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## Vacuum energy as a c-function for theories with dynamically generated masses

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## ABSTRACT

We argue that in asymptotically free non-Abelian gauge theories possessing the phenomenon of dynamical mass generation the  $\beta$  function is negative up to a value of the coupling constant that corresponds to a non-trivial fixed point, in agreement with recent AdS/QCD analysis. This fixed point happens at the minimum of the vacuum energy ( $\Omega$ ), which, as a characteristic of theories with dynamical mass generation, has the properties of a c-function.

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For unitary, renormalizable quantum field theories in two dimensions, Zamolodchikov has shown [1] that there exists a positive-definite real function of the coupling constant  $c(g)$  such that

$$-\beta \frac{\partial}{\partial g} c(g, \mu) \leq 0, \quad (1)$$

where  $\beta$  is the beta-function and  $\mu$  is the renormalization scale. This means that there exists a real function of the coupling constant that is monotonically decreasing along the renormalization group trajectories. The extension of the c-theorem to other dimensions was discussed by Cardy [2], where it was pointed all the difficulties to find such type of function in more than two dimensions. The attempts to demonstrate the existence of a c-theorem in four dimensions and its consequences led to several studies (see, for example, [3–5]), most of them following Cardy's proposal based on the Euler term in the trace of the energy–momentum tensor and relating it to the conformal anomaly coefficient.

It is particularly interesting the discussion of Ref. [3], where the renormalized free energy per unit volume,  $\mathcal{F}$ , is considered as a potential candidate for a c-function. In the high and low temperature limits the free energy can be used to characterize the number of degrees of freedom,  $f$ , of the theory in the infrared and ultraviolet regions:

$$f_{IR} \equiv - \lim_{T \rightarrow 0} \frac{\mathcal{F}}{T^4} \frac{90}{\pi^2}, \quad f_{UV} \equiv - \lim_{T \rightarrow \infty} \frac{\mathcal{F}}{T^4} \frac{90}{\pi^2}, \quad (2)$$

where  $T$  is the temperature. It was conjectured in Ref. [3] that

$$f_{IR} \leq f_{UV}, \quad (3)$$

and this inequality was able to constrain the low energy structure of supersymmetric and non-supersymmetric gauge theories [3,6]. A proof that the free energy,  $\mathcal{F}$ , plays the role of a c-function, as demonstrated in Ref. [3], fails in the case when the low energy theory is a gauge field theory governed by a free infrared fixed point.

We will argue that the existence or not of a c-function in more than two dimensions may be a property related to the gauge bosons and fermions dynamical mass generation (or breaking of the conformal symmetry), i.e. to theories where the mass generation mechanism is triggered by the non-trivial vacuum expectation value of composite operators. Therefore our discussion will make use of the vacuum energy,  $\Omega$ , defined many years ago by Cornwall and Norton [7] and detailed in Ref. [8]. Of course, the vacuum energy and the free energy are not fully distinct quantities, but there is an extensive research on the  $\Omega$  calculation as a function of the dynamical masses that will be quite useful for our purposes. We will show that  $\Omega$  is a good candidate for a c-function based on the fact that this quantity is always negative when a dynamical mass is generated, and we also verify that in this case the  $\beta$  function is negative, with the minimum of  $\Omega$  happening at a non-trivial fixed point.

The vacuum energy  $\Omega = \Omega(g, \mu)$  can be defined as

$$\Omega = V(G) - V_{pert}(G), \quad (4)$$

where we are subtracting from the effective potential for composite operators,  $V(G)$ , its perturbative counterpart, and  $V(G)$  is

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computed as a function of the nonperturbative Green functions (or complete propagators)  $G$ . The effective potential is obtained from the effective action,  $\Gamma(G)$ , when we consider translationally invariant (t.i.) field configurations,

$$V(G) \int d^4x = -\Gamma(G)|_{t.i.}$$

A very important point is that  $\Omega$  is a finite function of its arguments, because the perturbative contribution has been subtracted out [7,8]; being a physical quantity its anomalous dimension vanishes and  $\Omega$  satisfies a simple homogeneous renormalization group equation [9]

$$\left( \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \right) \Omega = 0. \quad (5)$$

