



Neutrino masses in a 5D $SU(3)_W$ TeV unification model

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Abstract

We study the generation of neutrino masses in the $SU(3)_W$ electroweak unified theory in $M_4 \times S_1/(Z_2 \times Z'_2)$ spacetime. By appropriate orbifolding, the bulk symmetry $SU(3)_W$ is broken into $SU(2)_L \times U(1)_Y$ at one of the fixed points, where the quarks reside. The leptons form $SU(3)_W$ triplets, localized at the other symmetric fixed point. The fermion masses arise from the bulk Higgs sector containing a triplet and an anti-sextet. We construct neutrino Majorana masses via 1-loop quantum corrections by adding a parity odd bulk triplet scalar. No right-handed neutrino is needed. The neutrino mass matrix is of the inverted hierarchy type. We show that the model can easily accommodate the bi-large mixing angle solution favored by the recent neutrino experiments without much fine tuning of parameters. The constraints from $\mu \rightarrow 3e$ transition and neutrinoless double β decays are discussed.

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1. Introduction

Recently a five dimension (5D) field theory on the orbifold $S_1/(Z_2 \times Z'_2)$ with bulk $SU(3)_W$ gauge symmetry was proposed to unify the electroweak gauge symmetries of $SU(2)_L$ and $U(1)_Y$ [1–3]. This is a higher dimension version of an earlier proposal [4]. The background geometry of the fifth dimension denoted by y is a circle S_1 with radius R moded out by two parities and has two fixed points at $y = 0$ and $y = \pi R/2$. At each fixed point a brane is located. The brane at $y = 0$ is $SU(3)_W$ symmetric. On the other hand the one at $\pi R/2$ is not. This is achieved by orbifold boundary conditions. However, on the second brane $SU(2)_L \times U(1)_Y$ still holds and it is broken by the usual Higgs mechanism. This unified theory gives a prediction of $\sin^2 \theta_W = 0.25$ at the tree level. The discrepancy with the observed value at M_Z of $\sin^2 \theta_W(M_Z) = 0.23$ can be accounted for by the coupling constant running from the cutoff scale M^* to M_Z . Electroweak unification scale at a few TeV was found to be phenomenologically viable [1,3].

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It is well known [4] that the SM doublet and singlet right-handed chiral lepton can be embedded into a $SU(3)_W$ triplet as given below¹

$$L_i = \begin{pmatrix} e_i \\ \nu_i \\ e_i^c \end{pmatrix}_L. \quad (1)$$

On the other hand, the hypercharges of the quarks are too small for similar embedding into $SU(3)_W$ multiplets. This suggests that the leptons and quarks be located at different fixed points in y [3,5]. Thus, the leptons can be located in the $SU(3)_W$ symmetric brane at the Z_2 fixed point $y = 0$ or in the $SU(3)_W$ symmetric bulk. For definiteness we focus on the brane lepton case. Since the quarks do not form complete multiplets they can only be placed on the $SU(2) \times U(1)$ brane which is at the Z'_2 fixed point $y = \pi R/2$. This intriguing set up of leptons points to a violation of the usual additive lepton number conservation scheme. It is more akin to the forgotten Konopinski–Mahmoud [6] assignments. Coupled with recent progress in orbifold field theories, new possibilities of studying neutrino masses are now opened. With the lepton number violation, radiatively generated neutrino masses, similar to the proposal in the Zee model [7], are possible in this scenario. In extra-dimensional models, it is customary to employ one or more right-handed SM singlet bulk field to generate a small Dirac neutrino masses [8]. We shall demonstrate here that using only the minimal number of chiral fermions contained in the SM and appropriate orbifolding, phenomenologically viable neutrino mass model can be constructed. Since no right-handed neutrinos are introduced, our construction is fundamentally different from the see-saw mechanism.

2. The 5D $SU(3)_W$ electroweak model

We reiterate that the model we study has only the minimal SM chiral matter fields and bulk $SU(3)_W$ gauge symmetry. However, the Higgs fields are drastically different and will be discussed in detail later. The extra-dimensional space is flat with orbifold compactification of $S_1/(Z_2 \times Z'_2)$. This means that the fifth dimension is the compactified space S_1 of range $[-\pi R, \pi R]$ moded out by two parities Z_2 and Z'_2 . Under Z_2 we have $y \leftrightarrow -y$ and $y = 0$ is clearly a fixed point. Now relabel the coordinate as $y' = y - \pi R/2$ and consider $y' \in [-\pi R/2, \pi R/2]$. The second Z'_2 is the transformation $y' \leftrightarrow -y'$. This has fixed points at $y = 0, \pi R/2$. The combination of the two Z_2 mappings is equivalent to the mapping $y \leftrightarrow y + \pi R$ which is a twist. These parities can be used to break the symmetry of the field theory by projecting out even or odd Kaluza–Klein states under Z_2 or Z'_2 [9]. This will be explicitly shown later. Having defined the geometry we now place the leptons families at $y = 0$ and the quark families are located $y = \pi R/2$.

