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Turbulent mixing of flue gases and hot air in combined high-rise constructions

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Abstract

An efficient numerical method for calculation of the turbulent mixing of hot air and flue gases in a vertical channel of arbitrary shape is developed in the paper. This method is used to study flows in high-rise constructions that combine a smokestack and a cooling tower. Flowfields, temperature and concentration distributions are calculated for various inlet conditions.

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Keywords: Flue gases; Turbulent mixing; Navier-Stokes equations; Finite-difference method

1. Introduction

Burning natural fuel with formation of flue gases has an adverse effect on the environment. In this case, the combustion products get mixed up with ambient air and acid rain can fall out. Heating up of the atmosphere by the high-grade products shows negative effect as well. Design of construction for natural-fuel combustion must meet certain requirements. The temperature of the flue gas must not fall below a certain limit at which condensation leads to corrosion of a smokestack. Sulfur content of the discharged gases should be no more than 200-400 mg/m^3 SO_2. The gas outlet velocity must be higher than 4 m/s to prevent downdraft. The concentrations of pollutants released into the atmosphere must be within the permissible limits. Thus, the development of pollution-free technologies for

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Nomenclature

U,V,W	axial, radial, and azimuthal velocity components in the z, r, φ directions respectively
T	temperature
E	admixture concentration
p	pressure
ρ	density
μ	dynamic viscosity
h, H	enthalpy and total enthalpy
c_p	specific heat
q	heat flux
γ	admixture mass flow-rate
σ, σ_α	Prandtl numbers
τ	friction force
g	gravity acceleration

natural-fuel combustion is a complex technical problem. One of the perspective directions to solve this problem is to build a high-rise constructions that combine a smokestack and a cooling tower. The scheme of such design is shown in Fig. 1 and the operating principle is as follows. At the base of the stack flue gas, from which the sulfur has been removed, is fed into a flow of air heated in a heat exchanger. As it moves through the stack, the gas mixes with the hot air and is carried into the atmosphere by the natural draft. This design has the following advantages. The energy consumption for re-heating of the smoke in this case is reduced. The pollutant concentrations and temperature of the gas at the stack outlet is significantly reduced, because the volume of heated air is much greater than the volume of flue gas (ratio 5:1 – 25:1). Combined tall structures provide better draft due to the mixing process in comparison with conventional smokestacks.

The efficiency of the discussed construction essentially depends on the gas-dynamic process of turbulent mixing of flue gas and hot air. In this paper, a mathematical model and an efficient numerical method for solution of the problem are presented.

2. Formulation of the problem and numerical procedure

We will consider the problem of mixing of hot gases in the axisymmetric pipe, whose lateral surface is specified in the cylindrical coordinate system r, φ, z by the equation $R(z)$. Flue gas is fed into the central part $(0 \le r \le R_1)$ of the inlet cross-section $(z = 0)$ of this pipe. Hot air flow is introduced into the peripheral part $(R_1 \le r \le R_0 = R(0))$. To investigate the gas dynamic processes of turbulent mixing of heated gases, we use the system of conservation equations for mixture mass, momentum, energy, and admixture mass in the form of boundary layer approximation

$$\begin{gathered}
\frac{\partial(r\rho U)}{\partial z} + \frac{\partial(r\rho V)}{\partial r} = 0, \\
\frac{\partial[(p+\rho U^2)r]}{\partial z} + \frac{\partial(\rho r U V)}{\partial r} = \frac{\partial}{\partial r}(r\tau) - \rho g r, \\
\frac{\partial(r\rho U H)}{\partial z} + \frac{\partial(r\rho V H)}{\partial r} = \frac{\partial(rq)}{\partial r}, \\
\frac{\partial(r\rho U E)}{\partial z} + \frac{\partial(r\rho V E)}{\partial r} = \frac{\partial(\mu r \gamma_\alpha)}{\partial r}, \\
h = c_p T, \quad H = h + \frac{U^2}{2} + gz, \quad q = \frac{1}{\sigma}\mu\frac{\partial}{\partial r}(h + 0.5\sigma U^2), \quad \gamma_\alpha = \frac{1}{\sigma_\alpha}\frac{\partial E}{\partial r}, \quad \sigma = \frac{1}{\lambda}\mu c_p, \quad \tau = \mu\frac{\partial U}{\partial r}.
\end{gathered} \tag{1}$$

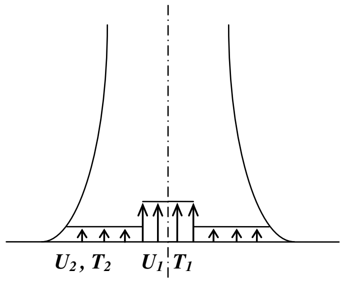

Fig. 1. Schematic of the problem

We will use Shkadov method of equal flow-rate surfaces [1]. In the cylindrical coordinate system r, φ, z we define smooth lines $r = \delta_n(z)$, $n = 0,1,2,...,N$, each of which is a streamline and satisfies the equation

$$U \frac{\partial \delta_n}{\partial z} = V \text{ for } r = \delta_n(z)\,. \tag{2}$$