Consider gauge theories with dynamically generated gauge boson masses ( $m$ ). In this case we can write  $m = \mu f(g)$  [9], from what follows that  $\mu(\partial m / \partial \mu) = m$  and, consequently,

$$m \frac{\partial \Omega}{\partial m} = -\beta(g) \frac{\partial \Omega}{\partial g}. \quad (6)$$

However, as a physical parameter in  $\epsilon$  dimensions,  $\Omega$  has a simple scaling behavior

$$\Omega(g, \mu) = \kappa m^\epsilon(g, \mu), \quad (7)$$

where  $\kappa$  is a calculable number, independent of  $g$ , consequently

$$\epsilon \Omega = -\beta(g) \frac{\partial \Omega}{\partial g}. \quad (8)$$

In the sequence we shall discuss the following points: (i)  $\partial \Omega / \partial m = 0$  is a stationary point for the vacuum energy, meaning that the minimum of energy happens at a fixed point, because  $\partial \Omega / \partial g \neq 0$  as demonstrated in Ref. [10]; (ii)  $\Omega$  is negative for theories with dynamically generated masses; therefore the right-hand side of Eq. (8) indicates that  $\Omega$  is a c-function; (iii) the  $\beta$  function is always negative when we have dynamical mass generation.

It is well known that the c-theorem holds in two dimensions; therefore it would be interesting to check if the vacuum energy of a two-dimensional theory, as the Gross–Neveu (GN) model, satisfies the necessary requirements to be considered as a c-function. The GN model contains  $N$  fermions and a four-fermion interaction with a coupling constant  $g$ , which can also be written in terms of an equivalent theory with a scalar field  $\sigma = g(\bar{\psi}\psi)$ . In the leading order of the  $1/N$  expansion, the effective potential of this model is equal to

$$V(\sigma) = \frac{1}{2}\sigma^2 + \frac{\lambda}{4\pi}\sigma^2 \left( \ln \frac{\sigma^2}{\mu^2} - 3 \right), \quad (9)$$

where  $\lambda = g^2 N$  is kept fixed,  $\mu$  is a renormalization point, and the minima of  $V$  occurs at

$$|\sigma_m| = \mu \exp(1 - \pi/\lambda).$$

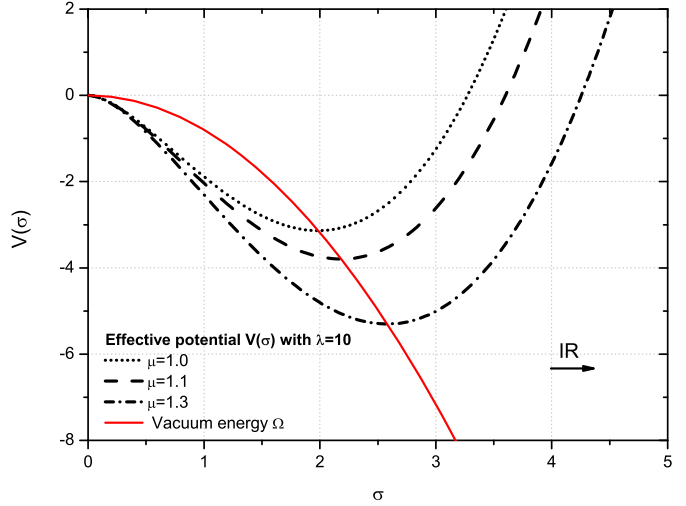
The vacuum energy is simply

$$\Omega \equiv V(\sigma_m), \quad (10)$$

and the chiral symmetry is dynamically broken generating a fermion mass

$$m \equiv m_F = g(\sigma) = \mu g \exp(1 - \pi/\lambda). \quad (11)$$

As required by condition (iii), the  $\beta$  function is negative, more specifically  $\beta(g) = -(g^3 N / 2\pi) < 0$ . In addition, substituting the expression for the vacuum energy in Eq. (6), we can easily verify that indeed  $m(\partial \Omega / \partial m) < 0$  satisfying therefore the condition



**Fig. 1.** Behavior of the vacuum energy in the case of the Gross–Neveu model. We show three curves of the effective potential as a function of the field  $\sigma$  and for different values of the renormalization point  $\mu$ .  $\Omega$  is the curve connecting all the minima and decreases towards the infrared values of the coupling constant.

