\phi\chi_j^2\rangle\right]=0. \tag{2}$$

We will use a linear response theory approach in which the field averages in Eq. (2) are expressed in terms of the respective field propagators $G_\phi(x,x')$ and $G_{\chi_j}(x,x')$. Also in the following, we derive the φ effective EOM from an adiabatic approximation. This approximation requires that all macroscopic motion is slow relative to the characteristic scales of the microscopic dynamics. In our model the time scale for microscopic dynamics is represented through the (inverse of the) particle decay widths Γ_ϕ, Γ_χ and for macroscopic dynamics is contained in $\varphi(t)$, with the basic consistency condition [4]

$$\dot{\varphi}/\varphi \ll \Gamma_\phi, \Gamma_\chi. \tag{3}$$

Turning to the derivation, consider first $\langle\chi_j^2\rangle$. This expectation value can be expressed in terms of the coincidence limit of the (causal) two-point Green's function for the χ_j field, $G_{\chi_j}^{++}(x,x')$. Recall that this Green's function is the $(1,1)$-component of the real time matrix of full propagators, all of which satisfy the appropriate Schwinger–Dyson equations (see, e.g., [4,7] for additional details)

$$\left[\Box+m_{\chi_j}^2+g_j^2\varphi^2(t)\right]G_{\chi_j}(x,x')+\int d^4z\,\Sigma_{\chi_j}(x,z)G_{\chi_j}(z,x')=i\delta(x,x'), \tag{4}$$

where Σ_{χ_j} is the χ_j field self-energy. The field frequencies appearing in these propagators depend on the background field configuration $\varphi(t)$. This field is decomposed as $\varphi(t)=\varphi_0+\delta\varphi(t)$, where φ_0 is a constant (the value of the field at say the initial time $t=t_0$) and $\delta\varphi(t)$ is treated perturbatively. This is just a linear response theory approach to calculating the averages of the fields appearing in Eq. (2). Following this procedure, we have that $\langle\chi_j^2\rangle$ can be written to lowest order as

$$\langle\chi_j^2\rangle\simeq\langle\chi_j^2\rangle_0-i\int_{-\infty}^{t}dt'\,\frac{g_j^2}{2}\left[\varphi^2(t')-\varphi_0^2\right]\left\langle\left[\chi_j^2(\mathbf{x},t),\chi_j^2(\mathbf{x},t')\right]\right\rangle, \tag{5}$$

where $\langle\ldots\rangle_0$ means the correlation function evaluated at the initial time. The φ^2 dependence in Eq. (5) emerges from expanding the two point function with respect to the $\delta\varphi$ dependent terms. Formally this can be done by treating $\delta\varphi$ dependent terms in the shifted potential as interaction vertices. This implies adding an interacting vertex quadratic in the χ_j field, with Feynman rule $-ig_j^2/2\ [\varphi^2(t)-\varphi_0^2]$, and is used in calculating the leading order one-loop bubble diagram that gives the two-point function. This method was first implemented to study dissipation in [12,13] and more recently in [7]. This is also analogous to the functional Schwinger closed time path formalism used in [4,10]. Using translational invariance we can now write $\langle[\chi_j^2(\mathbf{x},t),\chi_j^2(\mathbf{x},t')]\rangle$, appearing in Eq. (5), in terms of the two-point Green's function for the χ_j field, $G_{\chi_j}^{++}(x,x')$, as

$$\left\langle\left[\chi_j^2(\mathbf{x},t),\chi_j^2(\mathbf{x},t')\right]\right\rangle=2i\,\mathrm{Im}\left\langle T\chi_j^2(\mathbf{x},t)\chi_j^2(\mathbf{x},t')\right\rangle=4i\int\frac{d^3q}{(2\pi)^3}\,\mathrm{Im}\left[G_{\chi_j}^{++}(\mathbf{q},t-t')\right]_{t>t'}^2, \tag{6}$$

where $G^{++}_{\chi_j}(\mathbf{q}, t-t')$ is given by (see, e.g., [7] for the explicit expressions for both the scalar and fermion field propagators) $G^{++}_{\chi_j}(\mathbf{q}, t-t') = G^{>}_{\chi_j}(\mathbf{q}, t-t')\theta(t-t') + G^{<}_{\chi_j}(\mathbf{q}, t-t')\theta(t'-t)$. Here $G^{>}_{\chi_j}$, $G^{<}_{\chi_j}$ are

$$G^{>}_{\chi_j}(\mathbf{q}, t-t') = \frac{1}{2\omega_{\mathbf{q},\chi_j}(0)}\left\{e^{-i[\omega_{\mathbf{q},\chi_j}(0) - i\Gamma_{\chi_j}](t-t')}\theta(t-t') + e^{-i[\omega_{\mathbf{q},\chi_j}(0) + i\Gamma_{\chi_j}](t-t')}\theta(t'-t)\right\}$$
$$G^{<}_{\chi_j}(\mathbf{q}, t-t') = G^{>}_{\chi_j}(\mathbf{q}, t'-t), \tag{7}$$