The grid of lines $\delta_n(z)$ is constructed together with the solution. Obviously, $\delta_0 = 0$ is the symmetry axis and $\delta_N = R(z)$ is the pipe wall. The gas dynamic functions can be calculated on the intermediate lines

$$r = \delta_{n+1/2}(z) = 0.5(\delta_n + \delta_{n+1}),\ n = 0, 1, 2,..., N-1\,. \tag{3}$$

Each equation from the system (1) can be written in the form

$$\begin{gathered}\frac{\partial(r\rho\, UA)}{\partial z} + \frac{\partial(r\rho\, VA)}{\partial r} = \frac{\partial Q}{\partial r} - \varepsilon_A\, \omega\, r\,, \\ A = \{1, U, H, E\}\,,\ Q = \{0, r\tau, rq, r\mu\gamma_\alpha\}, \\ \varepsilon_A = 1\,,\ \omega = \frac{\partial p}{\partial z} + \rho g z \text{ for } A = U\,;\ \varepsilon_A = 1\,,\ \omega = 0 \text{ for } A = H\,;\ \varepsilon_A = 0 \text{ for } A = 1,\ A = E\,.\end{gathered} \tag{4}$$

Integrating equation (4) with respect to r from $r = \delta_n$ to $r = \delta_{n+1}$ and taking into account equation (2), we obtain

$$\frac{d}{dz}\int_{\delta_n}^{\delta_{n+1}} (r\rho UA)\, dr = -Q\Big|_{\delta_n}^{\delta_{n+1}} - \varepsilon_A\, \omega \frac{1}{2}(\delta_{n+1}^2 - \delta_n^2), \qquad \frac{d}{dz}\int_{\delta_n}^{\delta_{n+1}} (r\rho U)\, dr = 0. \tag{5}$$

Integrals are approximated by finite-difference expressions

$$\int_{\delta_n}^{\delta_{n+1}} (r\rho UA)\, dr = 0.5(\delta_{n+1}^2 - \delta_n^2)\,(\rho UA)_{n+1/2}\,. \tag{6}$$

Considering as unknown functions

$$f_{n+1/2} = 0.5(\delta_{n+1}^2 - \delta_n^2)\,,\ n = 0, 1, 2, ..., N-1\,, \tag{7}$$

we obtain expressions for $\delta_n(z)$

$$\delta_1^2 = 2 f_{1/2}, \quad \delta_2^2 = 2(f_{1/2} + f_{3/2}), \ \dots, \ \delta_N^2 = 2\sum_{n=1}^{N} f_{n-1/2} . \tag{8}$$

Taking this into account we deduce from (9) the system of ordinary differential equations on each line $r = \delta_{n+1/2}(z)$

$$\begin{aligned} U\dot{U} &= \frac{1}{\rho f} R_u - (1-\frac{1}{\gamma})\pi_T \frac{1}{\rho}\dot{p} - \pi_g , \\ U\dot{T} &= \frac{1}{\rho f} R_T - (1-\frac{1}{\gamma}) U \frac{1}{\rho}\dot{p} , \\ U\dot{E} &= \frac{1}{\rho f} R_E , \\ \frac{\dot{f}}{f} &= -\frac{\dot{p}}{p} + \frac{\dot{T}}{T} - \frac{\dot{U}}{U} . \end{aligned} \tag{9}$$

Here a dot denotes differentiation with respect to z . The system of equations (9) is written in the dimensionless form. The quantities U , T , ρ , E are scaled by their maximum values U_1, T_1, ρ_1, E_1, p_1 in the inner jet at the pipe inlet and f is scaled by R_0^2 . Dimensionless parameters in (9) $\pi_g = R_0 g / U_1^2$ is the Froude number and $\pi_T = c_p T_1 / U_1^2$ is the analog of the Mach number M: $\pi_T = (M^2(\gamma-1))^{-1}$.