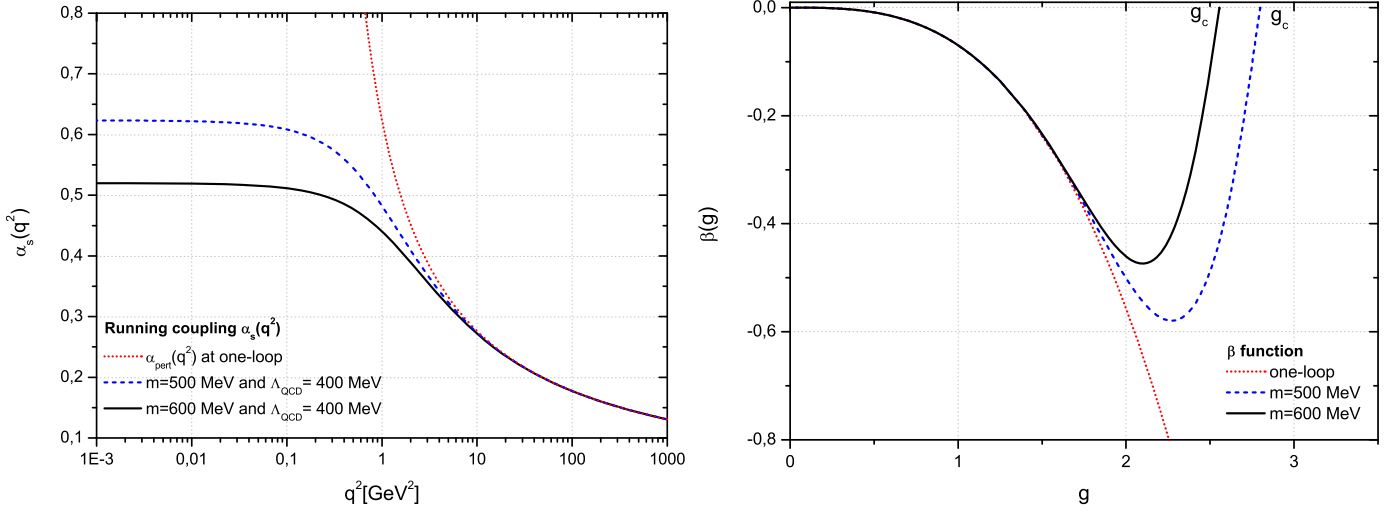
(ii). Note that the fixed point structure of the GN model (when  $\partial \Omega / \partial m \propto \beta(g) \rightarrow 0$ ) involves the knowledge of the  $\beta$  function at higher orders in the  $1/N$  expansion, when its vacuum energy depends on large  $m_F$  values, but there are evidences for such critical coupling [11].

To make clear, the  $\Omega$  behavior towards the infrared are illustrated in Fig. 1, where we show that the expected behavior of the Gross–Neveu model effective potential as the coupling constant is changing (increasing its value towards the IR). The minimum of energy, or  $\Omega$  value, monotonically decreases as we go to larger and larger fermion masses (or larger values of the coupling constant).

Now, let us come back for the case of non-Abelian theory in four dimensions. Note that we will be dealing specifically with asymptotically free  $SU(N)$  gauge theories where there is dynamical mass generation. However this does not exclude the possibility to extend the vacuum energy as a c-function for other groups as well as to the case of supersymmetric theories. In particular, all the SUSY cases treated in Ref. [3] should follow the same idea discussed here as long as the theory develops different phases or vacuum expectation values.

It was demonstrated in Ref. [10] that  $\partial \Omega / \partial g \neq 0$  when the theory has a condensate or develops a dynamically generated mass. Therefore Eq. (6) tell us that the  $\beta$  function has a zero, i.e. a non-trivial fixed point, at the minimum of the vacuum energy ( $\partial \Omega / \partial m = 0$ ). This point corresponds to the equality of Eq. (1) when  $\Omega$  plays the role of a c-function. Away from the fixed point the inequality ( $-\beta(g)[\partial \Omega / \partial g] < 0$ ) is due to the fact that a non-trivial dynamics leading to mass generation lowers the vacuum energy, i.e.  $\Omega$  is negative, as we shall discuss in detail ahead, in such a way that the left-hand side of Eq. (8) is negative, therefore  $-\beta(g)[\partial \Omega / \partial g] \leq 0$  and  $\Omega$  is a c-function. The inversion method discussed in Ref. [10] can also be used to show that  $\partial \Omega / \partial g$  is monotonic away from the fixed point.