Next, we list the bulk Higgs fields we require. First we need a triplet Higgs $\mathbf{3}$ in order to give lepton masses via Yukawa interactions. However, the resulting charged lepton masses are not realistic and an antisextet $\bar{\mathbf{6}}$ has to be employed [3]. For reasons which will be made clear later we also need a second $\mathbf{3}$. These bulk fields are represented by 3-columns ϕ_3, ϕ'_3 and a symmetric 3×3 matrix ϕ_6 and the bar is dropped for notational simplicity. The difference between $\mathbf{3}$ and $\mathbf{3}'$ is their parity assignments. We use 3×3 matrices P, P' to denote, respectively, their parities under Z_2 and Z'_2 :

$$\begin{aligned} \phi_3(y) &= P\phi_3(-y), & \phi_3(y') &= P'\phi_3(-y'), \\ \phi'_3(y) &= P\phi'_3(-y), & \phi'_3(y') &= -P'\phi_3(-y'), \\ \phi_6(y) &= P\phi_6(-y)P^{-1}, & \phi_6(y') &= -P'\phi_6(-y')P'^{-1}. \end{aligned} \quad (2)$$

¹ We use the middle Latin alphabets i, j to denote families $e_i = e, \mu, \tau$ and the early alphabets a, b, c to denote electroweak $SU(3)$ indices. In our notation the third component of weak isospin and the hypercharge is related to the Gell-Mann λ -matrices by $T_3 = -\frac{1}{2}\lambda_3, Y = -\frac{\sqrt{3}}{2}\lambda_8$ and $Q = T_3 + Y$.

P and P' are chosen to break bulk $SU(3)_W$ symmetry properly,

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad P' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (3)$$

With the above assignments, the components of the Higgs multiplets and their parities are

$$\phi_3 = \begin{pmatrix} \phi_3^{--}(++) \\ \phi_3^0(++) \\ h_3^+(+-) \end{pmatrix}, \quad \phi'_3 = \begin{pmatrix} \phi_3'^{--}(+-) \\ \phi_3'^0(+-) \\ h_3'^+(++) \end{pmatrix}, \quad (4)$$

and

$$\phi_6 = \begin{pmatrix} \phi_{\{11\}}^{++}(+-) & \phi_{\{12\}}^+(+-) & \phi_{\{13\}}^0(++) \\ \phi_{\{12\}}^+(+-) & \phi_{\{22\}}^0(+-) & \phi_{\{23\}}^-(++) \\ \phi_{\{31\}}^0(++) & \phi_{\{32\}}^-(++) & \phi_{\{33\}}^-(+-) \end{pmatrix}. \quad (5)$$

They are not used to break the $SU(3)_W$ symmetry spontaneously. Instead they play the role of generating fermion masses. The parities given above is engineered to give a reasonable mass pattern for the leptons in the lowest order. Under the assignment, only the parity positive ϕ_3^3 and $\phi_{\{13\}}^0$ could develop nonzero vacuum expectation value (VEV) and generate the charged lepton masses. This is the central ingredient in orbifold treatments of the flavor problem. To see this more clearly we need to construct the 5D Lagrangian density which is invariant under $SU(3)_W$ and the orbifold symmetry. It is given by

$$\begin{aligned} \mathcal{L}_5 = & -\frac{1}{4} G_{MN}^{(a)} G^{(a)MN} + \text{Tr}[(D_M \phi_6)^\dagger (D^M \phi_6)] + (D_M \phi_3)^\dagger (D^M \phi_3) + (D_M \phi'_3)^\dagger (D^M \phi'_3) \\ & + \delta(y) \left[\epsilon_{abc} \frac{f_{ij}^3}{\sqrt{M^*}} (\overline{L_i^a})^c L_j^b \phi_3^c + \epsilon_{abc} \frac{f_{ij}'^3}{\sqrt{M^*}} (\overline{L_i^a})^c L_j^b \phi_3'^c + \frac{f_{ij}^6}{\sqrt{M^*}} (\overline{L_i^a})^c L_j^b \phi_6^{\{ab\}} + \bar{L} \gamma^\mu D_\mu L \right] \\ & - V_0(\phi_6, \phi_3, \phi'_3) - \frac{m}{\sqrt{M^*}} \phi_3^T \phi_6 \phi_3' + \text{h.c.} + \text{quark sector}. \end{aligned} \quad (6)$$

The notations are self explanatory. The cutoff scale M^* is introduced to make the coupling constants dimensionless. In the literature, the strong coupling requirement is usually employed to fixed the ratio $M^* R \approx 100$ (see, for example, [1]). The quark sector is not relevant now and will be left out. The complicated scalar potential is gauge invariant and orbifold symmetric and will not be specified.

The 5D covariant derivatives are

$$D_M \phi_3 = (\partial_M + i g' A_M^a T^a) \phi_3, \quad (7)$$

$$D_M \phi_6 = \partial_M \phi_6 + i g' [A_M^a T^a \phi_6 + \phi_6 (A_M^a T^a)^T] \quad (8)$$

with generator $T^a = \frac{1}{2} \lambda^a$. The gauge matrix $\mathcal{A}_M \equiv A_M^a T^a$ is:

$$\mathcal{A} = \frac{1}{2} \begin{pmatrix} A^3 + \frac{1}{\sqrt{3}} A^8 & \sqrt{2} T^+ & \sqrt{2} U^+ \\ \sqrt{2} T^- & -A^3 + \frac{1}{\sqrt{3}} A^8 & \sqrt{2} V^+ \\ \sqrt{2} U^- & \sqrt{2} V^- & -\frac{2}{\sqrt{3}} A^8 \end{pmatrix}, \quad (9)$$

where

$$T^\pm = \frac{A^1 \mp i A^2}{\sqrt{2}}, \quad U^\pm = \frac{A^4 \mp i A^5}{\sqrt{2}}, \quad V^\pm = \frac{A^6 \mp i A^7}{\sqrt{2}}.$$

