Leptogenesis, $\mu-\tau$ symmetry and $\theta_{13}$

R.N. Mohapatra, S. Nasri, Hai-bo Yu

Department of Physics, University of Maryland, College Park, MD 20742, USA

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Abstract

We show that in theories where neutrino masses arise from type I see-saw formula with three right-handed neutrinos and where large atmospheric mixing angle owes its origin to an approximate leptonic $\mu-\tau$ interchange symmetry, the primordial lepton asymmetry of the Universe, $\epsilon_l$ can be expressed in a simple form in terms of low energy neutrino oscillation parameters as $\epsilon_l = (a/D\Delta m^2 + b/D\Delta m^2 \theta_{13}^2)$, where $a$ and $b$ are parameters characterizing high scale physics and are each of order $\leq 10^{-2}$ eV$^{-2}$. We also find that for the case of two right-handed neutrinos, $\epsilon_l \propto \theta_{13}$ as a result of which, the observed value of baryon to photon ratio implies a lower limit on $\theta_{13}$. For specific choices of the CP phase $\delta$ we find $\theta_{13}$ is predicted to be between 0.10–0.15.

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1. Introduction

There may be a deep connection between the origin of matter in the Universe and the observed neutrino oscillations. This speculation is inspired by the idea that the heavy right-handed Majorana neutrinos that are added to the Standard Model for understanding small neutrino masses via the see-saw mechanism [1] can also explain the origin of matter via their decay. The mechanism goes as follows [2]: CP violation in the same Yukawa interaction of the right-handed neutrinos, which go into giving nonzero neutrino masses after electroweak symmetry breaking, lead to a primordial lepton asymmetry via the out of equilibrium decay $N_R \to \ell + H$ (where $\ell$ are the known leptons and $H$ is the Standard Model Higgs field). This asymmetry subsequently gets converted to baryon–antibaryon asymmetry observed today via the electroweak sphaleron interactions [3], above $T \geq v_{wk}$ ($v_{wk}$ being the weak scale). Since this mechanism involves no new interactions beyond those needed in the discussion of neutrino masses, one would expect that better understanding of neutrino mass physics would clarify one of the deepest mysteries of cosmology both qualitatively as well as quantitatively. This question has been the subject of many investigations in recent years [4–11] in the context of different neutrino mass models and many interesting pieces of in-
formation about issues such as the spectrum of right-handed neutrinos, upper limit on the neutrino masses etc have been obtained. In a recent paper, [12], two of the authors showed that if one assumes that the lepton sector of minimal see-saw models has a leptonic \( \mu - \tau \) interchange symmetry [14, 15], then one can under certain plausible assumptions indeed predict the magnitude of the matter–antimatter asymmetry in terms of low energy oscillation parameter, \( \Delta m_{23}^2 \), and a high scale CP phase. The choice of \( \mu - \tau \) symmetry was dictated by the fact that it is the simplest symmetry of neutrino mass matrix that explains the maximal atmospheric mixing as indicated by data. Using present experimental value for \( \Delta m_{23}^2 \), one obtains the right magnitude for the baryon asymmetry of the Universe.

The results of the paper [12] were derived in the limit that \( \mu - \tau \) interchange symmetry is exact. If however a nonzero value for the neutrino mixing angle \( \theta_{13} \) is detected in future experiments, this would imply that this symmetry is only approximate. Also, since in the Standard Model \( v_\mu \) and \( v_\tau \) are members of the \( SU(2)_L \) doublets \( L_\mu \equiv (v_\mu, \mu) \) and \( L_\tau \equiv (v_\tau, \tau) \), any symmetry between \( v_\mu \) and \( v_\tau \) must be a symmetry between \( L_\mu \) and \( L_\tau \) at the fundamental Lagrangian level. The observed difference between the muon and tau masses would therefore also imply that the \( \mu - \tau \) symmetry has to be an approximate symmetry. In view of this, it is important to examine to what extent the results of Ref. [12] carry over to the case when the symmetry is approximate. We find two interesting results under some very general assumptions: (i) a simple formula is approximate. We find two interesting results under some very general assumptions: (i) a simple formula

\[
\begin{align*}
\mathcal{M}_v &= -Y_v^T \Gamma^{-1} Y_v \frac{v^2 \tan^2 \beta}{v_R}. \\
Y_v &= i M_R^{1/2} R(z_{ij}) (\mathcal{M}_v^0)^{1/2} U^T,
\end{align*}
\]

The constraints of \( \mu - \tau \) symmetry will manifest themselves in the form of the \( Y_v \) and \( M_R \). It has been pointed out that if we go to a basis where the right-handed neutrino mass matrix is diagonal, we can solve for \( Y_v \), in terms of the neutrino masses and mixing angles as follows [17]:

2. Introductory remarks on lepton asymmetry in type I see-saw models

We start with an extension of the minimal supersymmetric Standard Model (MSSM) for the generic type I see-saw model for neutrino masses. The effective low energy superpotential for this model is given by

\[
W = e^T Y_i L H_d + N^T Y_v L H_u + \frac{M_R}{2} N^T N c T.
\]
matrix defined by
\[ \mathcal{M}_\nu = U^* \mathcal{M}_\nu^T U. \]  
(4)
The complex orthogonal matrices \( R \) can be parameterized as:
\[ R(z_{12}, z_{23}, z_{13}) = R(z_{23})R(z_{13})R(z_{12}) \]  
(5)
with
\[ R(z_{12}) = \begin{pmatrix} \cos z_{12} & \sin z_{12} & 0 \\ -\sin z_{12} & \cos z_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \]  
(6)
and similarly for the other matrices. \( z_{ij} \) are complex angles.