To show that the  $\beta$  function is negative in the case of non-Abelian gauge theories with dynamical gauge boson mass generation, we may particularize the problem to the pure gauge theory. In this case the Lagrangian is given by  $\mathcal{L} = \frac{1}{2} G_{\mu\nu}^2$ . We can now rescale the fields  $A_\mu^a = g^{-1} \hat{A}_\mu^a$ ,  $G_{\mu\nu}^a = g^{-1} \hat{G}_{\mu\nu}^a$ , and regularize the vacuum energy (and the potential) setting its perturbative part equal to zero in order to obtain the generating functional ( $Z$ ) [12]



**Fig. 2.** Left panel: The IR finite QCD running coupling,  $\alpha_s(q^2)$ , given by Eq. (14) for a gluon mass of  $m = 500$  MeV (dashed blue curve) and  $m = 600$  MeV (continuous black curve) when  $\Lambda_{\text{QCD}} = 400$  MeV. The dotted red curve represents the one-loop perturbative behavior  $\alpha_{\text{pert}}(q^2)$ . Right panel: The corresponding  $\beta$  function, defined as  $\beta(g) = g(dg(q)/dq)$ , when  $m = 500$  MeV (dashed blue curve) and  $m = 600$  MeV (continuous black curve) compared to the one-loop perturbative value,  $\beta_{\text{pert}} = -bg^3$  (dotted red curve). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

$$Z = Z_p^{-1} \int d\hat{A}_\mu \exp \left[ -g^{-2} \int d^4x \frac{1}{4} \sum_a (\hat{G}_{\mu\nu}^a)^2 \right] = e^{-V\Omega}, \quad (12)$$

where  $V$  is the volume of Euclidean space–time and  $Z_p$  is the perturbative functional. Differentiating with respect to  $g$  it follows that

$$\frac{\partial \ln Z}{\partial g} = \frac{1}{2g} \int d^4x \left\langle \sum_a (\hat{G}_{\mu\nu}^a)^2 \right\rangle_{\text{reg}} = -\frac{V\partial\Omega}{\partial g}, \quad (13)$$

where  $\langle \sum_a (\hat{G}_{\mu\nu}^a)^2 \rangle$  is the gauge boson condensate and the subscript *reg* indicates that the regularization is by subtraction of the perturbative expectation value in the same way as indicated in Eq. (4). The factor  $V$  in the right-hand side is canceled with the one coming out from the  $x$  integration. It must be noted that Eq. (13) can be related to the Euclidean trace anomaly ( $\theta = -4\Omega$ ) and that  $-(\partial\Omega/\partial g)$  is a positive quantity [3,12]. Therefore, looking at Eq. (8), necessarily the  $\beta$  function is negative, as affirmed in the item (iii) above.

A central point in our discussion is the fact that  $\Omega$  is negative, or, as we discussed before, the presence of dynamically generated masses lowers the vacuum energy. It is well known that in asymptotically free non-Abelian gauge theories the vacuum has a minimum of energy when dynamical fermion masses are generated, which is supported, in the QCD case, by the chiral symmetry breaking phenomenology. Less known is the fact that such theories also generate gauge boson dynamical masses. It has already been observed through a variational calculation in four-dimensional  $SU(N)$  theories, that the vacuum energy is best minimized by a variational state characterized by a dynamically generated mass scale  $m$  [13], indicating that  $\Omega$  is indeed negative. In a different approach we can minimize the effective potential for composite operators (and consequently the vacuum energy) up to two loops, obtaining the Schwinger–Dyson equations for the gauge boson propagators. These equations can be reorganized in a gauge invariant (transversal) formulation, truncated and solved under certain approximations, resulting in a dynamically massive solution [12,14], which is in agreement with lattice simulations of  $SU(2)$  and  $SU(3)$  gauge theories in three and four dimensions [15].

A dynamical gauge boson mass displays the following asymptotic behavior

$$m(q^2) \rightarrow m(\equiv \text{const}) \quad \text{as } q \rightarrow +0,$$

and

$$m(q^2) \rightarrow 1/q^\eta \quad \text{as } q \rightarrow +\infty,$$

where  $\eta$  is some constant calculable from the SDE solution (obtained from the condition  $\partial\Omega/\partial m = 0$ ). For QCD it has been shown that  $\eta = 2$  [14–16]. The massive solution obtained in this procedure indeed minimizes the vacuum energy in the Hartree approximation [12,17]. This decreasing behavior with the momentum is typical of any dynamically generated mass and preserves unitarity.

