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$D^* D\rho$ vertex from QCD sum rules

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Abstract

We calculate the form factors and the coupling constant in the $D^* D\rho$ vertex in the framework of QCD sum rules. We evaluate the three-point correlation functions of the vertex considering D, ρ and D^* mesons off-shell. The form factors obtained are very different but give the same coupling constant: $g_{D^* D\rho} = 4.3 \pm 0.9\ \mathrm{GeV}^{-1}$.

Keywords: QCD sum rules; Charmonium in dense nuclear matter; Heavy quark theory

1. Introduction

Over the last years the strong interaction of charmed hadrons among themselves and with other species of hadrons has received increasing attention. From the discovery of charmed mesons in the seventies until the late eighties there was no motivation to study in detail the interactions of these particles. In the nineties there was a series of papers [1] trying to compute the cross section of a J/ψ with ordinary light hadrons. The motivation came from the heavy ion program running at CERN and later at RHIC. At that time J/ψ suppression was considered as a signature of quark gluon plasma (QGP) formation [2] and it was very important to know as accurately as possible the purely hadronic (non-QGP induced) charmonium suppression, which would be the background for the QGP signal. From 2000 on, while much more sophisticated

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calculations appeared [3], it became slowly clear that the physics of J/ψ is much more complex [4] than thought before and its simple suppression was no longer considered as a QGP signal and the subject lost interest. On the other hand, during those years, at the B factories the collaborations BABAR and BELLE started to produce results. One of the important decay channels of the B mesons is into J/ψ (plus other things). Moreover these collaborations found new charmonium states (the X, the Y's and the Z) [5], which also decay into J/ψ or into ψ'. It has been conjectured [6] that both B and the new charmonium states very often decay into an intermediate two body state with D's and/or D^*'s, which then undergoes final state interactions, with the exchange of one or more virtual mesons. In order to calculate the amplitudes of these processes we need to know the relevant vertices involving the charmed mesons. As an example of specific situation where a precise knowledge of the $D^*D\rho$ form factor is required, we may consider the decay $X(3872) \to J/\psi + \rho$. As suggested in [7], this decay proceeds in two steps. First the X decays into a D–D^* intermediate state and then these two particles exchange a D^* producing the final J/ψ and ρ. This is shown in Figs. 1b and 1f of [7]. In order to compute the effect of these interactions in the final decay rate we need the $D^*D\rho$ form factor. More generally, we need to know all the charm form factors to correctly calculate the interaction of J/ψ with light hadrons and the final state interactions in B decays. These form factors have been calculated in the framework of QCD sum rules (QCDSR) [8] techniques in a series of works on vertices involving charmed mesons, namely $D^*D\pi$ [9,10], $DD\rho$ [11], DDJ/ψ [12], D^*DJ/ψ [13], $D^*D^*\pi$ [14,15], D^*D^*J/ψ [16], D_sD^*K, D_s^*DK [17] , $DD\omega$ [18] and $D^*D^*\rho$ [19].

In the present paper we calculate the $D^*D\rho$ form factor with QCDSR. In the next section, for completeness we describe the QCDSR technique and in Section 3 we present the results and compare them with results obtained in other works.

2. The sum rule for the $D^*D\rho$ vertex

Following our previous works and especially Ref. [13], we write the three-point functions associated with the $D^*D\rho$ vertex, which are given by

$$\Gamma_{\mu\nu}^{(D)}(p,p') = \int d^4x\, d^4y\, e^{ip'\cdot x} e^{i(p'-p)\cdot y} \langle 0|T\{j_\mu^\rho(x) j^D(y) j_\nu^{D^*\dagger}(0)\}|0\rangle \tag{1}$$

$$\Gamma_{\mu\nu}^{(\rho)}(p,p') = \int d^4x\, d^4y\, e^{ip'\cdot x} e^{i(p'-p)\cdot y} \langle 0|T\{j^D(x) j_\mu^\rho(y) j_\nu^{D^{*\dagger}}(0)\}|0\rangle \tag{2}$$

and

$$\Gamma_{\mu\nu}^{(D^*)}(p,p') = \int d^4x\, d^4y\, e^{ip'\cdot x} e^{i(p'-p)\cdot y} \langle 0|T\{j^D(x) j_\mu^{D^*\dagger}(y) j_\nu^{\rho\dagger}(0)\}|0\rangle \tag{3}$$

These equations correspond to a D, a ρ and a D^* off-shell meson respectively. As it will be seen, the general expression for the correlators (1), (2) and (3) has only one Lorentz structure. These identities can be calculated in two different ways: using quark degrees of freedom – the theoretical or QCD side – or using hadronic degrees of freedom – the phenomenological side. In the QCD side the correlators are evaluated using the Wilson operator product expansion (OPE). The OPE incorporates the effects of the QCD vacuum through an infinite series of condensates of increasing dimension. On the other hand, the representation in terms of hadronic degrees of freedom is responsible for the introduction of the form factors, decay constants and masses. Both representations are matched invoking the quark–hadron global duality.