where $\omega_{\mathbf{q},\chi_j}(0) = \sqrt{\mathbf{q}^2 + m^2_{0\chi_j} + \mathrm{Re}\,\Sigma_{\chi_j}(q) + g_j^2\varphi_0^2}$, with $\Sigma_{\chi_j}(q)$ the χ_j field self-energy (recall that the field decay width Γ_{χ_j} is related to the imaginary part of the self-energy as $\Gamma_{\chi_j}(q) = -\mathrm{Im}\,\Sigma_{\chi_j}(q)/(2\omega_{\mathbf{q},\chi_j})$). Thus using Eq. (7) in Eq. (6), the explicit expression for Eq. (5) becomes

$$\langle\chi_j^2\rangle \simeq \int \frac{d^3q}{(2\pi)^3}\frac{1}{2\omega_{\mathbf{q},\chi_j}(0)} - g_j^2 \int_{-\infty}^{t} dt' \left[\varphi^2(t') - \varphi_0^2\right] \int \frac{d^3q}{(2\pi)^3} 2\frac{\exp(-2\Gamma_{\chi_j}|t-t'|)}{[2\omega_{\mathbf{q},\chi_j}(0)]^2} \sin\left[2\omega_{\mathbf{q},\chi_j}(0)|t-t'|\right]. \tag{8}$$

For the second term on the RHS of Eq. (8), after integrating by parts with respect to t', it becomes

$$\begin{aligned} -g_j^2 &\int_{-\infty}^{t} dt' \left[\varphi^2(t') - \varphi_0^2\right] \int \frac{d^3q}{(2\pi)^3} 2\frac{\exp(-2\Gamma_{\chi_j}|t-t'|)}{[2\omega_{\mathbf{q},\chi_j}(0)]^2} \sin\left[2\omega_{\mathbf{q},\chi_j}(0)|t-t'|\right] \\ &= -g_j^2\left[\varphi^2(t) - \varphi_0^2\right] \int \frac{d^3q}{(2\pi)^3} \frac{1}{4\omega_{\mathbf{q},\chi_j}(0)[\omega^2_{\mathbf{q},\chi_j}(0) + \Gamma^2_{\chi_j}]} \\ &\quad + g_j^2 \int_{-\infty}^{t} dt'\, \varphi(t')\dot{\varphi}(t') \int \frac{d^3q}{(2\pi)^3} \exp\left(-2\Gamma_{\chi_j}|t-t'|\right) \\ &\qquad \times \frac{\{\omega_{\mathbf{q},\chi_j}(0)\cos[2\omega_{\mathbf{q},\chi_j}(0)|t-t'|] + \Gamma_{\chi_j}\sin[2\omega_{\mathbf{q},\chi_j}(0)|t-t'|]\}}{2\omega^2_{\mathbf{q},\chi_j}(0)[\Gamma^2_{\chi_j} + \omega^2_{\mathbf{q},\chi_j}(0)]}. \end{aligned} \tag{9}$$

The first (local) terms on the RHS of both Eqs. (8) and (9), when perturbatively expanded in the coupling constant lead to quantum corrections from the χ_j-fields to $m^2_{0\phi}$ and λ, to order g_j^2 and g_j^4, respectively. These corrections are divergent but are renormalized by the usual procedure of adding mass and coupling constant counter-terms. The second term on the RHS of Eq. (9) is responsible for dissipation. In this study, we are interested in the regime where $\varphi(t)$ changes slowly relative to the relaxation time, in this case set by Γ_{χ_j}, which means the adiabatic approximation is valid. Under this approximation, similar to the treatment in [7], a Markovian, or equivalently time local, treatment can be used, which amounts to a derivative expansion of the field $\varphi(t)$ and in which the leading $\dot{\varphi}$ term only is retained. After implementing this approximation and substituting Eq. (9) back into Eq. (8), we obtain

$$\langle\chi_j^2\rangle \simeq \int \frac{d^3q}{(2\pi)^3 2\omega_{\mathbf{q},\chi_j}(t)}\left\{1 + \frac{g_j^2\varphi\dot{\varphi}\Gamma_{\chi_j}}{[\omega^2_{\mathbf{q},\chi_j}(t) + \Gamma^2_{\chi_j}]^2}\right\}. \tag{10}$$

In the above, note we have conveniently reintroduced the time dependence back into the field frequencies and when they are perturbatively expanded to order g_j^4, the above mentioned mass and coupling constant corrections are correctly reproduced.