The presence of the dynamically generated mass also modifies the IR behavior of the QCD running coupling,  $\alpha_s(q^2)$ , that can be modelled as

$$\alpha_s(q^2) = \frac{1}{4\pi b \ln[(q^2 + 4m^2(q^2))/\Lambda_{\text{QCD}}^2]}, \quad (14)$$

where  $b = 11C_A/48\pi^2$  is the first coefficient of the  $\beta$ -function,  $C_A$  is the Casimir eigenvalue of the adjoint representation ( $C_A = N$  for  $SU(N)$ ), and  $\Lambda_{\text{QCD}}$  is the characteristic QCD mass scale of a few hundred MeV.

From Eq. (14), we can easily see that  $\alpha_s(q^2)$  saturates in the deep infrared, reaching a finite value at  $q^2 = 0$ , which depends only on the ratio  $m/\Lambda_{\text{QCD}}$  [12,18]. In the left panel of Fig. 2, we show  $\alpha_s(q^2)$  for two different values of  $m$ , where we see that the higher is the ratio  $m/\Lambda_{\text{QCD}}$  the lower will be the value of  $\alpha_s(0) = g_c^2/4\pi$ . Moreover, for asymptotically large momentum, we recover the one-loop perturbative behavior  $\alpha_{\text{pert}}(q^2)$ , represented by the dotted red curve.

Notice that the appearance of  $m^2(q^2)$  not only allows for an infrared finite value for  $\alpha_s(q^2)$  but it also tames the Landau pole. As a consequence, such class of theories are not plagued by renormalon ambiguities, and calculations as the one defined by Eq. (4), where the finite result comes out by subtraction of the perturbative contributions are not ill-defined. Such theories may have a skeleton expansion, where the freezing of the running coupling constant at low energy scales could allow to capture at an inclusive level the nonperturbative effects in a reliable way as argued

in Ref. [19] (without renormalon ambiguities). The first steps to obtain such expansion are outlined in the last work in Ref. [14].

We now discuss a specific calculation of the vacuum energy for pure gauge theories. Cornwall [12] was the first one to compute  $\Omega$  in the QCD case with dynamically massive gluons, and these calculations are expected to be valid for any non-Abelian asymptotically free gauge theory. The vacuum energy,  $\Omega$ , was calculated in the Hartree approximation for pure  $SU(N)$  gauge theories, assuming hard gauge boson masses (i.e. neglecting all momentum dependence in the masses), with the divergent integral regulated as described in Ref. [12]. Note that these divergences appear only because the running of the masses is not considered (otherwise  $\Omega$  is finite), the result is

$$\Omega(m, g) = -\frac{3(N^2 - 1)}{2(2\pi)^4} \int d^4k \left[ \ln\left(\frac{k^2 + m^2}{k^2}\right) - \frac{m^2}{k^2 + m^2} \right] - \frac{3(N^2 - 1)}{4Ng^2} m^4. \quad (15)$$

It is easy to observe that  $\Omega$  is negative, which again confirms the assertion of item (ii) above. Therefore in theories with dynamically generated gauge boson masses  $\Omega$  is negative,  $\partial\Omega/\partial m = 0$  indicates the presence of a fixed point at  $g_c$  [10], the  $\beta$  function is negative, and near the IR fixed point, it approaches to zero as  $q^2/(q^2 + 4m^2)\beta_{pert}$ , where  $\beta_{pert} = -bg^3$  (see the right panel of Fig. 2). Notice that the same qualitative behavior for the  $\beta$  function is found in Ref. [20].