The parities of gauge field are assigned as:

$$\begin{aligned} \mathcal{A}_\mu(y) &= P \mathcal{A}_\mu(-y) P^{-1}, & \mathcal{A}_\mu(y') &= P' \mathcal{A}_\mu(-y') P'^{-1}, & \mu &= 0, 1, 2, 3, \\ \mathcal{A}_5(y) &= -P \mathcal{A}_5(-y) P^{-1}, & \mathcal{A}_5(y') &= -P' \mathcal{A}_5(-y') P'^{-1}. \end{aligned} \quad (10)$$

Explicitly, U, V are assigned (+, −) parities and their Kaluza–Klein decompositions are

$$\frac{2}{\sqrt{\pi R}} \sum_{n=0} A^{2n+1}(x) \cos \frac{(2n+1)y}{R}. \quad (11)$$

It can be seen that their wavefunctions vanish at the fixed point $y = (\pi R/2)$ where quarks live on. They have no zero modes and their masses are naturally heavy and of order $1/R$. The remaining entities A^3, A^8, T^\pm are endowed with even parities (+, +) and have zero modes and they decompose as

$$\frac{2}{\sqrt{\pi R}} \left[A_0/\sqrt{2} + \sum_{n=1} A^{2n}(x) \cos \frac{2ny}{R} \right]. \quad (12)$$

The zero modes are identified as the SM gauge bosons. The bulk Lagrangian still respect a restricted $SU(3)_W$ gauge symmetry with the gauge transformation parameters obeying the same boundary condition as the gauge fields. Hence, at the fixed point $y = (\pi R/2)$ the gauge symmetry $SU(3)_W$ is reduced to $SU(2)_L \times U(1)_Y$, allowing for the existence of quarks. The 4D effective Lagrangian can be obtained from Eq. (6) by integrating out y . In particular, we have the following gauge interactions

$$\begin{aligned} \mathcal{L}_g &= \frac{ig'}{\sqrt{2\pi M^* R}} \left[\bar{e}_L \gamma^\mu \left(A_\mu^3 + \frac{1}{\sqrt{3}} A_\mu^8 \right) e_L - \bar{\nu}_L \gamma^\mu \left(A_\mu^3 - \frac{1}{\sqrt{3}} A_\mu^8 \right) \nu_L \right. \\ &\quad \left. - \frac{2}{\sqrt{3}} \bar{e}_R \gamma^\mu e_R A_\mu^8 + \sqrt{2} \bar{e}_L \gamma^\mu \nu_L T_\mu^- + \text{h.c.} \right], \end{aligned} \quad (13)$$

and for the KK modes, there is a $\sqrt{2}$ enhancement factor. The 5D gauge coupling g' is now related to the $SU(2)$ gauge coupling g at low energy as $\frac{g'}{\sqrt{\pi M^* R}} = \frac{g}{\sqrt{2}}$. It is important to note that we also have the following interaction

$$\mathcal{L}_{UV} = \frac{ig'}{\sqrt{\pi M^* R}} \left[\sqrt{2} \bar{e}_L \gamma^\mu e_R^c U_{n\mu}^{-2} + \sqrt{2} \bar{\nu}_L \gamma^\mu e_R^c V_{n\mu}^{-1} + \text{h.c.} \right] \quad (14)$$

which can induce spectacular lepton number violating effects. The superscripts on U and V denotes their respective charges.

It can be seen from Eq. (4) and Eq. (5) that only the $SU(2)_L$ doublets in $\mathbf{3}$ and $\bar{\mathbf{6}}$ and the $SU(2)_L$ singlet in $\mathbf{3}'$ have zero modes. Parities and charges allow for the bulk fields $\mathbf{3}$ and $\bar{\mathbf{6}}$ to develop vacuum expectation values but not the $\mathbf{3}'$. Hence, we have

$$\langle \phi_3 \rangle = \frac{v_3^{3/2}}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \langle \phi_6 \rangle = \frac{v_6^{3/2}}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \quad (15)$$

A linear combination of the $SU(2)_L$ doublet in the $\mathbf{3}$ and the $\bar{\mathbf{6}}$ then breaks the SM gauge symmetry. Then the tree level W boson mass is given by

$$M_W^2 = \frac{g'^2}{2M^*} (v_3^3 + 2v_6^3) = \frac{g^2 \pi R (v_3^3 + 2v_6^3)}{4}. \quad (16)$$

The charged lepton mass matrix in the basis of (e, μ, τ) can be expressed as:

$$\begin{aligned} & \frac{v_3^{3/2}}{\sqrt{2M^*}} (\bar{e}_R, \bar{\mu}_R, \bar{\tau}_R) \begin{pmatrix} 0 & f_{12}^3 & f_{13}^3 \\ -f_{12}^3 & 0 & f_{23}^3 \\ -f_{13}^3 & -f_{23}^3 & 0 \end{pmatrix} \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} \\ & + \frac{v_6^{3/2}}{\sqrt{2M^*}} (\bar{e}_R, \bar{\mu}_R, \bar{\tau}_R) \begin{pmatrix} f_{11}^6 & f_{12}^6 & f_{13}^6 \\ f_{12}^6 & f_{22}^6 & f_{23}^6 \\ f_{13}^6 & f_{23}^6 & f_{33}^6 \end{pmatrix} \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} + \text{h.c.} \end{aligned} \quad (17)$$

Eq. (17) shows clearly that $\mathbf{3}$ alone gives the wrong mass pattern. The correct masses will require a detail numerical study of the Yukawa couplings f_{ij}^6 and f_{ij}^3 which is beyond our scope now. It suffices to note that a correct hierarchy for the charged lepton masses requires $(f_3 v_3 / f_6 v_6) \lesssim 0.1$. Thus, to a good approximation, the charged lepton mass matrix is dominated by the $\bar{\mathbf{6}}$:

$$\mathcal{M}_{ij} \sim f_{ij}^6 \frac{M_W}{g' \sqrt{2}} = \frac{f_{ij}^6}{\sqrt{\pi R M^*}} \frac{M_W}{g}, \quad v_6^{3/2} \sqrt{\pi R} \sim v_6^{3/2} \sqrt{\pi R} \sim v_0 = 250 \text{ GeV}. \quad (18)$$

Next, we turn our attention to neutrino masses.