Equation for the pressure is written as

$$\frac{\dot{p}}{p}\left[-\frac{1}{2\gamma} R^2 + \left(1 - \frac{1}{\gamma}\right)\pi_T p \sum_{n=0}^{N-1} \frac{f_{n+1/2}}{\rho U^2} \right] == R\dot{R} - \sum_{n=0}^{N-1}\left(\pi_g \frac{f_{n+1/2}}{U^2} + \frac{R_T}{\rho U T} - \frac{R_u}{\rho U^2} \right). \tag{10}$$

Density on each line is calculated by the formula

$$\rho(z, \delta_{n+1/2}) = \frac{p(z)}{T(z, \delta_{n+1/2})} . \tag{11}$$

Equations (9) and (10) contain the dissipative terms

$$R_u = [r\mu \frac{\partial U}{\partial r}], \quad R_E = \frac{1}{\sigma_\alpha}[r\mu \frac{\partial E}{\partial r}], \quad R_T = \frac{1}{\sigma}[r\mu \frac{\partial T}{\partial r}] + \frac{1}{\pi_T}\left([r\mu U \frac{\partial U}{\partial r}] - U[r\mu \frac{\partial U}{\partial r}] \right), \tag{12}$$

where the quantity in brackets means $[Q] = Q_{n+1} - Q_n$.

The boundary conditions on the flow axis for the unknown quantities $A = \{U, T, E\}$ of system (9) follow from the symmetry conditions. In the wall region we assume that the boundary layer is thin and the uniform flow zone extends to the wall. Therefore, we have

$$\frac{\partial A}{\partial r} = 0 \ \text{for} \ \delta = 0, \ \delta = R(z) . \tag{13}$$

The system of equations (9)–(10) must be closed by specifying a turbulence model. We will use an algebraic model based on the Prandtl mixing length l in the form

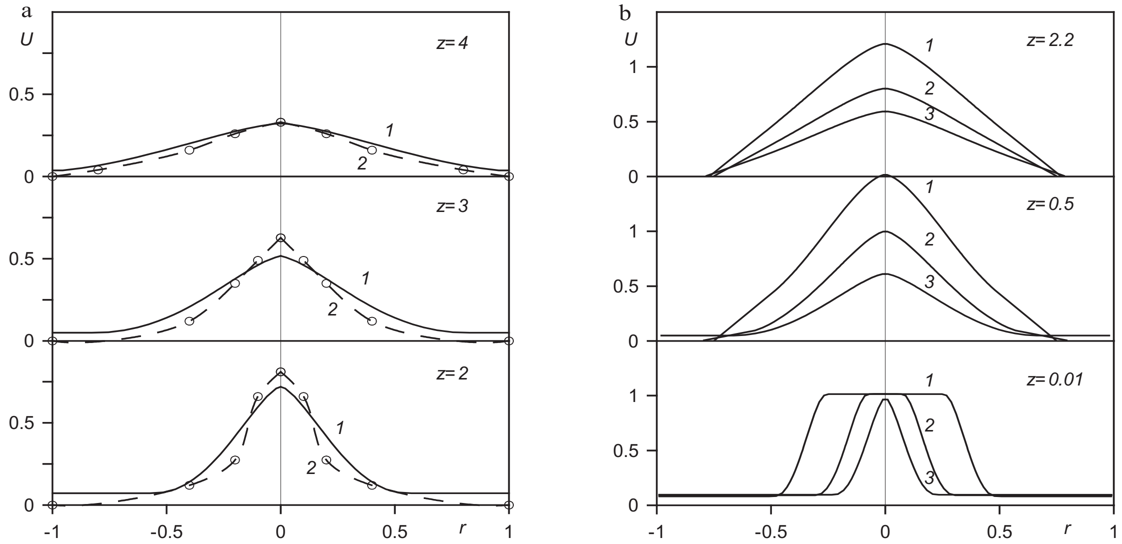

Fig. 2. Comparison of the calculated profiles (curves 1) with the data of [2] (curves 2) (a).
Velocity profiles for U_2=0.1, T_2=0.8, r_1=0.08, 0.15, 0.33 (curves 1-3) (b).

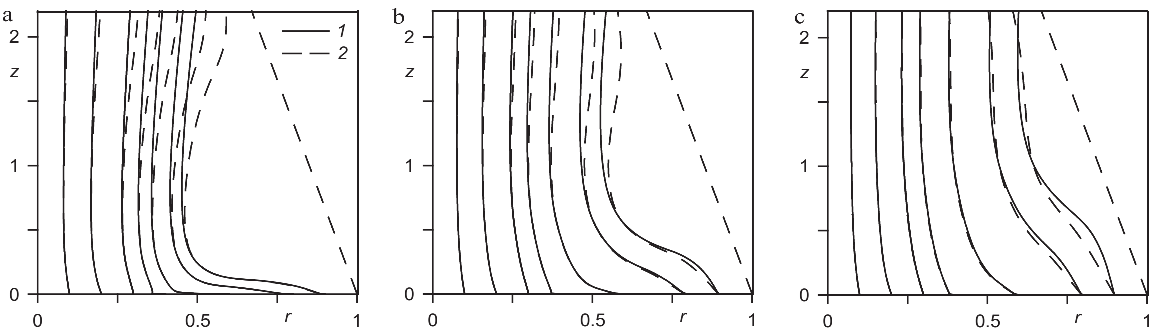

Fig. 3. Streamlines for T_2=0.8, r_1=0.33, U_2=0.1, 0.2, 0.3 (a, b, c), 1 – R(z)=1; 2 – R(z)=1–0.15z

$$\mu = \rho l^2 \left| \frac{\partial U}{\partial r} \right|, \quad l = c b_{1/2}(z)\,, \tag{14}$$

where $b_{1/2}(z)$ is half-width of mixing zone, $c = 0.23$ is an empirical constant.