2.1. The phenomenological side

The $D^* D\rho$ vertex can be studied with hadronic degrees of freedom. The corresponding three-point functions, Eqs. (1), (2) and (3), are written in terms of hadron masses, decay constants and form factors. This is the so-called phenomenological side of the sum rule and it is based on the interactions at the hadronic level, which are described here by the following effective Lagrangian [20,21]

$$\mathcal{L}_{D^* D\rho} = -g_{D^* D\rho}\epsilon^{\gamma\delta\alpha\beta}\big(D\partial_\gamma \rho_\delta \partial_\alpha \bar{D}^*_\beta + \text{h.c.}\big) \tag{4}$$

from where one can extract the matrix element associated with the $D^* D\rho$ vertex. In the above expression we have $\epsilon^{0123} = +1$. Saturating Eqs. (1), (2) and (3) with the appropriate D, D^* and ρ states and making all the contractions we arrive at:

$$\Gamma^{(M)}_{\mu\nu}(p, p') = \Lambda^{(M)}_{phen}\big(p^2, p'^2, q^2\big)\epsilon_{\alpha\beta\mu\nu} p^\alpha p'^\beta + \text{h.r.} \tag{5}$$

where h.r. means higher resonances and $q = p - p'$. The invariant amplitudes $\Lambda^{(M)}$ are given by

$$\Lambda^{(D)}_{phen} = -g^{(D)}_{D^* D\rho}\big(q^2\big)\frac{C}{(m^2_{D^*} - p^2)(m^2_D - q^2)(m^2_\rho - p'^2)} \tag{6}$$

$$\Lambda^{(\rho)}_{phen} = -g^{(\rho)}_{D^* D\rho}\big(q^2\big)\frac{C}{(m^2_{D^*} - p^2)(m^2_\rho - q^2)(m^2_D - p'^2)} \tag{7}$$

and

$$\Lambda^{(D^*)}_{phen} = -g^{(D^*)}_{D^* D\rho}\big(q^2\big)\frac{C}{(m^2_\rho - p^2)(m^2_{D^*} - q^2)(m^2_D - p'^2)} \tag{8}$$

for a D, a ρ and a D^* off-shell meson respectively. In the above expressions C is a constant defined as:

$$C = \frac{m^2_D f_D}{m_c} m_\rho f_\rho m_{D^*} f_{D^*}$$

In each expression the off-shell particle has virtuality q^2. The meson decay constants appearing in the equations above are defined by the vacuum to meson transition amplitudes:

$$\langle 0|j^D|D\rangle = \frac{m^2_D f_D}{m_c} \tag{9}$$

and

$$\big\langle V(p,\epsilon)\big|j^\dagger_\alpha|0\rangle = m_V f_V \epsilon^*_\alpha \tag{10}$$

for the vector mesons $V = D^*$ and $V = \rho$. The form factor which we want to estimate is defined through the vertex function for an off-shell ρ meson:

$$\big\langle D^*(p,\lambda)\big|D\big(p'\big)\rho\big(q,\lambda'\big)\big\rangle = i g^{(\rho)}_{D^* D\rho}\big(q^2\big)\epsilon^{\alpha\beta\gamma\delta}\epsilon^\lambda_\alpha(p)\epsilon^{\lambda'}_\gamma(q)p'_\beta q_\delta \tag{11}$$

where $\epsilon^\lambda_\alpha(p)$ and $\epsilon^{\lambda'}_\gamma(q)$ are the polarization vectors associated with the D^* and ρ respectively. Analogous expressions hold for an off-shell D and for an off-shell D^* mesons. As it will be seen in the next subsection, the contribution of higher resonances and continuum in Eq. (5) will be transferred to the OPE side.

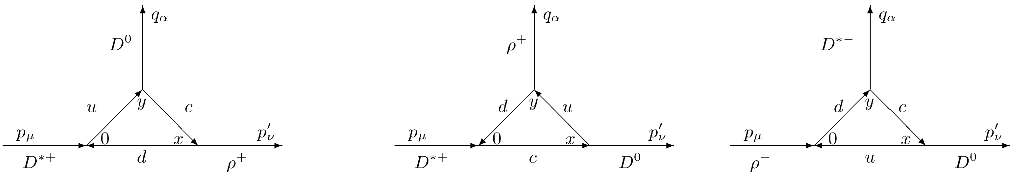

Fig. 1. Perturbative diagrams for the D off-shell (left), ρ off-shell (center) and D^* off-shell (right) correlators.

2.2. The OPE side

In the OPE or theoretical side each meson interpolating field appearing in Eqs. (1), (2) and (3) is written in terms of the quark field operators in the following form:

$$j^\rho_\mu(x) = \bar{d}(x)\gamma_\mu u(x) \tag{12}$$

$$j^D(y) = i\bar{u}(y)\gamma_5 c(y) \tag{13}$$

and

$$j^{D^*}_\nu(0) = \bar{d}(0)\gamma_\nu c(0) \tag{14}$$

where u, d and c are the up, down and charm quark field respectively. Each one of these currents has the same quantum numbers of the associated meson. The correlators (1), (2) and (3) receive contributions from all terms in the OPE. The first (and dominant) of these contributions comes from the perturbative term and it is represented in Fig. 1. Here we will consider the perturbative diagram and the quark condensate. We can write $\Gamma_{\mu\nu}$ in terms of the invariant amplitude:

$$\Gamma^{(M)}_{\mu\nu}(p, p') = \Lambda^{(M)}_{\mathrm{OPE}}(p^2, p'^2, q^2)\epsilon_{\alpha\beta\mu\nu}p^\alpha p'^\beta \tag{15}$$

where the meson M ($= D, \rho, D^*$) is off-shell. We can write a double dispersion relation for Λ, over the virtualities p^2 and p'^2 holding q^2 fixed:

$$\Lambda^{(M)}_{\mathrm{OPE}}(p^2, p'^2, q^2) = -\frac{1}{4\pi^2}\int ds \int du \, \frac{\rho^{(M)}(s, u, t)}{(s - p^2)(u - p'^2)} + \Lambda^{(M)}_{\langle\bar{q}q\rangle} \tag{16}$$

where $t = q^2$ and $\rho^{(M)}(s, u, t)$ is the double discontinuity of the amplitude $\Lambda^{(M)}(p^2, p'^2, q^2)$ when the meson M ($= D, \rho, D^*$) is off-shell. The perturbative contribution to the double discontinuity in (16) for an off-shell D meson is given by:

$$\rho^{(D)}(u, s, t) = \frac{3m_c}{\sqrt{\lambda}}\left[\frac{u(2m_c^2 - s - t + u)}{\lambda}\right] \tag{17}$$

with $\lambda = (u + s - t)^2 - 4us$. The integration limits in the integrals in (16) are:

$$0 < u < \frac{m_c^2(s + t - m_c^2) - st}{m_c^2}$$

and

$$m_c^2 < s < s_0$$

Evaluating the perturbative contribution for the double discontinuity for an off-shell ρ meson we find:

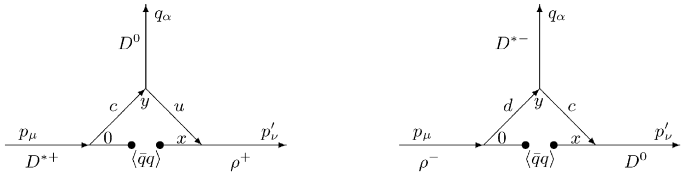

Fig. 2. Contribution of the $q\bar{q}$ condensate to the D off-shell correlator (left) and to the D^* off-shell correlator (right).

$$\rho^{(\rho)}(u,s,t) = \frac{3m_c t}{\lambda^{3/2}}\left[u + s - t - 2m_c^2\right] \tag{18}$$

and the corresponding integration limits in (16) are:

$$\frac{m_c^2(s - t - m_c^2)}{s - m_c^2} < u < u_0$$

and

$$m_c^2 < s < s_0$$

Finally, evaluating the perturbative contribution for the double discontinuity for an off-shell D^* meson we find:

$$\rho^{(D^*)}(u,s,t) = \frac{3m_c}{\lambda^{3/2}}\left[s\left(2m_c^2 + s - t - u\right)\right] \tag{19}$$

The integration limits in (16) are:

$$t < u < \frac{m_c^2(t - s - m_c^2)}{t - m_c^2}$$

and

$$0 < s < s_0$$

As usual, we have already transferred the continuum contribution from the hadronic side to the QCD side, through the introduction of the continuum thresholds s_0 and u_0 [22]. In doing so we made the assumption that at very large values of s and u the double discontinuity appearing in the phenomenological side coincides with that of the OPE side. This assumption is often called quark–hadron duality.

In order to improve the matching between the two sides of the sum rules we perform a double Borel transformation [22] in the variables $P^2 = -p^2 \to M^2$ and $P'^2 = -p'^2 \to M'^2$, on the invariant amplitude Λ_{OPE} and also on Λ_{phen} (which become $\bar{\Lambda}_{\mathrm{OPE}}$ and $\bar{\Lambda}_{phen}$ respectively). Incidentally, this double Borel transform will kill the contribution of the quark condensate $\bar{\Lambda}^{(\rho)}_{\langle\bar{q}q\rangle}$ leaving only $\bar{\Lambda}^{(D)}_{\langle\bar{q}q\rangle}$ and $\bar{\Lambda}^{(D^*)}_{\langle\bar{q}q\rangle}$ which are represented in Fig. 2 and are given by:

$$\bar{\Lambda}^{(D)}_{\langle\bar{q}q\rangle} = -\langle\bar{q}q\rangle e^{-m_c^2/M^2} \tag{20}$$

and

$$\bar{\Lambda}^{(D^*)}_{\langle\bar{q}q\rangle} = \langle\bar{q}q\rangle e^{-m_c^2/M'^2} \tag{21}$$

where $\langle\bar{q}q\rangle$ is the light quark condensate.