An analogous expression to Eq. (10) also follows for $\langle\phi^2\rangle$. Note, however, that for an initial (at $t = t_0$) zero temperature bath and for fields Φ and χ_j satisfying the mass constraint $m_{\chi_j} > 2m_{\psi_k} > m_\phi$, there only will be decay channels for χ_j into ψ_k particles. As a result, it implies $\Gamma_\phi(q) = 0$ and $\Gamma_{\psi_k}(q) = 0$, while we have that

$$\Gamma_{\chi_j}(q) = \sum_{k=1}^{N_\psi} h_{kj}^2 \frac{m_{\chi_j}^2}{8\pi \omega_{\mathbf{q},\chi_j}} \left(1 - \frac{4m_{\psi_k}^2}{m_{\chi_j}^2}\right)^{3/2}. \tag{11}$$

As such, in the adiabatic regime, dissipation will only involve the decay of χ_j particles. The other two averages in the EOM, $\langle\phi^3\rangle$ and $\langle\phi\chi_j^2\rangle$ can also be worked out in the linear response approach, and their leading contributions are at two-loop order [7]. Here, we will not consider them but restrict our calculation to leading one-loop order for simplicity. In this case, the only contribution to dissipation is Eq. (10), and this effect already will be adequate to demonstrate considerable radiation production from our model Lagrangian. Substituting Eq. (10) back into the effective EOM, Eq. (2), the second term on the RHS of Eq. (10) leads to a dissipative term in the EOM and the first term leads to Φ mass and coupling constant divergent corrections, that can be renormalized as usual by the introductions of counterterms in Eq. (1). This renormalization procedure is standard and will not be further addressed. In our final expressions, all mass parameters, $m_{0\phi}$, $m_{0\chi_j}$, $m_{0\psi_k}$, and coupling constants, λ, g_j, h_{kj} are then taken as the renormalized ones. The renormalized effective EOM for $\varphi(t)$ that finally emerges can be written as

$$\ddot{\varphi} + \frac{\partial V_{\rm eff}(\varphi)}{\partial\varphi} + \eta(\varphi)\dot{\varphi} = 0. \tag{12}$$

In the above equation, we have included in $V_{\rm eff}$ the quantum renormalization corrections to the mass and coupling constant for the Φ field, which are exactly the same as found in the calculation of a constant background φ-field effective potential. The dissipation coefficient $\eta(\varphi)$ in Eq. (12) comes from performing the momentum integral in Eq. (10) and using (11) to give

$$\eta(\varphi) = \varphi^2(t) \sum_{j=1}^{N_\chi} \frac{g_j^4 \alpha_{\chi,\psi}^2 (m_{\chi_j}^4 + \alpha_{\chi,\psi}^4)^{-1/2}}{32\pi (2\sqrt{m_{\chi_j}^4 + \alpha_{\chi,\psi}^4} + 2m_{\chi_j}^2)^{1/2}}, \tag{13}$$

where $\alpha_{\chi,\psi}^2 = \sum_{k=1}^{N_\psi} h_{kj,\chi}^2 m_{\chi_j}^2 (1 - 4m_{\psi_k}^2/m_{\chi_j}^2)^{3/2}/(8\pi)$ and m_{χ_j} in Eq. (13) denote the field dependent masses, $m_{\chi_j}^2 \equiv m_{\chi_j}^2(\varphi) = m_{0\chi_j}^2 + g_j^2\varphi^2(t)$. The dissipative mechanism Eq. (13) overcomes an underlying impediment to realizing robust warm inflation in the finite temperature calculations [4,6], where all mass scales were constrained by the temperature. In sharp contrast, a key feature about the dissipative mechanism of this paper is that irrespective of the magnitude of φ and m_{χ_j}, dissipation occurs unchecked by these severely limiting constraints.

For the dissipative mechanism derived in this Letter to be applicable to warm inflation, there must be some control in determining the quantum corrections in $V_{\rm eff}$ in Eq. (12). This is required mainly since, similar to supercooled inflation, in the warm inflation case also, treatment of density perturbations requires an ultraflat potential [2,3,15]. However, there are one-loop quantum corrections to the T = 0 effective potential arising in the Lagrangian Eq. (1) from the self-interaction of the ϕ-field and from its interactions with the χ-fields, which give [16]

$$V_1(\varphi) = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\left(E_{m_\phi} + \sum_{i=1}^{N_\chi} E_{m_{\chi_i}}\right), \tag{14}$$