3. 5D model of neutrino masses

The parities given in Eq. (4) and Eq. (5) disallow $\phi_{\{22\}}^0$ from developing a VEV and naturally forbid tree level neutrino masses. However, the model naturally generates neutrino masses via 1-loop quantum effects. The 4D effective interaction of the brane neutrinos and the bulk Higgs fields are given by the Yukawa terms of Eq. (6). It is

$$\mathcal{L}_{4Y} = \sum_n \frac{2(\sqrt{2})^{-\delta_{n,0}}}{\sqrt{\pi R M^*}} [\epsilon_{abc} f_{ij}^3 (\overline{L_i^a})^c L_j^b \phi_{3n}^c + \epsilon_{abc} f_{ij}^{\prime 3} (\overline{L_i^a})^c L_j^b \phi_{3n}^{\prime c} + f_{ij}^6 (\overline{L_i^a})^c L_j^b \phi_{6n}^{(ab)} + \text{h.c.}], \quad (19)$$

where n is the KK-number and there is a $\sqrt{2}$ factor enhancement for non-zero modes. Immediately, we notice that the extra space volume dilution factor $\sqrt{\pi R M^*}$ naturally show up to suppress the Yukawa couplings. The Feynman rules for these vertices are depicted in Fig. 1.

The next important ingredient is the $\mathbf{3} \mathbf{3}' \bar{\mathbf{6}}$ term in Eq. (6). It is this term that violates the usual additive lepton number conservation and makes the 1-loop Majorana mass possible. The effective Higgs mixing is derived to be

$$-\frac{\sqrt{2} m}{\sqrt{\pi R M^*}} \phi_{3p}^T \phi_{6q} \phi_{3r}^{\prime},$$

where indices p, q and r stand for the KK numbers which satisfy $|p \pm q \pm r| = 0$. When one of the fields develops a VEV it is replaced by $m(v_b^3 / 2M^*)^{1/2}$. These interactions induce three possible 1-loop diagrams for generating neutrino Majorana masses, see Fig. 2. The neutrino mass matrix is necessarily Majorana since only left-handed neutrinos exist in this model.

We first observe that the dominant contribution comes from Fig. 2(a) which is mediated by two Higgs zero modes. This by itself gives a neutrino mass matrix which is Zee model like [7] in its structure assuming the charged lepton mass matrix is diagonal. We can do this without loss of generality and including charged lepton rotations will only complicate the formulae without adding new insights to the physics.

Without further ado, the elements of the neutrino mass matrix calculated from this diagram is

$$(\mathcal{M})_{ij}^{(a)} = \frac{1}{16\pi^2} \frac{2m(v_3)^{3/2}}{(\pi R M^*) \sqrt{2M^*}} \sum_k \frac{m_k f_{ik}^{\prime 3} f_{jk}^6}{M_1^2 - M_2^2} \ln \frac{M_2^2}{M_1^2}, \quad (20)$$

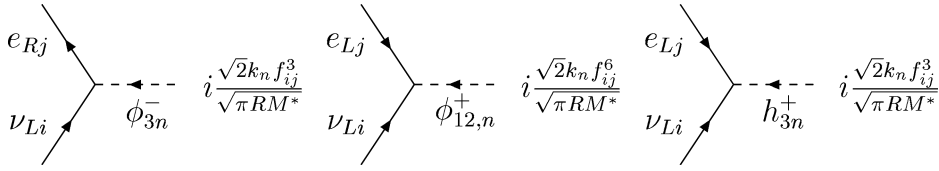


Fig. 1. The Feynman rules for the lepton Higgs couplings, i, j are the flavor indices, $f_{ij}^3 = -f_{ji}^3$, $f_{ij}^6 = f_{ji}^6$, n is the KK number and $k_n = 1$ for $n = 0$ and $k_n = \sqrt{2}$ for $n \neq 0$. For $\mathbf{3}'$, simply substitute the f_{ij}^3 by f'_{ij}^3 .

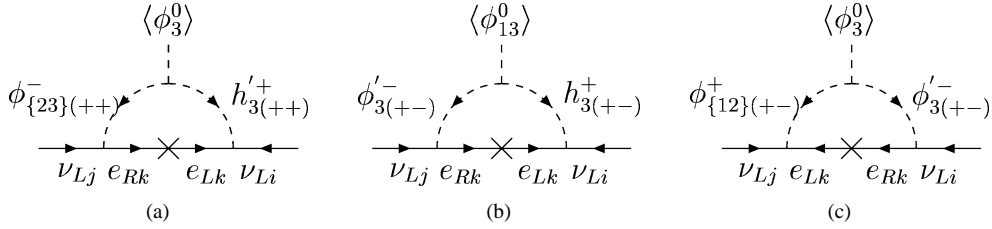


Fig. 2. The 1-loop neutrino mass through $\phi_3^T \phi_6 \phi_3'$ coupling.

