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Leptogenesis, μ–τ symmetry and θ_{13}

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Abstract

We show that in theories where neutrino masses arise from type I see-saw formula with three right-handed neutrinos and where large atmospheric mixing angle owes its origin to an approximate leptonic μ–τ interchange symmetry, the primordial lepton asymmetry of the Universe, ϵ_l can be expressed in a simple form in terms of low energy neutrino oscillation parameters as $\epsilon_l = (a\Delta m^2_\odot + b\Delta m^2_A \theta^2_{13})$, where a and b are parameters characterizing high scale physics and are each of order $\leqslant 10^{-2}$ eV^{-2}. We also find that for the case of two right-handed neutrinos, $\epsilon_l \propto \theta^2_{13}$ as a result of which, the observed value of baryon to photon ratio implies a lower limit on θ_{13}. For specific choices of the CP phase δ we find θ_{13} is predicted to be between 0.10–0.15.

1. Introduction

There may be a deep connection between the origin of matter in the Universe and the observed neutrino oscillations. This speculation is inspired by the idea that the heavy right-handed Majorana neutrinos that are added to the Standard Model for understanding small neutrino masses via the see-saw mechanism [1] can also explain the origin of matter via their decay. The mechanism goes as follows [2]: CP violation in the same Yukawa interaction of the right-handed neutrinos, which go into giving nonzero neutrino masses after electroweak symmetry breaking, lead to a primordial lepton asymmetry via the out of equilibrium decay $N_R \to \ell + H$ (where ℓ are the known leptons and H is the Standard Model Higgs field). This asymmetry subsequently gets converted to baryon–antibaryon asymmetry observed today via the electroweak sphaleron interactions [3], above $T \geqslant v_{\text{wk}}$ (v_{wk} being the weak scale). Since this mechanism involves no new interactions beyond those needed in the discussion of neutrino masses, one would expect that better understanding of neutrino mass physics would clarify one of the deepest mysteries of cosmology both qualitatively as well as quantitatively. This question has been the subject of many investigations in recent years [4–11] in the context of different neutrino mass models and many interesting pieces of in-

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formation about issues such as the spectrum of right-handed neutrinos, upper limit on the neutrino masses etc have been obtained. In a recent paper, [12], two of the authors showed that if one assumes that the lepton sector of minimal see-saw models has a leptonic μ–τ interchange symmetry [14,15], then one can under certain plausible assumptions indeed predict the magnitude of the matter–antimatter asymmetry in terms of low energy oscillation parameter, $\Delta m^2_{\odot}$ and a high scale CP phase. The choice of μ–τ symmetry was dictated by the fact that it is the simplest symmetry of neutrino mass matrix that explains the maximal atmospheric mixing as indicated by data. Using present experimental value for $\Delta m^2_{\odot}$, one obtains the right magnitude for the baryon asymmetry of the Universe.

The results of the paper [12] were derived in the limit that μ–τ interchange symmetry is exact. If however a nonzero value for the neutrino mixing angle θ_{13} is detected in future experiments, this would imply that this symmetry is only approximate. Also, since in the Standard Model ν_μ and ν_τ are members of the $SU(2)_L$ doublets $L_\mu \equiv (\nu_\mu, \mu)$ and $L_\tau \equiv (\nu_\tau, \tau)$, any symmetry between ν_μ and ν_τ must be a symmetry between L_μ and L_τ at the fundamental Lagrangian level. The observed difference between the muon and tau masses would therefore also imply that the μ–τ symmetry has to be an approximate symmetry. In view of this, it is important to examine to what extent the results of Ref. [12] carry over to the case when the symmetry is approximate. We find two interesting results under some very general assumptions: (i) a simple formula relating the lepton asymmetry and neutrino oscillation observables for the case of three right-handed neutrinos, i.e., $\epsilon_l = (a\Delta m^2_{\odot} + b\Delta m^2_A \theta^2_{13})$ and (ii) a relation of the form $\epsilon_l \propto \theta^2_{13}$ for the case of two right-handed neutrinos. Measurement of θ_{13} will have important implications for both the models; in particular, we show that in a class of models with two right-handed neutrinos with approximate μ–τ symmetry breaking, there is a lower limit on θ_{13}, which is between 0.1 to 0.15 depending on the values of the CP phase. These values are in the range which will be probed in experiments in near future [16].

The basic assumption under which the two results are derived are the following:

(A) type I see-saw formula is responsible for neutrino masses;

(B) μ–τ symmetry for leptons is broken only at high scale in the mass matrix of the right-handed neutrinos.