where $E_{m_\phi} = \sqrt{\mathbf{k}^2 + m_{0\phi}^2 + \lambda\varphi^2/2}$ and $E_{m_{\chi_i}} = \sqrt{\mathbf{k}^2 + m_{0\chi_i}^2 + g_i^2\varphi^2}$. To obtain the desired ultraflat potential, it requires λ to be tiny with $m_{0\phi}^2 \lesssim \lambda\varphi^2/2$. In this regime, the contribution from the E_{m_ϕ} term above is negligible. However, since in general we want $g_i^4 \gg \lambda$, the one-loop contributions from the $E_{m_{\chi_i}}$ terms lead to corrections $\sim g_i^4\varphi^4$ in $V_{\rm eff}$ and thus would ruin the flatness of the potential. Operationally these one-loop contributions can be controlled by adding to the Lagrangian Eq. (1) fermionic "partners" ψ^{χ} to the χ-fields, with one ψ^{χ}-field for every four χ-fields and coupling only to the Φ-field as $\sum_{i=1}^{N_\chi/4} g_i^{\chi}\bar{\psi}_i^{\chi}\psi_i^{\chi}\Phi$. The one-loop quantum corrections to the effective potential from these terms will yield [16]

$$V_1(\varphi) = -2\int \frac{d^3k}{(2\pi)^3}\sum_{i=1}^{N_\chi/4} E_{m_{\psi_i^{\chi}}}, \tag{15}$$

where $E_{m_{\psi_i^{\chi}}} = \sqrt{k^2 + (m_{0\psi_i^{\chi}} + g_i^{\chi}\varphi)^2}$. In particular, this fermionic contribution has the familiar opposite sign to the bosonic contribution. Thus with appropriately tuned parameters g_i, g_i^{χ} and with zero explicit masses $m_{0\psi_{\chi_i}} = m_{0\chi_i} = 0$, the one-loop quantum corrections to $V_{\rm eff}$ cancel to all orders in g_i, g_i^{χ}. This modification simply is mimicking supersymmetry. For realistic model building, the mechanism derived in this Letter must be examined in actual SUSY models, where the choice $g_i^4 \gg \lambda$ of coupling parameters could be obtained naturally, but that will not be pursued here.

3. Alternative derivation of dissipation—operator formalism

For completeness, here an alternative derivation of dissipation is presented using the canonical approach and following the formalism developed in [12,14]. In this approach, the fields ϕ, χ and ψ are expressed in terms of their mode decompositions and dynamics is determined with respect to the mode operators. Thus, for example, for the $\chi_j(\mathbf{x},t)$ field this means

$$\chi_j(\mathbf{x},t) = \int \frac{d^3q}{(2\pi)^{\frac{3}{2}}[2\omega_{\mathbf{q},\chi_j}(t)]^{\frac{1}{2}}}\big[a_{\mathbf{q},\chi_j}(t)e^{-i\mathbf{q}\cdot\mathbf{x}} + a^{\dagger}_{\mathbf{q},\chi_j}(t)e^{i\mathbf{q}\cdot\mathbf{x}}\big]. \tag{16}$$

Since there is a time dependent background field $\varphi(t)$, this induces time dependence in the frequencies and so in the creation/annihilation operators of the ϕ and χ_j fields. In the analysis that follows, we will focus on the χ_j fields, with similar considerations carrying over for the ϕ field.

The time dependent χ_j—frequency in Eq. (16) is given by $\omega_{\mathbf{q},\chi_j}(t) = [\mathbf{q}^2 + m_{0\chi_j}^2 + g_j^2\varphi^2(t)]^{1/2}$. From Eq. (16) it follows that

$$\langle\chi_j^2\rangle = \int \frac{d^3q}{(2\pi)^3 2\omega_{\mathbf{q},\chi_j}(t)}\big[2x_{\mathbf{q},\chi_j}(t) + 2\,\mathrm{Re}\big[y_{\mathbf{q},\chi_j}(t)\big] + 1\big], \tag{17}$$

where $x_{\mathbf{q},\chi_j}(t) = \langle a^{\dagger}_{\mathbf{q},\chi_j}(t)a_{\mathbf{q},\chi_j}(t)\rangle$ is the particle number density and $y_{\mathbf{q},\chi_j}(t) = \langle a_{\mathbf{q},\chi_j}(t)a_{-\mathbf{q},\chi_j}(t)\rangle$ is the off-diagonal correlation.

From the field equation for χ_j and Eq. (16) we can deduce the equations satisfied by $x_{\mathbf{q},\chi_j}$ and $y_{\mathbf{q},\chi_j}$. Taking also into account the possibility that the field χ_j can decay into lighter fields with a decay rate $\Gamma_{\chi_j}(q)$ as already given in Eq. (11), $x_{\mathbf{q},\chi_j}$ and $y_{\mathbf{q},\chi_j}$ can be shown to satisfy the coupled differential equations [12,14]

$$\dot{x}_{\mathbf{q},\chi_j} = \frac{\dot{\omega}_{\mathbf{q},\chi_j}}{\omega_{\mathbf{q},\chi_j}}\,\mathrm{Re}\,y_{\mathbf{q},\chi_j}, \qquad \dot{y}_{\mathbf{q},\chi_j} = \frac{\dot{\omega}_{\mathbf{q},\chi_j}}{\omega_{\mathbf{q},\chi_j} - i\Gamma_{\chi_j}(q)}\left[x_{\mathbf{q},\chi_j} + \frac{1}{2}\right] - 2i\big[\omega_{\mathbf{q},\chi_j} - i\Gamma_{\chi_j}(q)\big]y_{\mathbf{q},\chi_j}. \tag{18}$$