where m_k is the mass of charged lepton- k and M_1, M_2 are the masses of h_3^+ and ϕ_{23}^- . Substituting the f_{ij}^6 for lepton masses we get to the first order a neutrino mass matrix that is Zee model like:

$$\mathcal{M}_\nu \sim \frac{g}{16\pi^2} \frac{v_0}{M_W} \frac{\sqrt{2}m}{(\pi R M^*)} \frac{\ln \frac{M_2^2}{M_1^2}}{(M_1^2 - M_2^2)} \begin{pmatrix} 0 & f_{12}^3(m_\mu^2 - m_e^2) & f_{13}^3(m_\tau^2 - m_e^2) \\ f_{12}^3(m_\mu^2 - m_e^2) & 0 & f_{23}^3(m_\tau^2 - m_\mu^2) \\ f_{13}^3(m_\tau^2 - m_e^2) & f_{23}^3(m_\tau^2 - m_\mu^2) & 0 \end{pmatrix}. \quad (21)$$

If the Yukawa couplings observe the following hierarchy

$$f_{12}^3 : f_{13}^3 : f_{23}^3 \sim 1 : \epsilon : \epsilon^2, \quad \epsilon = m_\mu^2 / m_\tau^2$$

then it leads to bi-maximal mixing of neutrinos [10] which is close but do not explain the recent data [11]. It can serve as the leading order approximation to a more realistic mass matrix. With the volume dilution factor, it is very natural to have a small neutrino mass. As an example, the following parameter set,

$$(\pi R M^*) \sim 100.0, \quad M_2 \sim 300 \text{ GeV}, \quad M_1 \sim 900 \text{ GeV}, \quad m \sim 250 \text{ GeV}, \quad f_{12}^3 = -0.026 \quad (22)$$

gives the mass scale for neutrinos $m_{\nu 1} \sim 0.06 \text{ eV}$. We have normalized the Yukawa coupling with M_W , so that $|f_{12}^3| = 0.026$ is not unnatural compared with $f_{\tau\tau}^6 \sim 0.04$. Also, it has nothing to do with charged lepton masses and is basically a free parameter.

The model has a natural perturbation to the Zee mass pattern. They come from the diagrams of Fig. 2(b), (c). Because they involve KK-Higgs running in the loop, these diagrams are expected to be smaller compared to Fig. 2(a). Diagram (c) gives the same structure as diagram (a) but suppressed by the KK masses, $\mathcal{M}^{(c)} \sim 2M^2 R^2 \mathcal{M}^{(a)}$, where M represents the mass of zero mode Higgs boson in diagram (a). Diagram (b), on the other hand, exhibits different structure and hence can give the perturbation needed to account for the data. The contribution from diagram (b) can be calculated from the previous calculation Eq. (20) by replacing f_6 with f_3 , substituting the zero mode masses by n th KK masses, and inserting the factor $(\sqrt{2})^2$ for the normalization of KK

modes:

$$\mathcal{M}^{(b)} \sim \frac{1}{4\pi^2} \frac{m v_0 R^2}{(\pi R M^*)^{3/2}} \times \begin{pmatrix} 2(f_{12}^3 f_{21}^3 m_\mu + f_{13}^3 f_{31}^3 m_\tau) & (f_{13}^3 f_{32}^3 + f_{13}^3 f_{32}^3) m_\tau & (f_{12}^3 f_{23}^3 + f_{12}^3 f_{23}^3) m_\mu \\ (f_{13}^3 f_{32}^3 + f_{13}^3 f_{32}^3) m_\tau & 2(f_{21}^3 f_{12}^3 m_e + f_{23}^3 f_{32}^3 m_\tau) & (f_{21}^3 f_{13}^3 + f_{21}^3 f_{13}^3) m_e \\ (f_{12}^3 f_{23}^3 + f_{12}^3 f_{23}^3) m_\mu & (f_{21}^3 f_{13}^3 + f_{21}^3 f_{13}^3) m_e & 2(f_{31}^3 f_{13}^3 m_e + f_{32}^3 f_{23}^3 m_\mu) \end{pmatrix}. \quad (23)$$

For simplicity, we only include the contribution from $n = 1$ KK states. Assuming that f_{ij}^6 is nearly diagonal the six couplings f^3, f'^3 can be adjusted to fit the neutrino oscillation data. As a first step, we find that to fit all the data, including the recent KamLAND result [13], the couplings f^3, f'^3 have a pattern. We propose the following parameter set:

$$\begin{aligned} \{f_{12}^3, f_{13}^3, f_{23}^3\} &= 0.026 \times \{-1, 0.75\epsilon, 0.5\epsilon\}, \\ \{f_{12}^3, f_{13}^3, f_{23}^3\} &= 0.090 \times \{-0.1, -0.1, 1.0\}. \end{aligned} \quad (24)$$

Here we take $1/R = 2$ TeV [1] and keep the other parameters the same as in Eq. (22). It produces the following neutrino mass matrix

$$\mathcal{M}_\nu \sim \begin{pmatrix} 0.420 & 1.0 & 0.922 \\ 1.0 & 0.097 & -0.464 \\ 0.922 & -0.464 & 0.006 \end{pmatrix} \times 0.0441 \text{ (eV)}. \quad (25)$$

This translates into $\theta_{12} = 36.6^\circ$, $\theta_{23} = 42.4^\circ$, $\sin \theta_{13} = 0.064$, for the neutrino mixing angles in standard notation, and $\Delta M_\odot = 7.3 \times 10^{-5} \text{ (eV)}^2$ and $\Delta M_{\text{atm}} = 3.4 \times 10^{-3} \text{ (eV)}^2$, for mass square differences. This pattern is close to the phenomenologically studied inverted mass hierarchy with large mixing angle solution to the solar neutrino problem given in [12].