The Letter is organized as follows: in Section 2, we outline the general framework for our discussion; in Section 3, we rederive the result of Ref. [12] for the case of exact μ–τ symmetry; in Section 4, we derive the connection between ϵ_l and oscillation parameters for the case of approximate μ–τ symmetry. Section 4 is devoted to the case of two right-handed neutrinos, where we present the allowed range of θ_{13} dictated by leptogenesis argument. In Section 5, we describe a class of simple gauge models where these conditions are satisfied.

2. Introductory remarks on lepton asymmetry in type I see-saw models

We start with an extension of the minimal supersymmetric Standard Model (MSSM) for the generic the type I see-saw model for neutrino masses. The effective low energy superpotential for this model is given by

$$W = e^{cT}\mathbf{Y}_\ell L H_d + N^{cT}\mathbf{Y}_\nu L H_u + \frac{\mathbf{M_R}}{\mathbf{2}}\mathbf{N^{cT}}\mathbf{N^c}. \tag{1}$$

Here L, e^c, ν^c are leptonic superfields; $H_{u,d}$ are the Higgs fields of MSSM. Y_ν and M_R are general matrices where we choose a basis where Y_ℓ is diagonal. We do not display the quark part of the superpotential which is same as in the MSSM. After electroweak symmetry breaking, this leads to the type I see-saw formula for neutrino masses given by

$$\mathcal{M}_\nu = -\mathbf{Y}_\nu^{\mathbf{T}}\mathbf{f}^{-\mathbf{1}}\mathbf{Y}_\nu \frac{v^2_{\mathrm{wk}}\tan^2\beta}{v_R}. \tag{2}$$

The constraints of μ–τ symmetry will manifest themselves in the form of the Y_ν and M_R. It has been pointed out that if we go to a basis where the right-handed neutrino mass matrix is diagonal, we can solve for Y_ν in terms of the neutrino masses and mixing angles as follows [17]:

$$Y_\nu v = i {M_R^d}^{1/2} R(z_{ij}) \left(\mathcal{M}^d_\nu\right)^{1/2} U^\dagger, \tag{3}$$

where R is a complex matrix with the property that $RR^T = 1$. The unitary matrix U is the lepton mixing matrix defined by

$$\mathcal{M}_\nu = U^* \mathcal{M}_\nu^d U^\dagger. \tag{4}$$

The complex orthogonal matrices R can be parameterized as:

$$R(z_{12}, z_{23}, z_{13}) = R(z_{23}) R(z_{13}) R(z_{12}) \tag{5}$$

with

$$R(z_{12}) = \begin{pmatrix} \cos z_{12} & \sin z_{12} & 0 \\ -\sin z_{12} & \cos z_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{6}$$

and similarly for the other matrices. z_{ij} are complex angles.

Let us now turn to lepton asymmetry: the formula for primordial lepton asymmetry in this case, caused by right-handed neutrino decay is

$$\epsilon_l = \frac{1}{8\pi} \sum_j \frac{\mathrm{Im}[\tilde{Y}_\nu \tilde{Y}_\nu^\dagger]_{1j}^2}{(\tilde{Y}_\nu \tilde{Y}_\nu^\dagger)_{11}} F\left(\frac{M_1}{M_j}\right), \tag{7}$$

where $\tilde{Y}_\nu$ is defined in a basis where right-handed neutrinos are mass eigenstates and their masses are denoted by $M_{1,2,3}$ where $F(x) = -\frac{1}{x}[\frac{2x^2}{x^2-1} - \ln(1+x^2)]$ [18]. In the case where that the right-handed neutrinos have a hierarchical mass pattern, i.e., $M_1 \ll M_{2,3}$, we get $F(x) \simeq -3x$. In this approximation, we can write the lepton asymmetry in a simple form [19]

$$\epsilon_l = -\frac{3}{8\pi} \frac{M_1 \, \mathrm{Im}[Y_\nu \mathcal{M}_\nu^\dagger Y_\nu^T]_{11}}{v^2 (\tilde{Y}_\nu \tilde{Y}_\nu^\dagger)_{11}}, \tag{8}$$

where using the expression for Y_ν given above, we can rewrite ϵ_l as:

$$\epsilon_l = -\frac{3}{8\pi} \frac{\mathrm{Im}[{M_R^d}^{1/2} R(z_{ij}) {\mathcal{M}^d_\nu}^2 R(z_{ij}) {M_R^d}^{1/2}]_{11}}{v^2 |R(z_{ij}) \mathcal{M}_\nu R^\dagger(z_{ij})|_{11}^2}. \tag{9}$$

We will now apply this discussion to calculate the lepton asymmetry in the general case without any symmetries. In the following sections, we follow it up with a discussion of two cases: (i) the cases of exact μ–τ symmetry and (ii) the case where this symmetry is only approximate. Since the formula in Eq. (9) assumes that there are three right-handed neutrinos, we will focus on this case in the next two sections. In a subsequent section, we consider the case of two right-handed neutrinos (N_μ, N_τ), which transform into each other under the μ–τ symmetry. Both cases are in agreement with the observed neutrino mass differences and mixings.