The system (9)–(10) from 4N+1 differential equations is solved numerically by the Runge-Kutta method. In most calculations we used N=50.

3. Numerical results

Testing of the method has been carried out for the problem of the mixing of two uniform flows of the same gas in a cylindrical channel at a constant temperature without account for gravity. The results have been compared with the numerical solutions [2] obtained on the basis of the complete Navier-Stokes equations and the differential $k-\varepsilon$ turbulence model.

The velocity distribution over the inlet cross-section $z = 0$ is specified as follows

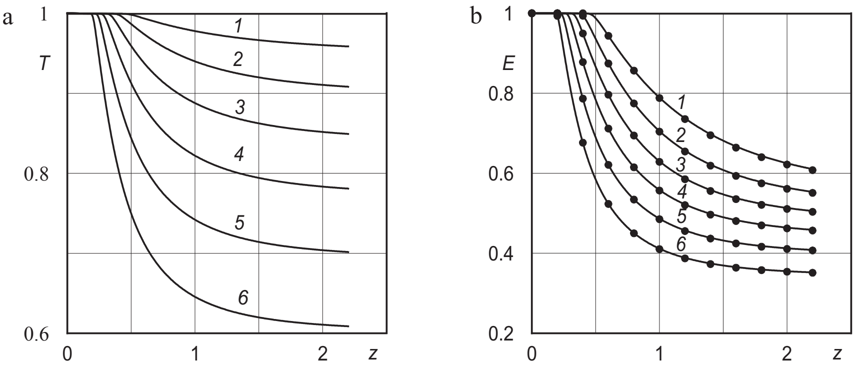

Fig. 4. Distributions of the temperature (a) and the concentration (b) over the axis r=0 for U_2=0.1, r_1=0.33, E_2=0, T_2=0.9, 0.8, 0.7, 0.6, 0.5, 0.4 (curves 1-6)

$$U = U_1 = 1, \quad 0 \le r \le r_1; \qquad U = U_2 = 0.1, \quad r_1 < r \le 1, \tag{15}$$

where $r_1 = 0.12$. The velocity profiles obtained from a numerical solution of problem (9)–(10) with conditions (15) are presented in Fig. 2a. The velocity distributions are only slightly different from the results of [2].

Typical values of the parameters for tall structures are as follows: base diameter is 90 m, height is 100 m, flow-rate of the flue gases in the inner flow is 300 m^3/s at a gas temperature of 120°C, and air flow-rate in the outer flow is 5000 m^3/s at a gas temperature of 70°C. Therefore, in the calculations the values of the dimensionless quantities are equal to $\sigma_\alpha = \sigma = 0.72$, $\pi_g = 6.45$, $\pi_T = 5754$.

Most of the calculations have been performed for the following distributions over the inlet cross-section $z = 0$

$$\begin{aligned} &U(r) = U_1 = 1, \ T(r) = T_1 = 1, \ E(r) = E_1 = 1, \quad 0 \le r \le r_1, \\ &U(r) = U_2, \quad T(r) = T_2, \quad E(r) = E_2, \quad r_1 < r \le 1, \end{aligned} \tag{16}$$

where $r_1 = R_1 / R_0$. The solution domain is determined by the length $z_0 = 2.2 R_0$ and the lateral surface is either assumed to be cylindrical $R_0 = 1$ or specified by the equation $R(z) = 1 - 0.15z$.

The calculation examples of problem (9)–(10) with conditions (16) are presented at Fig. 3-4. A graphic representation of the flow pattern inside the ventilation pipe is shown at Fig. 3. The streamlines converge fairly rapidly toward the center with increase of the distance z . This information can be used in profiling the stack walls in order to reduce the dimensions of the structure, cut costs, and make the structure more stable. In all cases, the temperature decreases with increase of z (Fig. 4a). A favorable effect is detected in the variation of the admixture concentration along the flow axis (Fig. 4b) which is almost halved at the channel outlet as compared with its initial value. For flows in a channel with an inclined lateral surface specified by the equation $R(z) = 1 - 0.15z$, the calculation results are shown by the points in Fig. 4b.

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