The introduction of fermions in the non-Abelian gauge theory does not modify our arguments. We can follow Ref. [17] to compute the fermionic vacuum energy, adding to  $\Omega$  the fermionic contribution ( $\equiv \Omega_f$ ), which, in one approximation where we neglected the running of dynamical fermion masses ( $m_f$ ), is equal to

$$\Omega_f \approx 2Nn_f \int d^4k \left[ -\ln\left(\frac{k^2 + m_f^2}{k^2}\right) + \frac{m_f^2}{k^2 + m_f^2} + \frac{m_f^4}{2k^4} \right]. \quad (16)$$

The calculation was performed for a  $SU(N)$  gauge theory with  $n_f$  fermions. We are assuming that the dynamical gauge boson mass ( $m$ ) is larger than the dynamical fermion mass ( $m_f$ ). This means that the fermionic contribution is just a small perturbation in the full vacuum, and allows the simple sum of different contributions to be reliable despite the rough approximations to compute the effective action [17]. Of course, this is justified as long as we maintain asymptotic freedom, in such a way that the instability due to the fermionic contribution is small, and also by the fact that, at least in QCD, the dynamical gauge boson masses are twice the fermionic ones (see, for instance, Refs. [12,21] where it is pointed out that  $m \approx 2\Lambda_{QCD}$  whereas we expect  $m_f \approx \Lambda_{QCD}$ ).

It can be verified that the vacuum energy that we discussed up to now, when computed at finite temperature, is equivalent to the renormalized free energy discussed in Ref. [3], as it is possible to see if we compare the free energy obtained by Freedman and McLerran [22] with the loop expansion of Refs. [8,23]. We expect that at very high temperatures all dynamical masses are erased and we end up with an almost non-interacting  $SU(N)$  theory with  $(N^2 - 1)$  gauge bosons and  $n_f$  fermions. The vacuum or free energy will just be  $kT$  factors times the number of degrees of freedom, which will be  $[2(N^2 - 1) + \frac{7}{8}(4Nn_f)]$ .

At low temperatures we shall not have exactly the vacuum energy that we calculated above, because confinement is missing from the Green's functions in the above calculation. However we know that the vacuum energy will be approximated by the same  $kT$  factors times  $[(n_f^2 - 1)]$ , which is the number of degrees of freedom of the Goldstone bosons resulting from the chiral

$(SU(n_f) \times SU(n_f))$  symmetry breaking. This is the dominant contribution to the vacuum energy, but it must be remembered that all the other excitations (fermionic as well as the ones formed by gauge bosons) also contribute to the vacuum energy at higher order. In Ref. [3] it was obtained a constraint on the number of fermions, that for QCD reads,  $n_f < 12$ ; therefore, taking into account the effects of other excitations that we discussed above, which, unfortunately, have to be computed with the help of models for low temperature QCD, since we do not know how to introduce confinement in the  $\Omega$  calculation, may lead to an even tighter constraint on the number of fermions. Notice that the vacuum energy also takes into account the thinning of degrees of freedom as we move towards the infrared values of the coupling constant, similarly to what is usually discussed in the standard renormalization group framework.

We may have other constraints on the particle spectrum without the need of comparing the extreme  $T \rightarrow 0$  and  $T \rightarrow \infty$  limits. The confinement and chiral symmetry breaking phase transitions separate different regions of the vacuum energy for asymptotically free non-Abelian gauge theories. In the QCD case it is expected that the confinement and chiral transitions for quarks in the fundamental representation are quite close, while the chiral transition for quarks in the adjoint representation would happen at a temperature different of the confinement one [24]. As the coupling constant increases with the decrease of the temperature we may have a specific phase transition happening at some critical temperature ( $T_c$ ), and we may find temperature regions above ( $T_a > T_c$ ) and below ( $T_b < T_c$ ) the critical temperature such that

$$\Omega_{T_b} \leq \Omega_{T_a}. \quad (17)$$

At zero temperature we can also expect that gauge theories with similar  $\beta$  function, i.e. with approximately the same value for the coefficients of the  $\beta$  function for the same gauge group, but fermions in different representations, for instance in the fundamental ( $f$ ) and adjoint representation ( $A$ ), will have [25]

$$\Omega^{(f)} \leq \Omega^{(A)}. \quad (18)$$

This is a consequence of the monotonic behavior of  $\Omega$  with the coupling constant and the fact that the vacuum energy (or the effective potential for composite operators that generates the gap equation) scales with the coupling constant times the Casimir operator ( $\propto g^2 C$ ).