It follows from Eq. (9) that

$$\epsilon_l = -\frac{3M_1}{8\pi} \frac{\mathrm{Im}[m_1^2 R_{11}^2 + m_2^2 R_{12}^2 + m_3^2 R_{13}^2]}{v^2 |R(z_{ij}) \mathcal{M}_\nu R^\dagger(z_{ij})|_{11}^2}. \tag{10}$$

Since the matrix R is an orthogonal matrix, we have the relation

$$R_{11}^2 + R_{12}^2 + R_{13}^2 = 1. \tag{11}$$

Using this equation in Eq. (10), we get

$$\epsilon_l = -\frac{3M_1}{8\pi} \frac{\mathrm{Im}[\Delta m_\odot^2 R_{12}^2 + \Delta m_A^2 R_{13}^2]}{v^2 \sum_j (|R_{1j}|^2 m_j)}. \tag{12}$$

This relation connects the lepton asymmetry to both the solar and the atmospheric mass difference square [5]. To make a prediction for the lepton asymmetry, we need to the lengths of the complex quantities R_{1j}. The out of equilibrium condition does provide a constraint on $|R_{1j}|$ as follows:

$$\sum_{j=1,2,3} \left(|R_{1j}|^2 m_j\right) \leqslant 10^{-3} \text{ eV}. \tag{13}$$

Naively interpreted, this would have meant a strong constraint on the degenerate neutrino spectrum. However, as has been shown in Ref. [5] the preferred range for $\sum_{j=1,2,3}(|R_{1j}|^2 m_j)$ is from 10^{-3} to 0.1 eV with no strict upper bound, although an upper bound of on $(\sum m_i^2)^{1/2}$ of 0.1 eV can be deduced from washout processes. It is clear from Eq. (13) that if neutrinos are quasidegenerate based on this argument, we conclude that a degenerate mass spectrum with $m_0 \geqslant 0.1$ eV will most likely be in conflict with observations, if type I see-saw is responsible for neutrino masses. It must however be noted that a more appealing and natural scenario for degenerate neutrino masses is type II see-saw formula [20], in which case the above considerations do not apply. Therefore, it is not possible to conclude based on the leptogenesis argument alone that a quasi-degenerate neutrino spectrum is inconsistent.

In a hierarchical neutrino mass picture, Eq. (13) implies that $|R_{13}|^2 \leqslant 0.02$ and $|R_{12}|^2 \leqslant 0.1$. If we assume that the upper limit in the Eq. (13) is saturated, then we get the atmospheric neutrino mass difference square in Eq. (12) to give the dominant contribution.

We will see below that if one assumes an exact μ–τ symmetry for the neutrino mass matrix, the situation becomes different and it is the solar mass difference square that dominates.

3. Three right-handed neutrinos and exact μ–τ symmetry

In this section, we consider the case of three right-handed neutrino with an exact μ–τ symmetry in the Dirac mass matrix as well as the right-handed neutrino mass matrix. In this case, the right-handed neutrino mass matrix M_R and the Dirac–Yukawa coupling Y_ν can be written, respectively, as:

$$\mathbf{M_R} = \begin{pmatrix} M_{11} & M_{12} & M_{12} \\ M_{12} & M_{22} & M_{23} \\ M_{12} & M_{23} & M_{22} \end{pmatrix},$$
$$\mathbf{Y}_\nu = \begin{pmatrix} h_{11} & h_{12} & h_{12} \\ h_{21} & h_{22} & h_{23} \\ h_{21} & h_{23} & h_{22} \end{pmatrix}, \tag{14}$$

where M_{ij} and h_{ij} are all complex. An important property of these two matrices is that they can be cast into a block diagonal form by the same transformation matrix $U_{23}(\pi/4) \equiv \left(\begin{smallmatrix} 1 & 0 \\ 0 & U(\pi/4) \end{smallmatrix}\right)$ on the ν's and N's. Let us denote the block diagonal forms by a tilde, i.e., $\tilde{Y}_\nu$ and $\tilde{M}_R$. We then go to a basis where the $\tilde{M}_R$ is subsequently diagonalized by the most general 2×2 unitary matrix as follows:

$$V^T(2\times 2)U_{23}^T(\pi/4)M_R U_{23}(\pi/4)V(2\times 2) = M_R^d, \tag{15}$$

where $V(2\times 2) = \left(\begin{smallmatrix} V & 0 \\ 0 & 1 \end{smallmatrix}\right)$ where V is the most general 2×2 unitary matrix given by $V = e^{i\alpha}P(\beta)R(\theta)P(\gamma)$. The 3×3 case therefore reduces to a 2×2 problem. The third mass eigenstate in both the light and the heavy sectors play no role in the leptogenesis as well as generation of solar mixing angle [12]. Note also that we have $\theta_{13} = 0$. The see-saw formula in the 1–2 subsector has exactly the same form except that all matrices in the left- and right-hand side of Eq. (9) are 2×2 matrices. The formula for the Dirac–Yukawa coupling in this case can be inverted to the form:

$$\tilde{Y}_\nu(2\times 2) = i {M_R^d}^{1/2}(2\times 2)R(z_{12})\big(\mathcal{M}_\nu^d\big)^{1/2}(2\times 2)\tilde{U}^\dagger, \tag{16}$$

where $U = U_{23}(\pi/4)\left(\begin{smallmatrix} \tilde{U} & 0 \\ 0 & 1 \end{smallmatrix}\right)$. Using this, we can cast ϵ_l in the form:

$$\epsilon_l = \frac{3}{8\pi}\frac{M_1}{v^2}\frac{\mathrm{Im}(\cos^2 z_{12})\Delta m_\odot^2}{(|\cos z_{12}|^2 m_1 + |\sin z_{12}|^2 m_2)}. \tag{17}$$

This could also have been seen from Eq. (12) by realizing that for the case of exact μ–τ symmetry, we have $z_{13} = 0$ and $z_{23} = \pi/4$.

The above result reproduces the direct proportionality between ϵ_l and solar mass difference square found in Ref. [12]. To simplify this expression further, let us note that out of equilibrium condition for the decay of the lightest right-handed neutrino leads to the condition:

$$\frac{M_1^2}{v_{\mathrm{wk}}^2}\big[m_1|\cos z_{12}|^2 + m_2|\sin z_{12}|^2\big] \leqslant 14\frac{M_1^2}{M_{P\ell}}, \tag{18}$$

which implies that

$$\big|m_1|\cos z_{12}|^2 + m_2|\sin z_{12}|^2\big| \leqslant 2\times 10^{-3}\ \mathrm{eV}. \tag{19}$$

Since solar neutrino data require that in a hierarchical neutrino mass picture $m_2 \simeq 0.9 \times 10^{-2}$ eV, in Eq. (19), we must have $|\sin z_{12}|^2 \sim 0.2$. If we parameterize $\cos^2 z_{12} = \rho e^{i\eta}$, we recover the conclusions of Ref. [12]. This provides a different way to arrive at the conclusions of Ref. [12].

4. Lepton asymmetry and μ–τ symmetry breaking

In this section, we consider the effect of breaking of μ–τ symmetry on lepton asymmetry. Within the see-saw framework, this breaking can arise either from the Dirac mass matrix for the neutrinos or from the right-handed neutrino sector or both. We focus on the case, when the symmetry is broken in the right handed sector only. Such a situation is easy to realize in see-saw models where the theory obeys exact μ–τ symmetry at high scale (above the see-saw scale) prior to B–L symmetry breaking as we show in a subsequent section. We will also show that in this case there is a simple generalization of the lepton asymmetry formula that we derived in the exact μ–τ symmetric case [12].[1]

[1] Leptogenesis in a specific μ–τ symmetric model where the Dirac Yukawa coupling has the form $Y_\nu = \mathrm{diag}(a, b, b)$ has been discussed in Ref. [13]. Our discussion applies more generally.