It is interesting that the model we constructed is naturally of the inverted hierarchy kind and a mass at 0.06 eV without excessive fine tuning of parameters. It is interesting to note that the model cannot accommodate the normal hierarchy even with fine tuning.

4. Rare μ decays

The model has lepton number violating gauge interactions (see Eq. (14)) as well as Higgs interactions. The latter arise because the charged leptons get their masses from the VEV's of both the $\mathbf{3}$ and the $\bar{\mathbf{6}}$ as given in Eq. (17). Diagonalization of $\mathcal{M}_{\text{lept}}$ in general does not separately diagonalize the matrices f^3 or f^6 . If we denote the bi-unitary rotations that diagonalize $\mathcal{M}_{\text{lept}}$ by $U_{L/R}$, the interaction of neutral Higgs boson with the charged lepton mass eigenstates is

$$\left(\frac{\sqrt{2}}{\sqrt{\pi R M^*}} \right) \bar{l}'_R [(U_R^\dagger \{f_{ij}^3\} U_L) \phi_3^0 + (U_R^\dagger \{f_{ij}^6\} U_L) \phi_{(13)}^0] l'_L + \text{h.c.} \quad (26)$$

In our scenario, we assume that $v_3 \sim v_6$ and the charged lepton mass hierarchy is due to $f^3 \ll f^6$ which admittedly is fine tuning. The U rotations approximately diagonalize the f^6 matrix. Hence, the only flavor changing neutral current comes from the $\mathbf{3}$ and will be suppressed by f^3/f^6 .

Consider the rare decay $\mu \rightarrow 3e$. It can proceed through neutral Higgs exchange or the doubly charged KK gauge boson $U^{\pm 2}$ as seen in Fig. (3).

We estimate the contribution due to ϕ_3^0 to be given by

$$\frac{\text{Br}(\mu \rightarrow 3e)}{\text{Br}(\mu \rightarrow e \bar{\nu}_e \nu_\mu)} \sim 6.3 \times 10^{-17} |\xi_{\mu e}|^2 \left(\frac{f^3 M_W}{f^6 M_H} \right)^4, \quad (27)$$

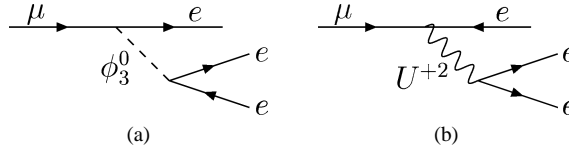


Fig. 3. The tree level diagram for $\mu \rightarrow 3e$ induced by (a) the off diagonal couplings of ϕ_3 and (b) the KK U^{+2} gauge boson.

where we have used $(\pi RM^*) = 100.0$. Without taking account of the suppression of mixing, $\xi_{\mu e}$, and smallness of Yukawa couplings this estimate is already way below the current experimental bound, $\text{Br}(\mu \rightarrow 3e) < 1 \times 10^{-12}$ [14].

The contribution of $U^{\pm 2}$ can be gleaned from Eq. (14). We note that in the limit of $f^3 = 0$ the mass matrix of charged leptons is symmetric which totally comes from the VEV of Higgs sextet. In that limit, $U_L = U_R^*$ and there is no FCNC mediated by doubly charged $U^{\pm 2}$ boson, namely,

$$(U_L^\dagger U_R^*)_{ij} = \delta_{ij}.$$

When the Yukawa coupling of Higgs triplet is turned on, we expect the off-diagonal couplings are proportional to (f^3/f^6) . Thus, we can give an order of magnitude estimate for the branching ratio

$$\frac{\text{Br}(\mu \rightarrow 3e)}{\text{Br}(\mu \rightarrow e\bar{\nu}_e\nu_\mu)} \sim \frac{|\frac{g^2}{M_U^2}(U_L^\dagger U_R^*)_{ee}(U_L^\dagger U_R^*)_{\mu e}|^2}{|\frac{g^2}{M_W^2}|^2} \sim (RM_W)^4 \left(\frac{f^3}{f^6}\right)^2 < 10^{-12}. \quad (28)$$

Thus this decay can be suppressed by either the compactification scale and/or the ratio f_3/f_6 . The compactification scale is usually determined by requiring the coupling constant running between M^* and M_Z gives the correct prediction of $\sin^2 \theta_W(M_Z)$. For non-supersymmetric version, M_c is predicted to be a few TeV [1]. There is not much room to maneuver. To stay below the experimental bound will require $f_3/f_6 \lesssim 6 \times 10^{-4}$ or a special Yukawa pattern which leads to small μ - e mixing after mass diagonalization. However, for the supersymmetric scenario, M_c could be as large as 100 TeV [5], though the exact number depends on the detail of sparticle spectrum.

5. Neutrinoless double beta decays

Neutrinoless double beta decay is an important tool in the study of neutrino masses. A recent analysis taken into account all the recent neutrino data is given in [16]. For our model there are three possible sources that can lead to the decay:

1. The first entry in the Majorana neutrino mass matrix;
2. The triple coupling of $W^- W^- \phi_{\{11\}}^{++}$;
3. The triple coupling of $W^- W^- U^{+2}$.

These are depicted in Fig. 4.