In conclusion, we are arguing that asymptotically free non-Abelian gauge theories possessing the phenomenon of dynamical mass generation, for gauge bosons and fermions, have a fixed point at the minimum of the vacuum energy, the  $\beta$  function is negative up to the fixed point and the vacuum energy is a good candidate for a  $c$ -function. The  $\beta$  function behavior for theories with dynamically generated gauge boson and fermion masses displays the same qualitative behavior as the one found in an AdS/QCD analysis [20]. We believe that the  $c$ -theorem can be better understood as a property of theories with dynamical mass generation, and, consequently, be related in this way to the breaking of the conformal symmetry.

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## References

- [1] A.B. Zamolodchikov, JETP Lett. 43 (1986) 730.
- [2] J.L. Cardy, Phys. Lett. B 215 (1988) 749.

- [3] T. Appelquist, A.G. Cohen, M. Schmaltz, Phys. Rev. D 60 (1999) 045003.
- [4] R.D. Ball, P.H. Damgaard, Phys. Lett. B 510 (2001) 341.
- [5] K. Intriligator, Nucl. Phys. B 730 (2005) 239.
- [6] T. Appelquist, Z. Duan, F. Sannino, Phys. Rev. D 61 (2000) 125009.
- [7] J.M. Cornwall, R.E. Norton, Phys. Rev. D 8 (1973) 3338.
- [8] J.M. Cornwall, R. Jackiw, E. Tomboulis, Phys. Rev. D 10 (1974) 2428.
- [9] D.J. Gross, in: R. Balian, J. Zinn-Justin (Eds.), Methods in Field Theory, Les Houches, Session XXVIII, North-Holland Pub. Company, 1975, p. 141.
- [10] A.C. Aguilar, A.A. Natale, P.S. Rodrigues da Silva, Phys. Rev. Lett. 90 (2003) 152001.
- [11] T. Muta, Phys. Rev. D 18 (1978) 2196;  
J.F. Schonfeld, Nucl. Phys. B 95 (1975) 148;  
J.A. Gracey, Nucl. Phys. B 341 (1990) 403;  
C. Luperini, P. Rossi, Ann. Phys. (NY) 212 (1991) 371.
- [12] J.M. Cornwall, Phys. Rev. D 26 (1982) 1453.
- [13] I.I. Kogan, A. Kovner, Phys. Rev. D 52 (1995) 3719.
- [14] A.C. Aguilar, J. Papavassiliou, JHEP 0612 (2006) 012;  
A.C. Aguilar, J. Papavassiliou, Eur. Phys. J. A 35 (2008) 189;  
A.C. Aguilar, J. Papavassiliou, Phys. Rev. D 81 (2010) 034003;  
D. Binosi, J. Papavassiliou, JHEP 0811 (2008) 063;  
D. Binosi, J. Papavassiliou, Phys. Rep. 479 (2009) 1.
- [15] A.C. Aguilar, D. Binosi, J. Papavassiliou, Phys. Rev. D 78 (2008) 025010;  
A.C. Aguilar, D. Binosi, J. Papavassiliou, Phys. Rev. D 81 (2010) 125025.
- [16] M. Lavelle, Phys. Rev. D 44 (1991) 26.
- [17] E.V. Gorbar, A.A. Natale, Phys. Rev. D 61 (2000) 054012.
- [18] J.M. Cornwall, Phys. Rev. D 80 (2009) 096001;  
J.M. Cornwall, J. Papavassiliou, Phys. Rev. D 40 (1989) 3474.
- [19] S.J. Brodsky, Acta Phys. Polon. B 32 (2001) 4013;  
S.J. Brodsky, Fortschr. Phys. 50 (2002) 503;  
S.J. Brodsky, E. Gardi, G. Grunberg, J. Rathsmann, Phys. Rev. D 63 (2001) 094017.
- [20] S.J. Brodsky, G.F. de Teramond, A. Deur, Phys. Rev. D 81 (2010) 096010,  
arXiv:1002.4660 [hep-ph].
- [21] A.A. Natale, PoS QCD-TNT09 (2009) 031, arXiv:0910.5689 [hep-ph].
- [22] B.A. Freedman, L.D. McLerran, Phys. Rev. D 16 (1977) 1130.
- [23] J.M. Cornwall, R.C. Shellard, Phys. Rev. D 18 (1978) 1216.
- [24] J.M. Cornwall, Talk at the Symposium "Approaches to Quantum Chromodynamics", Oberwölz, September 2008, arXiv:0812.0395 [hep-ph].
- [25] A. Doff, A.A. Natale, Phys. Rev. D 81 (2010) 097702.