In this case the neutrino Yukawa matrix is given in the mass eigenstates basis of the right-handed neutrinos by

$$\tilde{Y}_\nu = V_{1/3}^{+} V_{1/2}^{+} V_{2/3}^{+} Y_\nu, \tag{20}$$

where Y_ν is the neutrino Dirac matrix in the flavor basis; The notation $V_{i/j}^{+}$ denotes a unitary 2×2 matrix in the (i, j) subspace. In the above equation, $V_{2/3} = V_{2/3}(\pi/4)$. Now if we substitute for $\tilde{Y}_\nu$ the expression in Eq. (3) and use maximal mixing for the atmospheric neutrino we obtain

$$\begin{bmatrix} \tilde{Y}_{2\times 2} & 0 \\ 0 & \tilde{y}_3 \end{bmatrix} = V_{1/3} M_R^{1/2} R_{1/2} R_{1/3} m_\nu^{1/2} U_{1/2}^{+} U_{1/3}^{+}. \tag{21}$$

Since the μ–τ symmetry breaking is assumed to be small and from reactor neutrino experiments $\theta_{13} \ll 1$ we will expand the mixing matrices in the 1–3 subspace to first order in mixing parameter:

$$(V, R, U)_{1/3} \simeq 1 + (\epsilon, z, \theta)_{13} E, \tag{22}$$

where

$$E = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}. \tag{23}$$

To first order in ϵ_{13}, z_{13} and θ_{13} we have

$$\begin{aligned} & z_{13} M_R^{1/2} R_{1/2} E m_\nu U_{1/2}^{+} + \epsilon_{13} E M_R^{1/2} R_{1/2} m_\nu^{1/2} U_{1/2}^{+} \\ & \quad - \theta_{13} M_R^{1/2} R_{1/2} m_\nu^{1/2} U_{1/2}^{+} E = 0. \end{aligned} \tag{24}$$

It is straightforward to show that the perturbation parameters should satisfy the following equations

$$\begin{aligned} & \epsilon_{13} M_{R_3} m_3 + z_{13} M_{R_1} m_3 R_{11} \\ & \quad - \theta_{13} e^{-i\delta} M_{R_1} c_\theta (m_1 R_{11} - m_2 R_{12}) \simeq 0, \\ & \epsilon_{13} M_{R_2} (m_2 R_{12} s_\theta - m_1 R_{11} c_\theta) \\ & \quad - z_{13} M_{R_3} m_1 c_\theta - \theta_{13} e^{-i\delta} M_{R_3} m_3 \simeq 0, \\ & \epsilon_{13} M_{R_2} (m_1 R_{11} s_\theta + m_2 R_{12} c_\theta) + z_{13} M_{R_3} m_1 s_\theta \simeq 0, \\ & z_{13} M_{R_2} m_3 R_{21} \\ & \quad - \theta_{13} e^{-i\delta} M_{R_2} c_\theta (m_1 R_{21} - m_2 R_{22}) \simeq 0. \end{aligned} \tag{25}$$

Where R_{ij} are the matrix elements of $R_{1/2}$ and c_θ and s_θ are the sine and cosine of the solar neutrino mixing angle. Hence one can see that the parameter z_{13} is proportional to the θ_{13} neutrino mixing angle and is given to first order by

$$z_{13} = \left[\left(\frac{m_1}{m_3} \right) R_{21} - \left(\frac{m_2}{m_3} \right) R_{22} \right] \theta_{13} e^{-i\delta} c_\theta. \tag{26}$$

This proves that the matrix element R_{13} that goes into the leptogenesis formula is directly proportional to the physically observable parameter θ_{13}. This enables us to write $\epsilon_l = a \Delta m_\odot^2 + b \Delta m_A^2 \theta_{13}^2$. A consequence of this is that if the coefficient of proportionality is chosen to be of order one, then as experimental upper limit goes down, unlike the generic type I see-saw case in Section 2, the solar mass difference square starts to dominate for the LMA solution to the solar neutrino problem.

5. Lepton asymmetry for two right-handed neutrinos

In this section, we consider the case of two right-handed neutrinos which transform into one another under μ–τ symmetry. The leptogenesis in this model with exact μ–τ symmetry was discussed in [12] and was shown that it vanishes. In this model therefore, a vanishing or very tiny θ_{13} would not provide a viable model for leptogenesis. Turning this argument around, enough leptogenesis should provide a lower limit on the value of θ_{13}.

To set the stage for our discussion, let us first review the argument for the exact μ–τ symmetry case [12]. The symmetry under which $(N_\mu \leftrightarrow N_\tau)$ and $L_\mu \leftrightarrow L_\tau$ whereas the $m_\mu \neq m_\tau$ constrains the general structure of $\mathbf{Y}_\nu$ and $\mathbf{M_R}$ as follows:

$$\begin{aligned} \mathbf{M_R} &= \begin{pmatrix} M_{22} & M_{23} \\ M_{23} & M_{22} \end{pmatrix}, \\ \mathbf{Y}_\nu &= \begin{pmatrix} h_{11} & h_{22} & h_{23} \\ h_{11} & h_{23} & h_{22} \end{pmatrix}. \end{aligned} \tag{27}$$