Now we can argue that only the one through neutrino mass is important. The six fermions operator responsible for the process is $(\bar{u}d)(\bar{u}d)e^c e$ and the coefficient associated with can be estimated. From the neutrino mass term we have

$$G_{(a)} \sim \frac{g^4}{M_W^4} \frac{m_\nu}{\langle p^2 \rangle}, \quad (29)$$

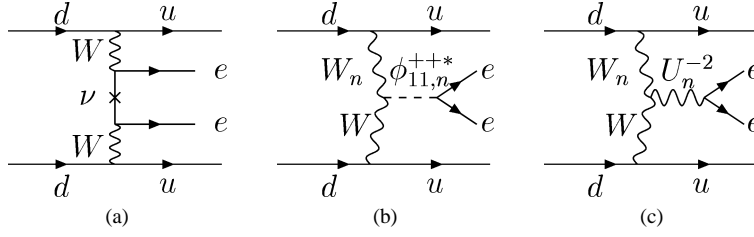


Fig. 4. The tree level diagram for $0\nu\beta\beta$ decays through (a) neutrino Majorana mass and the mediation of KK modes of (b) $\phi_{\{11\}}^{+++}$ Higgs boson and (c) U^{-2} gauge boson.

where p is the momentum transfer in this process, and similarly we have

$$G_{(b)} \sim \frac{g^2}{M_W^2} (R^2) \left(\frac{gM_W}{1/R^2} \right) \left(g \frac{m_e}{M_W} \right) = g^4 \frac{m_e R^4}{M_W^2}, \quad (30)$$

$$G_{(c)} \sim \frac{g^2}{M_W^2} (R^2) \left(\frac{g\langle p \rangle}{1/R^2} \right) (g) = g^4 \frac{\langle p \rangle R^4}{M_W^2} \quad (31)$$

for diagram (b) and (c). So their relative size compared to diagram (a) are

$$\frac{G_{(b)}}{G_{(a)}} \sim \left(\frac{m_e}{m_\nu} \right) M_W^2 \langle p^2 \rangle R^4, \quad \frac{G_{(c)}}{G_{(a)}} \sim \left(\frac{\langle p \rangle}{m_\nu} \right) M_W^2 \langle p^2 \rangle R^4. \quad (32)$$

Where $\langle p^2 \rangle$ is around the order of $10^3 - 10^4$ (MeV) 2 , $m_\nu \sim 0.06$ eV, and taking $1/R \sim 2$ TeV as example, we have the ratios:

$$\frac{G_{(b)}}{G_{(a)}} \sim 6 \times 10^{-6}, \quad \frac{G_{(c)}}{G_{(a)}} \sim 2.5 \times 10^{-3}. \quad (33)$$

Thus, the neutrinoless double beta decays rate is mainly controlled by the (11) component of neutrino mass matrix. Due to the charged lepton mass suppression, the contribution from the physical charged Higgs bosons can be ignored. The example we have given in Eq. (25) has $m_{11} \sim 2 \times 10^{-2}$ eV. This is well within the current experimental limit of [15]

$$|\langle m_\nu \rangle| = \begin{pmatrix} 0.39 & +0.45 \\ & -0.34 \end{pmatrix} \text{eV}.$$

Interestingly this may be within reach of the next generation of these experiments.

6. Conclusions

We have investigated neutrino masses in the framework of brane world scenario with $SU(3)_W$ bulk symmetry and orbifold symmetry breaking. Since the leptons are localized on a brane and form a complete triplet, the masses of the charged leptons call for the use of $\mathbf{3}$ and $\bar{\mathbf{6}}$ Higgs boson when Yukawa interactions are employed. However, the necessary fine tuning of Yukawa couplings is not explained by the model and is no better understood than in the SM. One possible way to rationalize the charged lepton mass hierarchy is to incorporate the split fermion [17] with this model which is beyond our scope now. To generate neutrino masses via orbifold mechanism we have to introduce another triplet, $\mathbf{3}'$, which is odd under the Z'_2 parity. This is a scalar field and do not develop VEV. In this way the neutrino masses arise from a 1-loop process. The overall scale is in the 0.01 eV range. Here the suppression comes from the loop integration, the bulk volume dilution and the smallness of Yukawa couplings, required since the charged lepton masses are small compared to the weak scale. No other fine tuning is required.

The mass pattern we obtained is of the inverted mass hierarchy type. At the 1-loop level the dominant structure of the neutrino mass matrix is Zee model like. However, unlike the Zee model the diagonal elements are not vanishing but only subleading. This is due to the fact that they arise from virtual KK particle exchanges. They have the effect of inducing a perturbation to the dominant structure and gives the necessary perturbation in order to account for the data [12]. The unknown parameters here are the Yukawa couplings of the $\mathbf{3}$ and $\mathbf{3}'$. Since they are not involved with charged lepton mass generation, which is assumed to be done mainly by a diagonal Yukawa couplings of the $\bar{\mathbf{6}}$, these parameters are not constrained. We found that all the current neutrino oscillation data can be easily accommodated. One possible pattern was given in Eq. (24). We digress here for the need to introduce $\mathbf{3}'$. Actually $\mathbf{3}$ could play the role of $\mathbf{3}'$ in all the loop diagrams. However, the triple scalar term $\mathbf{3} \bar{\mathbf{6}} \mathbf{3}$ is odd under $Z_2 \times Z_2'$. Even if we allow for such terms, we cannot generate a correct MNS neutrino mixing matrix [18].