A solution for Eq. (18) can be found in the quasi-adiabatic regime as follows. Let us consider the case of a slowly changing configuration $\varphi(t)$. We can therefore suppose that the number of produced particles at time t is $x_{\mathbf{q},\chi_j}(t) \ll 1$. Consequently we also have that $\omega_{\mathbf{q},\chi_j}$ and its time derivative slowly change. We then find for $y_{\mathbf{q},\chi_j}$ in Eq. (18) the result

$$y_{\mathbf{q},\chi_j}(t) = -i\frac{\dot{\omega}_{\mathbf{q},\chi_j}\{1 - \exp[-2i(\omega_{\mathbf{q},\chi_j} - i\Gamma_{\chi_j})t]\}}{4[\omega_{\mathbf{q},\chi_j} - i\Gamma_{\chi_j}(q)]^2}, \tag{19}$$

which in the limit $t \gg \Gamma_{\chi_j}^{-1}$ yield

$$\mathrm{Re}\, y_{\mathbf{q},\chi_j}(t) = \frac{g_j^2}{2}\varphi\dot{\varphi}\frac{\Gamma_{\chi_j}}{(\omega_{\mathbf{q},\chi_j}^2 + \Gamma_{\chi_j}^2)^2}. \tag{20}$$

Using Eq. (20) in Eq. (17), once again we get Eq. (10), from which the effective EOM Eq. (12) follows. A shortcoming of this approach is that interactions are added to the set of Eq. (18) in a somewhat ad-hoc way. This point was discussed recently in [8], where the complete kinetic equations where derived for the single field self-interacting ϕ^4 model. Nevertheless, the final answer from the approach of this section agrees with that from the Lagrangian based approach of the previous section, where interactions can be added consistently through the appropriate set of Schwinger–Dyson equations for the propagators [7]. Thus it suggests the results by this canonical approach are acceptable, but missing gaps in the formalism of [12] must still be resolved. For our purposes, due to the importance of the dissipative mechanism studied in this Letter, we felt it was important to point out the agreement between independently developed formalisms, even if there remain shortcomings in one of them. The practical significance of the results in this Letter provide motivation to address these difficult problems in the course of future work.

4. Physical interpretation and an explicit application

We now turn to an application of the equations derived above, using an explicit set of model parameter values, which are consistent with simple inflationary models. But before that, let us address briefly the physical interpretation of dissipation in Eq. (12).

We note that the evolving background field $\varphi(t)$ changes the masses of the χ_j bosons. As a consequence, the positive and negative frequency components of the χ_j-fields mix. This in turn results in the coherent production of χ_j particles which then decohere through decay into lighter ψ_k-fermions. This picture can be confirmed by checking energy balance. This is done by examining the time evolution of the χ_j-particle number density. For this, their number density is expressed in terms of time dependent creation and annihilation operators as $\mathcal{N} \equiv \sum_j \langle a_{\chi_j}^\dagger(t) a_{\chi_j}(t)\rangle$. By relating the time dependent operators $a_{\chi_j}^\dagger(t)$ and $a_{\chi_j}(t)$ to the initial, time independent, creation and annihilation operators through a Bogoliubov transformation, the total particle production rate then can be computed in general. Thus, the time evolution of the total production rate is

$$\dot{\mathcal{N}} = \sum_{j=1}^{N_\chi} \int \frac{d^3k}{(2\pi)^3} \dot{x}_{\mathbf{q},\chi_j}, \tag{21}$$

which using Eqs. (18) and (20), leads to

$$\dot{\mathcal{N}} = \dot{\varphi}^2 \sum_{j=1}^{N_\chi} \int \frac{d^3k}{(2\pi)^3} \frac{g_j^4}{2\omega_{\mathbf{q},\chi_j}} \frac{\Gamma_{\chi_j}}{(\omega_{\mathbf{q},\chi_j}^2 + \Gamma_{\chi_j}^2)^2}. \tag{22}$$

It can now be checked from Eqs. (2), (10) and (12), that the above result, Eq. (22) is precisely equal to the vacuum energy loss rate, $\eta\dot{\varphi}^2$, as obtained from the effective EOM, Eq. (12).

Let us now examine the application of the results in this Letter to warm inflation and also understand their significance. The scope of the present calculation is limited since dissipation at zero temperature necessarily implies a non-equilibrium state, which is evolving to some statistical state containing particles. Thus the estimates made below only give some idea of the magnitude of particle production. However, provided the magnitude is significant, as will be shown, it reveals that on scales relevant to inflation, quantum field theory with generic interactions has robust tendency to dissipate. For our estimates, we have set same all $\Phi - \chi$ couplings $g_{\chi_j} = g$ as well all $\chi - \psi$ couplings, $h_{kj} = h$.