Table 1
Parameters used in the calculation with their errors.

m_c (GeV)	m_{D*} (GeV)	m_D (GeV)	m_ρ (GeV)	f_{D*} (GeV)	f_D (GeV)	f_ρ (GeV)	$\langle\bar{q}q\rangle$ (GeV)^3
1.27 ± 0.1	2.01	1.86	0.775	0.24 ± 0.02	0.18 ± 0.02	0.16 ± 0.005	$(-0.23 \pm 0.01)^3$

2.3. The sum rule

After performing the Borel transformation on both invariant amplitudes $\Lambda^{(M)}_{\mathrm{OPE}}$ and $\Lambda^{(M)}_{phen}$ we identify (16) with (6), with (7) and then with (8). In doing so we obtain three equations (the sum rules) for the form factors $g^{(D)}_{D^*D\rho}(q^2)$, $g^{(\rho)}_{D^*D\rho}(q^2)$ and $g^{(D^*)}_{D^*D\rho}(q^2)$ respectively. We get the following sum rules:

$$C\,\frac{g^{(\rho)}_{D^*D\rho}(q^2)}{(q^2-m_\rho^2)}e^{-\frac{m_D^2}{M'^2}}e^{-\frac{m_{D^*}^2}{M^2}} = \frac{1}{4\pi^2}\int ds\int du\,\rho^{(\rho)}(u,s,t)e^{-\frac{s}{M^2}}e^{-\frac{u}{M'^2}} \tag{22}$$

$$C\,\frac{g^{(D)}_{D^*D\rho}(q^2)}{(q^2-m_D^2)}e^{-\frac{m_\rho^2}{M'^2}}e^{-\frac{m_{D^*}^2}{M^2}} = \frac{1}{4\pi^2}\int ds\int du\,\rho^{(D)}(u,s,t)e^{-\frac{s}{M^2}}e^{-\frac{u}{M'^2}} + \langle\bar{q}q\rangle e^{-m_c^2/M^2} \tag{23}$$

and

$$C\,\frac{g^{(D^*)}_{D^*D\rho}(q^2)}{(q^2-m_{D^*}^2)}e^{-\frac{m_D^2}{M'^2}}e^{-\frac{m_\rho^2}{M^2}} = \frac{1}{4\pi^2}\int ds\int du\,\rho^{(D^*)}(u,s,t)e^{-\frac{s}{M'^2}}e^{-\frac{u}{M'^2}} - \langle\bar{q}q\rangle e^{-m_c^2/M'^2} \tag{24}$$

The sum rules above refer to ρ, D and D^* off-shell meson respectively. Choosing different values of q^2 we can control their virtualities and choosing $q^2 = m_M^2$ (where $M = \rho$, D or D^*) we put them on the mass shell. From now on we shall study the form factors in terms of the Euclidean variable $Q^2 = -q^2$.

3. Numerical results and discussion

3.1. The form factors

Table 1 shows the values of the parameters used in the present calculation. We used the experimental value for f_ρ and for the meson masses [23] and took f_D and f_{D^*} from Refs. [14,24,25]. The continuum thresholds are given by $s_0 = (m_i + \Delta_s)^2$ and $u_0 = (m_o + \Delta_u)^2$, where m_i and m_o are the masses of the incoming and outgoing meson respectively.

In this work we use the following relations between the Borel masses M^2 and M'^2: $\frac{M^2}{M'^2} = \frac{m_{D^*}^2 - m_c^2}{m_\rho^2}$ for a D off-shell, $\frac{M^2}{M'^2} = \frac{m_{D^*}^2}{m_D^2}$ for a ρ off-shell and $\frac{M^2}{M'^2} = \frac{m_D^2 - m_c^2}{m_\rho^2}$ for a D^* off-shell.

Using $\Delta_s = 0.5$ GeV and $\Delta_u = 0.7$ GeV for the continuum thresholds and fixing $Q^2 = 1\ \mathrm{GeV}^2$, we found a sum rule for $g^{(D)}_{D^*D\rho}$ as a function of M^2 which is very stable with respect to M^2 in the interval $20 < M^2 < 50\ \mathrm{GeV}^2$. This can be seen in Fig. 3. In what follows we choose the value $M^2 = 30\ \mathrm{GeV}^2$ as a reference. In Fig. 4 we show the M^2 dependence of the form factor $g^{(\rho)}_{D^*D\rho}$. Here the threshold parameters were taken to be $\Delta_s = \Delta_u = 0.5$ GeV. Also

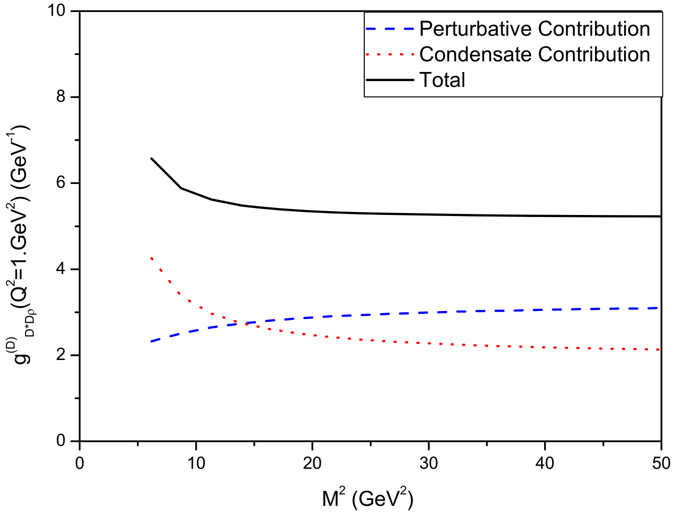

Fig. 3. $g^{(D)}_{D^*D\rho}(Q^2 = 1.0\ \text{GeV}^2)$ as a function of the Borel mass M^2.