In order to calculate the lepton asymmetry using Eq. (7), we first diagonalize the right-handed neutrino mass matrix and change the Y_ν to $\tilde{Y}_\nu$. Since $\mathbf{M_R}$ is a symmetric complex 2×2 matrix, it can be diagonalized by a transformation matrix $U(\pi/4) \equiv \frac{1}{\sqrt{2}} \left(\begin{smallmatrix} 1 & 1 \\ -1 & 1 \end{smallmatrix} \right)$, i.e., $U(\pi/4) \mathbf{M_R} U^T(\pi/4) = \mathrm{diag}(M_1, M_2)$ where $M_{1,2}$ are complex numbers. In this basis we have $\tilde{\mathbf{Y}}_\nu = U(\pi/4) \mathbf{Y}_\nu$. We can therefore rewrite the formula for n_ℓ as

$$\epsilon_l \propto \sum_j \mathrm{Im}\big[U(\pi/4)\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger U^T(\pi/4)\big]_{12}^2 F\left(\frac{M_1}{M_2}\right). \tag{28}$$

Now note that $\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger$ has the form $\left(\begin{smallmatrix} A & B \\ B & A\end{smallmatrix}\right)$ which can be diagonalized by the matrix $U(\pi/4)$. Therefore it follows that $\epsilon_\ell = 0$.

Let us now introduce μ–τ symmetry breaking. If we introduce a small amount of μ–τ breaking in the right-handed neutrino sector as follows: we keep the Y_ν symmetric but choose the right-handed neutrino mass matrix as:

$$\mathbf{M_R} = \begin{pmatrix} M_{22} & M_{23} \\ M_{23} & M_{22}(1+\beta) \end{pmatrix}. \tag{29}$$

After the right-handed neutrino mass matrix is diagonalized, the 3×2 Y'_ν takes the form (for $\theta_{13} \ll 1$ and in the basis where the light neutrino masses are diagonal):

$$\begin{pmatrix} A & B & w\theta_{13} \\ x\theta_{13} & y\theta_{13} & D \end{pmatrix}. \tag{30}$$

Here B, D, x, y, w are of order one and $\theta_{13} \propto \beta$.

To first order in the small mixing θ_{13}, the complex parameters A, B, D satisfy the constraint

$$A \sim \theta_{13}, \qquad Bv^2 \simeq m_2 M_1,$$
$$Dv^2 \simeq m_3 M_2. \tag{31}$$

Using these order of magnitude values, we now find that

$$\epsilon_l \simeq \frac{3}{8\pi}\frac{M_1}{v^2}\frac{\sin\eta[m_3^2\theta_{13}^2\xi]}{m_2}, \tag{32}$$

where ξ is a function of order one. It is clear that very small values for θ_{13} will lead to unacceptably small ϵ_l. In Fig. 1, we have plotted η_B against θ_{13} for values of the parameters in the model that fit the oscillation data and find a lower bound on $\theta_{13} \geqslant 0.1$–$0.15$ for two different values of the CP phases (Fig. 1). In this figure, we have chosen, $M_1 \simeq 7 \times 10^{11}$ GeV. For higher values of M_1 the allowed range θ_{13} moves to the lower range and goes down like $M_1^{-1/2}$. It must however be noted that in Ref. [23] an upper bound on M_1 of about 10^{14} GeV has been derived from the constraint $\tilde{m}_1 \geqslant 10^{-5}$ eV that follows from the requirement that there must be a large enough density of the lightest right-handed neutrinos to lead to sufficient lepton asymmetry. If we take this upper bound, then we get an absolute lower bound on $\theta_{13} \geqslant 0.015$–$0.008$. Also we note that for values of $M_1 < 7 \times 10^{11}$ GeV, the baryon asymmetry becomes lower than the observed value.

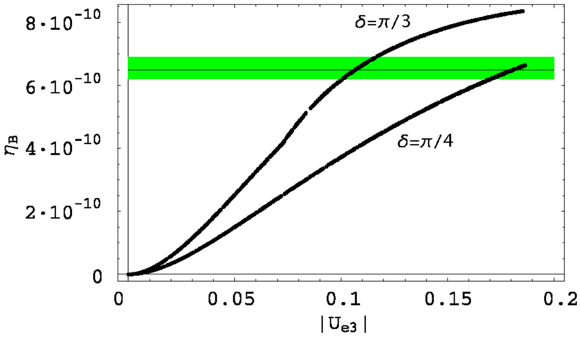

Fig. 1. Plot of η_B vs θ_{13} for the case of two right-handed neutrinos with approximate μ–τ symmetry and CP phases $\delta = \pi/4$ and $\pi/3$. The values of θ_{13} are predicted to be 0.1 and 0.15, respectively. The horizontal line corresponds to $\eta_B^{\mathrm{obs}} = (6.5^{+0.4}_{-0.3}) \times 10^{-10}$ [21].