We are interested in overdamped motion for the inflaton $\varphi(t)$, which requires (i) $m_\phi^2 \equiv m_\phi^2(\varphi) = m_{0\phi}^2 + \lambda\varphi^2/2 < \eta^2(\varphi)/4$. The adiabatic approximation Eq. (3) requires (ii) $m_\phi^2(\varphi)/\eta(\varphi) < \Gamma_\chi$. Although our derivation was for Minkowski spacetime, provided the time scale of microscopic dynamics is faster than the Hubble time scale, then within sub-Hubble length scales, this Minkowski spacetime calculation should be valid. For this to hold, it requires (iii) $H = \sqrt{8\pi V_{\rm eff}/3m_{\rm pl}^2} \simeq \sqrt{8\pi(\lambda/4!)\varphi^4/3m_{\rm pl}^2} < \Gamma_\chi$, where $m_{\rm pl}$ is the Planck mass. Also, so that the macroscopic motion of φ is governed by the dissipative term it requires (iv) $\eta(\varphi) > 3H$. Thus combining all four of the above consistency conditions leads to parametric constraints. To obtain these, we will treat $m_\chi \geqslant \alpha_{\chi,\psi}$, where from below Eq. (13) we have, by setting $m_\chi^2 \sim g^2\varphi^2$, $\alpha_{\chi,\psi}^2 \approx g^2h^2N_\psi\varphi^2/(8\pi)$, which thus requires $h^2N_\psi/(8\pi) < 1$. In this regime, we have from Eq. (13) $\eta \approx g^3h^2N_\chi N_\psi\varphi/(512\pi^2)$ and from Eq. (11) $\Gamma_\chi \approx gh^2N_\psi\varphi/(8\pi)$. The parametric constraints that follow from the four conditions given above are, respectively,

$$\text{(i)}\quad \lambda < \frac{g^6h^4N_\chi^2N_\psi^2}{2(512\pi^2)^2}, \qquad \text{(ii)}\quad \lambda < \frac{g^4h^4N_\chi N_\psi^2}{2048\pi^3},$$
$$\text{(iii)}\quad \lambda < \frac{9g^2h^4N_\psi^2}{64\pi^3}\frac{m_{\rm pl}^2}{\varphi^2}, \qquad \text{(iv)}\quad \lambda < \frac{g^6h^4N_\chi^2N_\psi^2}{\pi(512\pi^2)^2}\frac{m_{\rm pl}^2}{\varphi^2}. \tag{23}$$

To yield large dissipation, we are usually interested in the regime where the couplings g, h are big. To remain within a well defined perturbative region, we will then further require that $g^4N_\chi \lesssim 1$ and $h^2N_\psi \lesssim 1$ and will base our estimates on the upper bounds here. Also, in general $\varphi \lesssim m_{\rm pl}$, but to obtain the tightest constraints on λ in (iii) and (iv), we will set this at the equality point. Under these conditions, we find for the constraints (i)–(iv) in Eq. (23), respectively, $\lambda < \min(10^{-8}g^2N_\chi, 10^{-5}, 10^{-3}g^2, 10^{-8}g^2N_\chi)$. Recalling that constraints imposed by density fluctuations give typically $\lambda < 10^{-14}$ [2,3,15], we see that the above constraints introduce no stricter limitations.

As shown in Eq. (22), radiation production is determined by

$$\dot\rho_r(t) = \eta(\varphi)\dot\varphi^2 = -\frac{dV_{\rm eff}}{d\varphi}\dot\varphi \approx V_{\rm eff}(\varphi)\frac{m_\phi^2(\varphi)}{\eta}. \tag{24}$$

The zero temperature calculation should be valid for a time period $\sim 1/\Gamma_\chi$, in which time the magnitude of radiation produced is

$$\rho_r(1/\Gamma_\chi) \approx V_{\rm eff}(\varphi)m_\phi^2(\varphi)/(\eta\Gamma_\chi) < V_{\rm eff}(\varphi). \tag{25}$$