Returning to our model, it predicts a small neutrinoless double decay rate which can be seen from the neutrino mass matrix. This comes from an overall scale discussed above and a small (11) element of the neutrino mass matrix. Our solution also indicated a small value for $U_{e3} = 0.064$ of the MNS matrix [18] and a small $m_{\nu e} \sim 0.06$ eV which is well below the upper bound $m_{\nu e} < 2.2$ eV from tritium beta decay [19]. In the course of this study we find no solution that accommodates the data with a normal mass hierarchy. Indeed this is a generic feature of this scenario. Although we have not achieved an understanding of charged lepton masses our modest attempt in building a model for the masses of brane neutrinos is nevertheless interesting since no right-handed neutrinos are introduced. It is an alternative to the current models of neutrino masses either in four or higher dimensions.

Viewed in the usual additive lepton number picture, this model has many sources that can lead to lepton flavor violating neutral current processes. These include KK modes of the bulk Higgs as well as doubly charged gauge bosons originating from the $SU(3)_W$ symmetry. Since unification and compactification is to take place at the TeV region we expect $\mu \rightarrow 3e$ to occur not far from the current experimental limit. There are many other phenomenologically interesting signatures in the production and decay of these particles. As an example the U^{-2} particle can be produced in a e^-e^- collider as a resonance. It then decays into $l_i l_j$ pair predominantly. The quark decay modes are absent since they are located on an $SU(2) \times U(1)$ brane. Furthermore the two W^- channel is forbidden by KK number conservation. We can also have $l^-l^- \rightarrow W^-V^-$, where the V^- behaves as an excited W^- . Many such phenomena will be reserved for future study.

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References

- [1] L.J. Hall, Y. Nomura, Phys. Lett. B 532 (2002) 111.
- [2] T. Li, W. Liao, Phys. Lett. B 545 (2002) 147.
- [3] S. Dimopoulos, D.E. Kaplan, Phys. Lett. B 531 (2002) 127;
S. Dimopoulos, D.E. Kaplan, N. Weiner, Phys. Lett. B 534 (2002) 124;
For an earlier speculation based on string theories see, I. Antoniadis, K. Benakli, Phys. Lett. B 326 (1994) 69.
- [4] S. Weinberg, Phys. Rev. D 2 (1975) 1962.
- [5] H.D. Kim, J.E. Kim, H.M. Lee, JHEP 06 (2002) 48.
- [6] E.J. Konopinski, H.M. Mahmoud, Phys. Rev. 92 (1953) 1045.
- [7] A. Zee, Phys. Lett. B 93 (1980) 389;

- A. Zee, *Phys. Lett. B* 161 (1985) 141;
D. Chang, A. Zee, *Phys. Rev. D* 61 (2000) 071303.
- [8] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, J. March-Russell, *Phys. Rev. D* 65 (2002) 024032;
K.R. Dienes, E. Dudas, T. Gherghetta, *Nucl. Phys. B* 557 (1999) 25;
G. Dvali, Yu. Smirnov, *Nucl. Phys. B* 563 (1999) 63;
R.N. Mohapatra, S. Nandi, A. Perez-Lorenzana, *Phys. Lett. B* 466 (1999) 115;
C.S. Lam, J.N. Ng, *Phys. Rev. D* 64 (2001) 113006;
K.K. Dienes, I. Sarcevic, *Phys. Lett. B* 500 (2001) 133;
A. Fraggi, M. Pospelov, *Phys. Lett. B* 458 (1999) 237;
A. Das, O. Kong, *Phys. Lett. B* 470 (1999) 149;
G. McLaughlin, J.N. Ng, *Phys. Lett. B* 470 (1999) 157;
G. McLaughlin, J.N. Ng, *Phys. Lett. B* 493 (2000) 88;
G. Bhattacharya, H.V. Klapdor-Kleingrothaus, H. Päs, A. Pilaftsis, hep-ph/0212169.
- [9] Y. Kawamura, *Prog. Theor. Phys.* 103 (2000) 613.
- [10] C. Jarlskog, M. Matsuda, S. Skadhauge, M. Tanimoto, *Phys. Lett. B* 449 (1999) 240;
P.H. Frampton, S.L. Glashow, *Phys. Lett. B* 461 (1999) 95;
D.A. Dicus, H.-J. He, J.N. Ng, *Phys. Rev. Lett.* 87 (2001) 111803.
- [11] S. Fukuda, et al., Super-Kamiokande Collaboration, *Phys. Rev. Lett.* 86 (2001) 5651;
Q.R. Ahmad, et al., SNO Collaboration, *Phys. Rev. Lett.* 89 (2002) 011301.
- [12] H.-J. He, D.A. Dicus, J.N. Ng, *Phys. Lett. B* 536 (2002) 83.
- [13] K. Eguchi, et al., KamLAND Collaboration, hep-ex/0212021.
- [14] K. Hagiwara, et al., *Phys. Rev. D* 66 (2002) 010001.
- [15] H.V. Klapdor-Kleingrothaus, et al., *Mod. Phys. Lett.* 16 (2001) 2409.
- [16] S. Pascoli, S.T. Petcov, W. Rodejohann, hep-ph/0212113.
- [17] N. Arkani-Hamed, M. Schmaltz, *Phys. Rev. D* 61 (2000) 033005;
N. Arkani-Hamed, Y. Grossman, M. Schmaltz, *Phys. Rev. D* 61 (2000) 115004;
W.-F. Chang, I.-L. Ho, J.N. Ng, *Phys. Rev. D* 66 (2002) 076004;
W.-F. Chang, J.N. Ng, *JHEP* 12 (2002) 077.
- [18] Z. Maki, M. Nakagawa, S. Sakata, *Prog. Theor. Phys.* 28 (1962) 870.
- [19] J. Bonn, et al., *Nucl. Phys. (Proc. Suppl.)* 91 (2001) 273.