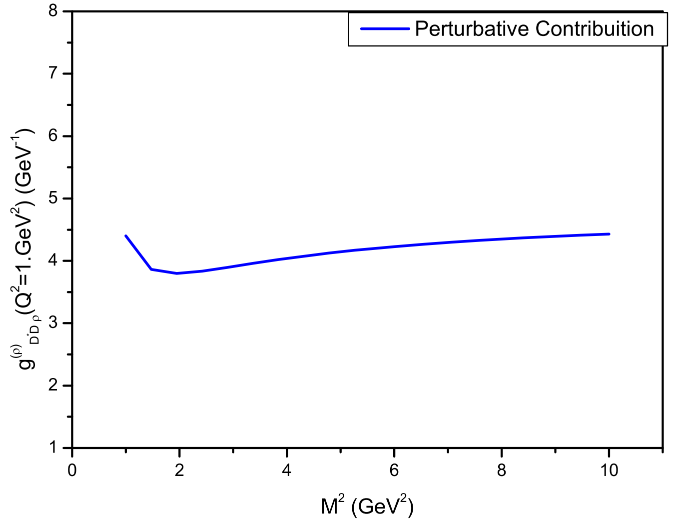

Fig. 4. $g^{(\rho)}_{D^*D\rho}(Q^2 = 1\ \text{GeV}^2)$ as a function of the Borel mass M^2.

in this case we find a good stability for a wide range of M^2 values. We have chosen the Borel mass to be $M^2 = 3\ \text{GeV}^2$. In Fig. 5 we show the M^2 dependence of the form factor $g^{(D^*)}_{D^*D\rho}$. Here the threshold parameters were taken to be $\Delta_s = 0.7$ GeV and $\Delta_u = 0.6$ GeV. Also in this case we find a good stability for a wide range of M^2 values. We have chosen the Borel mass to be $M^2 = 3\ \text{GeV}^2$.

Having determined M^2, we can calculate the Q^2 dependence of the form factors. At this point a remark is in order. At first sight, it might appear strange that we obtain reasonable results in spite of the fact that only very few terms in the OPE were taken into account. However, we must

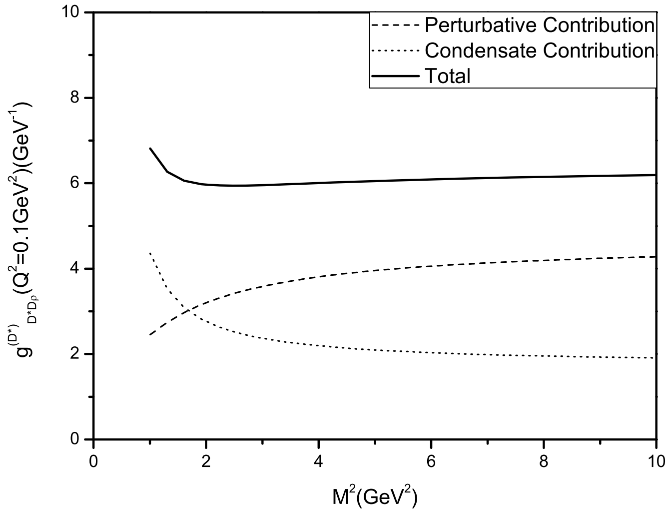

Fig. 5. $g^{(D^*)}_{D^*D\rho}$ ($Q^2 = 0.1$ GeV^2) as a function of the Borel mass M^2.

keep in mind that the OPE has a better convergence for mesons than for multiquark systems. Moreover, the convergence is better if there are heavy quark lines. When the non-perturbative corrections become less important, as in the present case, one might worry about the size of α_s corrections. In [26] these corrections were computed for the vertices $B^*B\pi$ and $D^*D\pi$. In both cases they were found to be of the order of 15% of the leading order results. Based on this previous experience, we shall assume here that the uncertainty related to α_s corrections is of the same magnitude of the other uncertainties associated, for example, with variations in the continuum thresholds or the extrapolation procedure. Hence, we leave for a future work the explicit computation of these corrections.

We present the results in Fig. 6. The triangles, squares and circles are the results for the $g^{(\rho)}_{D^*D\rho}(Q^2)$, $g^{(D)}_{D^*D\rho}(Q^2)$ and $g^{(D^*)}_{D^*D\rho}(Q^2)$ form factors respectively. In the case of an off-shell D meson, our numerical results can be fitted by the following monopolar parametrization (shown by the dashed line in Fig. 6):

$$g^{(D)}_{D^*D\rho}(Q^2) = \frac{234.4}{Q^2 + 44.1} \tag{25}$$

where the function $g^{(D)}_{D^*D\rho}(Q^2)$ has the units of GeV^{-1}, as we could anticipate from (4). Following our previous works [11–13,15,16], we define the coupling constant as the value of the form factor at $Q^2 = -m^2_M$, where m_M is the mass of the meson M. Therefore, using $Q^2 = -m^2_D$ in Eq. (25), the resulting coupling constant is $g^{(D)}_{D^*D\rho} = 5.76$ GeV^{-1}. For an off-shell ρ meson our sum rule results can be fitted by an exponential parametrization, which is represented by the solid line in Fig. 6:

$$g^{(\rho)}_{D^*D\rho}(Q^2) = 5.12e^{-Q^2/4.33} \tag{26}$$

Using $Q^2 = -m^2_\rho$ in Eq. (26) we get $g^{(\rho)}_{D^*D\rho} = 5.89$ GeV^{-1}. In the case of an off-shell D^* meson, our numerical results can be fitted by the following monopolar parametrization (shown by the dotted line in Fig. 6):