Based on Eqs. (11) and (13) and the above constraints on λ, there is considerable freedom in choosing the ratio $\mathcal{R} \equiv m_\phi^2/(\eta\Gamma_\chi)$ appearing in Eq. (25). Considering then an ultraflat potential, as necessary for observationally consistent density perturbations, which requires typical values of $\lambda \lesssim 10^{-14}$, this implies $\mathcal{R} \lesssim 10^{-10}/(g^4h^4N_\psi^2N_\chi)$. For unexceptional values of the perturbative coupling parameters, say $g \sim h \sim 0.1$, and small number of χ and ψ fields, $N_\chi, N_\psi \sim 1$–10, this leads to $\mathcal{R} \sim 10^{-(2-5)}$. Also note these parameters choices are consistent with the conditions on λ given above Eq. (24). Thus for a typical scale for inflation, where the potential energy is at the GUT scale, $V_{\rm eff}(\varphi)^{1/4} \sim 10^{15-16}$ GeV, it implies a generated radiation component which, if expressed in terms of temperature, is at the scale $T \sim 10^{13-16}$ GeV, and this is non-negligible. This is a significant result not only because the magnitude of produced radiation is large, but also because it emerges from a very generic interaction, scalar $\to$ heavy scalar $\to$ light fermions, which is very common in many particle physics models. Moreover, we expect similar robust radiation production for decay of the heavy scalars into gauge bosons. Finally, although we did this zero temperature calculation first simply due to its tractability, an interesting fact emerges for inflationary cosmology, that even if the initial state of the universe before inflation is at zero temperature, the dynamics itself could bootstrap the universe to a higher temperature during inflation.

5. Extension to expanding spacetime

The extension of this calculation is formally straightforward to Friedmann–Robertson–Walker (FRW) spacetime, $ds^2 = dt^2 - a^2(t)\,d\mathbf{x}^2$, where $a(t)$ is the cosmic scale factor and t is cosmic time. In this case, the extension of Eq. (1), for the Lagrangian density of the matter fields coupled to the gravitational field tensor $g_{\mu\nu}$, is given by

$$\begin{aligned}\mathcal{L} = \sqrt{-g}\Bigg\{ & \frac{1}{2} g^{\mu\nu}\partial_\mu \Phi \partial_\nu \Phi - \frac{m_{0\phi}^2}{2}\Phi^2 - \frac{\lambda}{4!}\Phi^4 - \frac{\xi}{2}R\Phi^2 \\ & + \sum_{j=1}^{N_\chi}\left[\frac{g^{\mu\nu}}{2}\partial_\mu \chi_j \partial_\nu \chi_j - \frac{m_{0\chi_j}^2}{2}\chi_j^2 - \frac{f_j}{4!}\chi_j^4 - \frac{g_j^2}{2}\Phi^2\chi_j^2 - \frac{\xi}{2}R\chi_j^2\right] \\ & + \sum_{k=1}^{N_\psi}\left[i\bar{\psi}_k \gamma^\mu(\partial_\mu + \omega_\mu)\psi_k - \bar{\psi}_k\left(m_{0\psi_k} + \sum_{j=1}^{N_\chi} h_{kj}\chi_j\right)\psi_k\right]\Bigg\},\end{aligned} \tag{26}$$

where R is the curvature scalar and ξ is the dimensionless parameter describing the coupling of the matter fields to the gravitational background. In the last terms involving the fermion fields, the γ^μ matrices are related to the vierbein e_μ^a (where $g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab}$, with η_{ab} the usual Minkowskii metric tensor) by $\gamma^\mu(x) = \gamma^a e_a^\mu(x)$ [17], where γ^a are the usual Dirac matrices and $\omega_\mu = -(i/4)\sigma^{ab} e_a^\nu \nabla_\mu e_{b\nu}$, with $\sigma^{ab} = i/2[\gamma^a, \gamma^b]$.

It is easy to show that the Lagrangian Eq. (26) in conformal time, t_c, where $dt = a\,dt_c$, remains unchanged from Eq. (1) except that all masses obtain time dependence related to $a(t_c)$ (see, for example, [14] for a similar problem). In particular, for the bosonic fields we have that $m_{\chi_j}^2(t_c) = m_{0\chi_j}^2 a^2(t_c) - d^2a/2a\,dt_c^2 + \xi a^2 R/2$ and similar for the ϕ field, and for the fermionic fields $m_{\psi_k}(t_c) = m_{0\psi_k} a(t_c)$. These time dependent parameters can be treated within the linear response formalism used in this Letter. Moreover, since the time dependence is associated with $a(t_c)$, it is easy to show that provided $H < \Gamma_\chi$, the time dependence of the mass terms is slow relative to microscopic dynamics and thus an appropriate adiabatic approximation should be applicable.

The observations made above are adequate to establish that, for the mechanism of central interest in this Letter, the robust dissipative properties found above for Minkowski spacetime also will hold for expanding spacetime. However, the exact form of the effective φ-EOM is a more involved matter. The problem is there are three relevant time scales H, Γ_{χ_j} and $\dot{\varphi}/\varphi$, where for the slow-roll motion of interest, we seek solutions with $\dot{\varphi}/\varphi < H$. Moreover, ultimately we require the evolution equation in cosmic time, and the relation between that and conformal time is in general very non-linear. For example, for the case of prime interest, de Sitter space, $t \propto \ln(1 - bt_c)$. Thus power law ambiguities can have non-trivial relevance in relating between conformal and cosmic time, and such ambiguities are prevalent in adiabatic approximations and derivative expansions. This is a serious matter and to learn more about this mechanism in expanding spacetime beyond what already has been understood from the above Minkowski spacetime calculation requires application of more complete non-equilibrium methods, such as [18]. We will consider the details of this derivation in the FRW spacetime in a future work.