6. A model for μ–τ symmetry for neutrinos

In this section, we present a simple extension of the minimal supersymmetric standard model (MSSM) by adding to it specific high scale physics that at low energies can exhibit μ–τ symmetry in the neutrino sector as well as real Dirac masses for neutrinos.

First we recall that MSSM needs to be extended by the addition of a set of right-handed neutrinos (either two or three) to implement the see-saw mechanism for neutrino masses [1]. We will accordingly add three right-handed neutrinos (N_e, N_μ, N_τ) to MSSM. We then assume that at high scale, the theory has μ–τ S_2 symmetry under which $N_\pm \equiv (N_\mu \pm N_\tau)$ are even and odd combinations; similarly, we have for leptonic doublet superfields $L_\pm \equiv (L_\mu \pm L_\tau)$ and leptonic singlet ones $\ell^c_\pm \equiv (\mu^c \pm \tau^c)$; two pairs of Higgs doublets $(\phi_{u,\pm}$ and $\phi_{d,\pm})$, and a singlet superfields $S_\pm$. Other superfields of MSSM such as N_e, L_e, e^c as well as quarks are even under the μ–τ S_2 symmetry. Now suppose that we write the superpotential involving the S fields as follows:

$$W_S = \lambda_1\phi_{u,-}\phi_{d,+}S_- + \lambda_2\phi_{u,-}\phi_{d_-}S_+, \tag{33}$$

then when we give high scale vevs to $\langle S_\pm\rangle = M_\pm$, then below the high scale there are only the usual MSSM Higgs pair $H_u \equiv \phi_{u,+}$ and $H_d \equiv (c\phi_{d,+} + s\phi_{d,-})$ that survive whereas the other pair becomes superheavy and decouple from the low energy Lagrangian. The effective coupling at the MSSM level is then given by:

$$\begin{aligned}W = {} & h_e L_e H_d e^c + h_1 L_e H_d \ell^c_+ + h_2 L_e H_d m^c_- \\ & + h_3 L_+ H_d e^c + h_4 L_- H_d e^c + h_5 L_+ H_d \ell^c_+ \\ & + h_6 L_- H_d m^c_- + h_7 L_- H_d \ell^c_+ + f_1 L_e H_{u,+} N_e \\ & + f_2 L_e H_{u,+} N_+ + f_3 L_+ H_{u,+} N_e \\ & + f_4 L_+ H_{u,+} N_+ + f_5 L_- H_{u,+} N_-. \end{aligned} \tag{34}$$

Note that the μ–τ symmetry is present in the Dirac neutrino mass matrix whereas it is not in the charged lepton sector as would be required to.

We show below that it is possible to have a high scale supersymmetric theory which would lead to real Dirac–Yukawa couplings (f_i) if we require the high scale theory to be left–right symmetric. To show how this comes about, consider the gauge group to be $\mathrm{SU}(2)_L \times SU(2)_R \times U(1)_{B-L}$ with quarks and leptons assigned to left- and right-handed doublets as usual [22], i.e., $Q(2,1,1/3)$, $Q^c(1,2,-1/3)$; $L(2,1,-1)$ and $L^c(1,2,+1)$; Higgs fields $\Phi(2,2,0)$; $\chi(2,1,+1)$; $\bar\chi(2,1,-1)$; $\chi^c(1,2,-1)$ and $\bar\chi_c(1,2,-1)$. The new point specific to our model is that we have two sets of the Higgs fields with the above quantum numbers, one even and the other odd under the μ–τ S_2 permutation symmetry, i.e., $\Phi_\pm$, $\chi_\pm$, $\bar\chi_\pm$, $\chi^c_\pm$ and $\bar\chi^c_\pm$ (plus for fields even under S_2 and '$-$' for fields odd under S_2). Furthermore, we will impose the parity symmetry under which $Q \leftrightarrow Q^{c*}$, $L \leftrightarrow L^{c*}$, $(\chi, \bar\chi \leftrightarrow \chi^{c*}, \bar{\chi}^{c*})$, $\Phi \leftrightarrow \Phi^\dagger$.