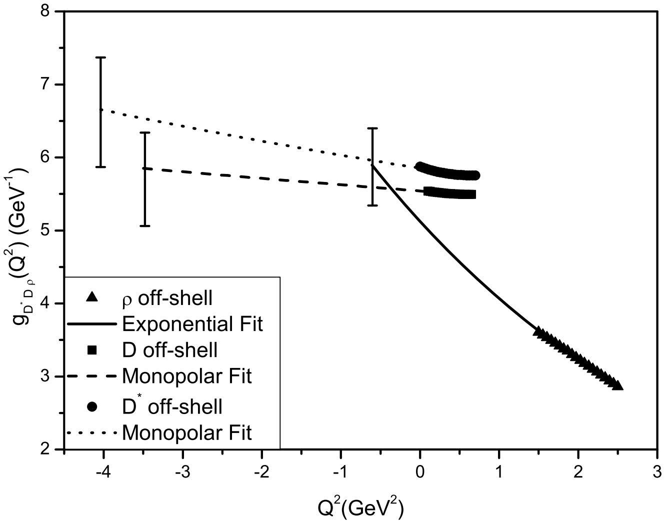

Fig. 6. $g^{(D)}_{D^*D\rho}$ (squares), $g^{(\rho)}_{D^*D\rho}$ (triangles) and $g^{(D^*)}_{D^*D\rho}$ (circles) form factors as a function of Q^2. The dotted, solid and dashed lines correspond to the parametrizations discussed in the text. The vertical bars show the theoretical errors in the coupling constants once all variations in the parameters are taken into account, as explained in the text.

$$g^{(D^*)}_{D^*D\rho}\left(Q^2\right)=\frac{195.8}{Q^2+33.5} \tag{27}$$

Evaluating this form factor at $Q^2=-m^2_{D^*}$ we find the coupling $g^{(D^*)}_{D^*D\rho}=6.65\ \text{GeV}^{-1}$.

Looking at Fig. 6 we can observe that the D off-shell form factor is much harder (i.e., the curve in the figure is much flatter) than the ρ off-shell one. This agrees with the conclusions found in most of our previous works: the heavier is the off-shell meson, the harder is its form factor. Following this same trend, we would expect the D^* off-shell form factor to be even harder than the D off-shell one. However, comparing the dashed and dotted lines in Fig. 6, this seems not to be the case: the slope of the D^* curve is slightly bigger than the one of the D curve. Since their mass difference is relatively small ($\simeq$ 150 MeV) the two curves should have almost the same slope. The observed difference is an indication of the limited precision of our method. This subject will be discussed in more detail in the next subsection.

3.2. Uncertainties

The form factors (25), (26), (27) and their extrapolations to the on-shell points leading to the coupling constants do not contain error bars. In fact, a careful and systematic study of errors in QCDSR calculations is hard to find in the literature. We took Refs. [27,28] as a guide.

In Fig. 6 we can see the theoretical error bars at the endpoints of the three curves. In what follows we describe how we obtain them.

We compute the sum rules (22), (23) and (24) extensively, taking into account the errors in the masses, decay constants, condensates, choice of the Borel mass and continuum threshold parameters. In each computation all the parameters are kept fixed, except one, which is changed according to its intrinsic error. The errors in the quark condensate, in the masses and decay constants are listed in Table 1. The three Borel masses were chosen in the interval $2.7 \leqslant M^2 \leqslant 3.3\ \text{GeV}^2$ for an off-shell ρ and an off-shell D^* and in the interval $27 \leqslant M^2 \leqslant 33\ \text{GeV}^2$ for an

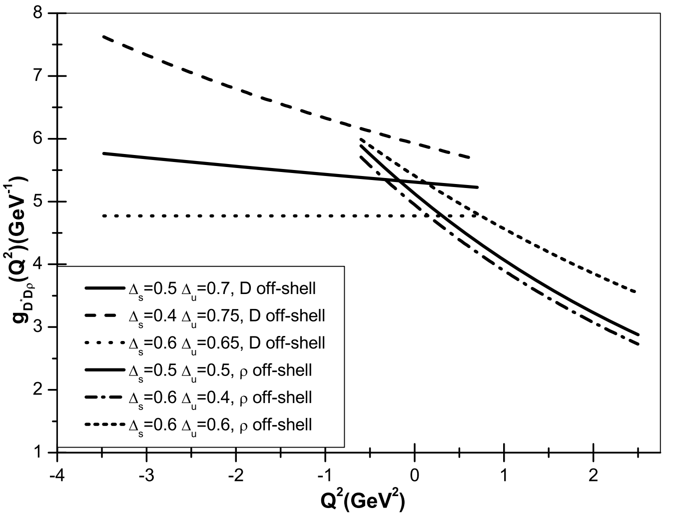

Fig. 7. Dependence of the form factors on the continuum thresholds.

off-shell D. After each round of calculation of the three sum rules, we obtain three sets of points which are then fitted and extrapolated to the respective on-shell points.