6. Conclusions

The relevance of the analysis in this Letter extends beyond warm inflation, since the interactions studied here are exactly the same as found in supercooled inflation models. In fact, in the context of the model studied here, with couplings around the ones studied in the example of Section 4, reheating becomes irrelevant, since our analysis showed the model is inconsistent with supercooling in the first stage, and the entire dynamics is warm throughout. Thus, as originally suggested [2,15], warm inflation dynamics is inherently intertwined with the general problem of inflationary dynamics.

Since the first principle results in this Letter give support to the warm inflation picture, it is worth recalling here other features that also have made this picture compelling. First, warm inflation overcomes a conceptual barrier that the supercooled picture has never shaken away, which is that in warm inflation there is no quantum-classical transition problem, since the macroscopic dynamics of the background field and fluctuations [15] are classical from the onset. Second, in warm inflation models, in regimes relevant to observation, the mass of the inflaton field is typically much larger than the Hubble scale, thus these models do not suffer from what is sometimes called the "eta problem". Finally, accounting for dissipative effects may be important in alleviating the initial condition problem of inflation [19,20].

The emerging picture is that warm inflation remains a hopeful direction toward a complete and consistent dynamical description of the early universe. However, considerable work remains in understanding the quantum field theory of this picture. Two areas were already identified in the paper. One is resolving the gaps in the canonical dissipative formalism of [12], thus permitting this approach to be a viable cross-check to the Lagrangian approach. The other area is a full investigation of the dissipative formalism in expanding spacetime. Beyond this, the more difficult problem is extending the adiabatic contraints in the present formalisms to treat nonequilibrium conditions. Steps along this direction already have begun, using operator methods [9] and the even more ambitious attempt in [8] to derive the Boltzmann-like kinetic equation for interacting fields.

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References

[1] K.A. Olive, Phys. Rep. 190 (1990) 307.
[2] A. Berera, Phys. Rev. Lett. 75 (1995) 3218;
A. Berera, Phys. Rev. D 54 (1996) 2519;
A. Berera, Phys. Rev. D 55 (1997) 3346.
[3] H.P. de Oliveira, R.O. Ramos, Phys. Rev. D 57 (1998) 741;
W. Lee, L.-Z. Fang, Phys. Rev. D 59 (1999) 083503;
J.M.F. Maia, J.A.S. Lima, Phys. Rev. D 60 (1999) 101301;
M. Bellini, Class. Quant. Grav. 16 (1999) 2393;
A.N. Taylor, A. Berera, Phys. Rev. D 62 (2000) 083517;
H.P. De Oliveira, S.E. Joras, Phys. Rev. D 64 (2001) 063515;
L.P. Chimento, A.S. Jakubi, D. Pavn, N.A. Zuccala, Phys. Rev. D 65 (2002) 083510.
[4] A. Berera, M. Gleiser, R.O. Ramos, Phys. Rev. D 58 (1998) 123508.
[5] A. Berera, M. Gleiser, R.O. Ramos, Phys. Rev. Lett. 83 (1999) 264.
[6] J. Yokoyama, A. Linde, Phys. Rev. D 60 (1999) 083509.
[7] A. Berera, R.O. Ramos, Phys. Rev. D 63 (2001) 103509.
[8] I.D. Lawrie, Phys. Rev. D 66 (2002) 041702.
[9] G. Flores-Hidalgo, R.O. Ramos, Physica A 326 (2003) 159, hep-th/0206022.
[10] M. Gleiser, R.O. Ramos, Phys. Rev. D 50 (1994) 2441.
[11] M. Morikawa, M. Sasaki, Phys. Lett. B 165 (1985) 59;
M. Morikawa, Phys. Rev. D 33 (1986) 3607.
[12] M. Morikawa, S. Sasaki, Prog. Theor. Phys. 72 (1984) 782.
[13] A. Hosoya, M. Sakagami, Phys. Rev. D 29 (1984) 2228.
[14] A. Ringwald, Ann. Phys. (N.Y.) 177 (1987) 129.
[15] A. Berera, L.Z. Fang, Phys. Rev. Lett. 74 (1995) 1912;
A. Berera, Nucl. Phys. B 585 (2000) 666.

[16] L. Dolan, R. Jackiw, Phys. Rev. D 9 (1974) 3320.
[17] N. Banerjee, S. Mallik, Phys. Rev. D 45 (1992) 701.
[18] I.D. Lawrie, Phys. Rev. D 40 (1989) 3330.
[19] A. Berera, C. Gordon, Phys. Rev. D 63 (2001) 063505.
[20] R.O. Ramos, Phys. Rev. D 64 (2001) 123510.