The Yukawa couplings of this theory invariant under the gauge group as well as parity are given by the superpotential:

$$\begin{aligned}W = {} & h_{11} L_e^T \Phi_+ L_e^c \\ & + h_{++} L_+^T \Phi_+ L_+^c h_{--} L_-^T \Phi_+ L_-^c h_{e+} L_e^T \Phi_+ L_+^c \\ & + h^*_{e+} L_+^T \Phi_+ L_e^c + h_{e-} L_e^T \Phi_- L_-^c \\ & + h^*_{e-} L_-^T \Phi_- L_e^c + h_{+-} L_+^T \Phi_- L_-^c \\ & + h^*_{+-} L_-^T \Phi_- L^c_, \end{aligned} \tag{35}$$

where h_{11}, h_{++}, h_{--} are real.

The Higgs sector of the low energy superpotential is determined from this theory after left–right gauge group is broken down to the Standard Model gauge group by the vevs of χ^c. The phenomenon of doublet–doublet spitting leaves only two Higgs doublets out of the four in $\Phi_\pm$ and is determined by a generic superpotential of type

$$\begin{aligned}W_{DD} = {} & \sum_{i,j,k} \lambda_{ijk} \chi_i \Phi_j \chi^c_k + \lambda'_{ijk} \bar\chi_i \phi_j \bar\chi^c_k \\ & + M_1\big(\chi_\pm \bar\chi_\pm + \chi^c_\pm \bar\chi^c_\pm\big), \end{aligned} \tag{36}$$

where i, j, k go over '$+$' and '$-$' for even and odd and only even terms are allowed by μ–τ invariance, e.g., $\lambda_{+++}, \lambda_{+--}, \ldots$ are nonzero. Now suppose that $\langle\chi^c_+\rangle = 0$ but $\langle\chi^c_-\rangle \neq 0$ and $\langle\bar\chi^c_\pm\rangle \neq 0$. These vevs break the left–right group to the Standard Model gauge group. It is then easy to see that below the $\langle\chi^c\rangle$ scale, there are only one Higgs pair where $H_u = \phi_{u,+}$ and $H_d = \sum_{i=+,-,3,4} a_i \phi_{d,i}$. Here we have denoted the $\Phi \equiv (\phi_u, \phi_d)$ and $\phi_{d,3,4} = \chi_\pm$. The upshot of all these discussions is that the right-handed neutrino Yukawa couplings are μ–τ even and therefore have the form:

$$Y_\nu = \begin{pmatrix} h_{11} & h_{e+} & 0 \\ h^*_{e+} & h_{++} & 0 \\ 0 & 0 & h_{--} \end{pmatrix}. \tag{37}$$

It is easy to see that redefining the fields appropriately, we can make Y_ν real. So the only source of complex phase in this model is in the RH neutrino mass matrix, which in this model are generated by higher dimensional couplings of the form $L^c L^c \bar\chi^c \bar\chi^c$ as we discuss now.

The most general nonrenormalizable interactions that can give rise to right-handed neutrino masses are of the form:

$$\begin{aligned}W_{NR} = {} & \frac{1}{M}\Big[\big(L^c_e \bar\chi^c_+\big)^2 + \big(L^c_e \bar\chi^c_-\big)^2 \\ & + \big(L^c_+ \bar\chi^c_+\big)^2 \big(L^c_- \bar\chi^c_-\big)^2 + \big(L^c_- \bar\chi^c_+\big)^2 \\ & + \big(L^c_+ \bar\chi^c_-\big)^2 \big(L^c_+ \bar\chi^c_-\big)\big(L^c_- \bar\chi^c_+\big)\Big]. \end{aligned} \tag{38}$$

Note that since both $\bar\chi^c_\pm$ acquire vevs, the last term in the above expression will give rise to μ–τ breaking in the RH neutrino sector while preserving it in the

Y_ν. The associated couplings in the above equations are in general complex. This leads to a realistic three generation model with approximate μ–τ symmetry as analyzed in the previous sections.

In summary, we have studied the implications for leptogenesis in models where neutrino masses arise from the type I see-saw mechanism and where the near maximal atmospheric mixing angle owes its origin to an approximate μ–τ symmetry. We derive a relation of the form $\epsilon_l = (a\Delta m^2_\odot + b\Delta m^2_A \theta^2_{13})$ for the case of three right-handed neutrinos, which directly connects the neutrino oscillation parameters with the origin of matter. We also show that if θ_{13} is very small or zero, only the LMA solution to the solar neutrino puzzle would provide an explanation of the origin of matter within this framework. Finally for the case of two right-handed neutrinos with approximate μ–τ symmetry, we predict values for θ_{13} in the range 0.1–0.15 for specific choices of the high energy phase between $\pi/4$ and $\pi/3$.

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