Every extrapolation introduces some ambiguity in the final results, since we have the freedom to fit a set of points with different parametrizations. In our case *this freedom is strongly reduced because we require that all the three parametrizations lead to approximately the same coupling constant*. In Fig. 6 this requirement forces the three endpoints of (25), (26) and (27), which are taken at the squared masses of the corresponding particles, to coincide, i.e., to have the same height in the figure. Of course, due to the approximations used, we cannot expect this matching to be perfect. Once this procedure is completed and we determine the three coupling constants with an error corresponding solely to the variation of one parameter, we move to the next parameter to be varied, keeping all others fixed and repeat the procedure. In each step we can have an idea of how sensitive is each coupling constant to the parameter under consideration. In the end, for each coupling constant we take the average of all encountered values and calculate also the global error, which is shown in Fig. 6 as an error bar at the on-shell point. The final number is then obtained taking the average of the three couplings found and the final error is also obtained from the errors of each coupling.

Among the sources of errors, one deserves a special discussion. Very often in QCDSR calculations, appreciable uncertainties in the results come from the lack of knowledge on the continuum threshold parameters. In order to study the dependence of our results with these parameters, we vary $\Delta_{s,u}$ between $0.4\ \text{GeV} \leqslant \Delta_{s,u} \leqslant 0.6\ \text{GeV}$ in the sum rule (22), between $0.4\ \text{GeV} \leqslant \Delta_s \leqslant 0.6\ \text{GeV}$ and $0.65\ \text{GeV} \leqslant \Delta_u \leqslant 0.75\ \text{GeV}$ in the sum rule (23) and between $0.65\ \text{GeV} \leqslant \Delta_s \leqslant 0.75\ \text{GeV}$ and $0.50\ \text{GeV} \leqslant \Delta_u \leqslant 0.70\ \text{GeV}$ in the sum rule (24). This variation produces new sets of curves which are shown in Fig. 7 and give us an uncertainty range in the resulting coupling constants $g^{(D)}_{D^*D\rho}$ and $g^{(\rho)}_{D^*D\rho}$. For the sake of clarity we did not include the lines corresponding to the coupling $g^{(D^*)}_{D^*D\rho}$. Surprisingly, in the case of the form factor $g^{(\rho)}_{D^*D\rho}$, we observe a "convergence" of the extrapolation lines, which reduces the final error. Due to this accident, the continuum threshold parameters are not, in the $g^{(\rho)}_{D^*D\rho}$ case, the ultimate source of

Table 2
Changes in $g^{(\rho)}_{D*D_\rho}$ induced by changes in different quantities.

Quantity	$\langle g^{(\rho)}_{D*D_\rho}\rangle$	σ	$\sigma\%$
Δ	5.86	0.08	1.4
f_ρ	5.89	0.01	0.1
f_D	5.41	0.65	12.0
f_{D*}	5.90	0.40	6.8
M^2	5.90	0.10	1.7
m_c	5.97	0.40	7.4

Table 3
Changes in $g^{(D)}_{D*D_\rho}$ induced by changes in different quantities.

Quantity	$\langle g^{(D)}_{D*D_\rho}\rangle$	σ	$\sigma\%$
Δ	5.95	0.87	14.7
f_ρ	5.76	0.01	0.1
f_D	5.30	0.64	12.0
f_{D*}	5.80	0.40	6.8
M^2	5.76	0.05	0.8
m_c	5.70	0.30	5.6
$\langle\bar{q}q\rangle$	5.77	0.04	0.8

Table 4
Changes in $g^{(D^*)}_{D*D_\rho}$ induced by changes in different quantities.

Quantity	$\langle g^{(D^*)}_{D*D_\rho}\rangle$	σ	$\sigma\%$
Δ	7.00	1.00	14.3
f_ρ	6.61	0.07	1.1
f_D	6.11	0.74	12.0
f_{D^*}	6.69	0.46	6.8
M^2	6.65	0.19	2.8
m_c	6.61	0.06	0.8
$\langle\bar{q}q\rangle$	6.66	0.08	1.3

error. Their contribution (denoted by Δ in Tables 2–4) is still significant, as it can be seen in Tables 2 and 3, but now they have less impact on the final error than the uncertainties in the decay constants f_D and f_{D^*} and in the charm quark mass. In the tables we show in the first column the quantity which was varied, in the second the average coupling constant resulting from that variation, in the third the standard deviation and in the fourth the percentual significance of σ.

After scanning the space of reasonable values of all the parameters, we conclude that, in spite of the inherent uncertainties, the sum rules really point to a value of the coupling constant! Of course, as in most of QCDSR calculations, the lack of precision is due to the "usual suspects", i.e., continuum threshold parameters, decay constants, heavy quark masses and condensates. A comparison of the tables shows an intriguing aspect, namely that some of the input quantities affect each of the three sum rules in a quite different way. This may be a signal that some of the sum rules are less reliable than others. A deeper investigation of this question would involve several refinements, such as the calculation of α_s corrections and higher order terms in the OPE, which go beyond the scope of this paper.

Table 5
$g_{D^*D\rho}$ in GeV^{-1} obtained in previous works.

This work	LCSR [29]	LCSR [14]	VDM [21]	$SU(4)$ [19]
4.3 ± 0.9	4.17 ± 1.04	3.56 ± 0.6	2.82 ± 0.1	3.28 ± 0.1

3.3. The coupling constant

Considering the results presented in the tables, the couplings are:

$$g^{(D)}_{D^*D\rho} = 5.71 \pm 0.62\ \mathrm{GeV}^{-1}$$
$$g^{(\rho)}_{D^*D\rho} = 5.87 \pm 0.53\ \mathrm{GeV}^{-1}$$

and

$$g^{(D^*)}_{D^*D\rho} = 6.63 \pm 0.73\ \mathrm{GeV}^{-1}$$

We can see that the three cases considered here, off-shell D, ρ and D^*, give compatible results for the coupling constant. Considering all the uncertainties and taking the average between the obtained values we have:

$$g_{D^*D\rho} = (6.1 \pm 1.3)\ \mathrm{GeV}^{-1} \tag{28}$$

Our results were obtained for a concrete choice of currents, Eqs. (12), (13) and (14), which represent charged states. Consequently the obtained couplings are for charged states and from them we can get the generic coupling appearing in the Lagrangian (4) through the relation:

$$g_{D^*D\rho} = \frac{g_{\rho^+ D^0 D^{*+}}}{\sqrt{2}} = \frac{g_{\rho^- D^0 D^{*+}}}{\sqrt{2}} \tag{29}$$

Therefore the value of the coupling constant is:

$$g_{D^*D\rho} = (6.1 \pm 1.3)/\sqrt{2} = (4.3 \pm 0.9)\ \mathrm{GeV}^{-1}$$

In Table 5 we compare this value with others discussed in previous works [14,20,21,29,30]. For us the comparison between our results and those found in Refs. [14] and [29] is especially meaningful, since both approaches use QCD sum rules, although in a different implementation. As it can be seen in Table 5, these two works arrive at somewhat different values of the coupling constant, which are, within the errors, compatible with each other. We use the standard SVZ sum rules and the authors of [14,29] work with QCD Light Cone Sum Rules (LCSR). We use the three-point function, whereas they use the two-point function with the ρ as an external field. The advantage of using the three-point function is that it allows us to treat the ρ meson as an off-shell particle and compute not only the coupling constant but also the form factor. Our results have non-perturbative corrections coming from condensates whereas in [14,29] the authors perform a twist expansion. In view of these differences it is reassuring to see that we obtain values of $g_{D^*D\rho}$ which are compatible with each other. Moreover, it seems that the precision of both methods is similar, as it can be seen from the errors, which vary from 16% to 25%.

In Ref. [21] the authors made an estimate of the $D^*D\rho$ coupling constant applying the Vector Dominance Model (VDM) to the radiative decay $D^* \to D\gamma$ and using experimental information. The obtained value is somewhat smaller than the others. We should take this estimate with caution, since it has been known since long ago [31] that the application of VDM to the charm sector is not always reliable.

Another way to estimate unknown charm coupling constants is to connect them with known couplings through $SU(4)$ relations. In the present case, we could use the relation:

$$g_{D^*D\rho} = \frac{g_{D^*D^*\rho}}{m_{D^*}} = \frac{6.6 \pm 0.3}{2.01} = (3.28 \pm 0.15)\ \text{GeV}^{-1} \tag{30}$$

This number is smaller the QCDSR results. In our previous works [12,16] we found that, in QCDSR, the $SU(4)$ relation $g_{J/\psi D^*D^*} = g_{J/\psi DD}$ is satisfied. However, from [11] and [19] we observe that other $SU(4)$ relations, such as $g_{\rho D^*D^*} = g_{\rho DD}$ and $g_{\rho D^*D^*} = \frac{\sqrt{6}}{4} g_{J/\psi D^*D^*}$ are violated at the level of 50%. This is not surprising since the mass difference starts to play an important role when we go from the heavier vector mesons to ρ.

In conclusion, we have calculated the form factors of the $D^*D\rho$ vertex and also the coupling constant. We have used QCD sum rules to explore the properties of the three-point Green function of this vertex. The form factors $g^{(D)}_{D^*D\rho}(Q^2)$ (25) and $g^{(\rho)}_{D^*D\rho}(Q^2)$ (26) were obtained for the first time and, as mentioned in the introduction, they can be used in several phenomenological applications. The coupling constant extracted from the form factors is $g_{D^*D\rho} = 4.3 \pm 0.9\ \text{GeV}^{-1}$ and it is in agreement with other QCDSR estimates.

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