Let us now turn to lepton asymmetry: the formula for primordial lepton asymmetry in this case, caused by right-handed neutrino decay is
\[ \epsilon_l = \frac{1}{8\pi} \sum_j \text{Im}[\tilde{Y}_e \tilde{Y}_e^\dagger_{1j}] F(M_1 \left| M_j \right|), \]  
(7)
where \( \tilde{Y}_e \) is defined in a basis where right-handed neutrinos are mass eigenstates and their masses are denoted by \( M_{1,2,3} \) where \( F(x) = -\frac{1}{4} \left[ \frac{2}{x^2} - \ln(1 + x^2) \right] \) [18]. In the case where that the right-handed neutrinos have a hierarchical mass pattern, i.e., \( M_1 \ll M_{2,3} \), we get \( F(x) \simeq -3x \). In this approximation, we can write the lepton asymmetry in a simple form [19]
\[ \epsilon_l = -\frac{3}{8\pi} \frac{M_1 \text{Im}[Y_e \mathcal{M}_\nu^T Y_e^\dagger]_{11}}{v^2 (\tilde{Y}_e \tilde{Y}_e^\dagger)_{11}}, \]  
(8)
where using the expression for \( Y_e \) given above, we can rewrite \( \epsilon_l \) as:
\[ \epsilon_l = -\frac{3}{8\pi} \frac{\text{Im}[M_R^{1/2} \mathcal{M}_{\nu R}^{-1}(z_{1j}) M_R^{1/2}]_{11}}{v^2 (R(z_{1j}) \mathcal{M}_\nu R(z_{1j})^{-1})_{11}}. \]  
(9)
We will now apply this discussion to calculate the lepton asymmetry in the general case without any symmetries. In the following sections, we follow it up with a discussion of two cases: (i) the cases of exact \( \mu-\tau \) symmetry and (ii) the case where this symmetry is only approximate. Since the formula in Eq. (9) assumes that there are three right-handed neutrinos, we will focus on this case in the next two sections. In a subsequent section, we consider the case of two right-handed neutrinos \( (N_\mu, N_\tau) \), which transform into each other under the \( \mu-\tau \) symmetry. Both cases are in agreement with the observed neutrino mass differences and mixings.

It follows from Eq. (9) that
\[ \epsilon_l = -\frac{3M_1 \text{Im}[m^2_1 R_{11}^2 + m^2_2 R_{12}^2 + m^2_3 R_{13}^2]}{8\pi v^2 |R(z_{1j})\mathcal{M}_\nu R(z_{1j})^{-1}|_{11}}. \]  
(10)
Since the matrix \( R \) is an orthogonal matrix, we have the relation
\[ R_{11}^2 + R_{12}^2 + R_{13}^2 = 1. \]  
(11)
Using this equation in Eq. (10), we get
\[ \epsilon_l = -\frac{3M_1 \text{Im}[\Delta m^2_{\odot} R_{12}^2 + \Delta m^2_{\text{atm}} R_{13}^2]}{8\pi v^2 \sum_j (|R_{1j}|^2 m_j)}. \]  
(12)
This relation connects the lepton asymmetry to both the solar and the atmospheric mass difference square [5]. To make a prediction for the lepton asymmetry, we need to know the lengths of the complex quantities \( R_{1j} \). The out of equilibrium condition does provide a constraint on \( |R_{1j}| \) as follows:
\[ \sum_{j=1,2,3} \left( |R_{1j}|^2 m_j \right) < 10^{-3} \text{ eV}. \]  
(13)
Naively interpreted, this would have meant a strong constraint on the degenerate neutrino spectrum. However, as has been shown in Ref. [5] the preferred range for \( \sum_{j=1,2,3} (|R_{1j}|^2 m_j) \) is from \( 10^{-3} \) to 0.1 eV with no strict upper bound, although an upper bound of on \( \sum m^2_{12}^{1/2} \) of 0.1 eV can be deduced from washout processes. It is clear from Eq. (13) that if neutrinos are quasi-degenerate based on this argument, we conclude that a degenerate mass spectrum with \( m_0 \geq 0.1 \text{ eV} \) will most likely be in conflict with observations, if type I see-saw is responsible for neutrino masses. It must however be noted that a more appealing and natural scenario for degenerate neutrino masses is type II see-saw formula [20], in which case the above considerations do not apply. Therefore, it is not possible to conclude based on the leptogenesis argument alone that a quasi-degenerate neutrino spectrum is inconsistent.

In a hierarchical neutrino mass picture, Eq. (13) implies that \( |R_{13}|^2 \lesssim 0.02 \) and \( |R_{12}|^2 \lesssim 0.1 \). If we assume that the upper limit in the Eq. (13) is saturated, then we get the atmospheric neutrino mass difference square in Eq. (12) to give the dominant contribution.
We will see below that if one assumes an exact $\mu-\tau$ symmetry for the neutrino mass matrix, the situation becomes different and it is the solar mass difference square that dominates.

3. Three right-handed neutrinos and exact $\mu-\tau$ symmetry

In this section, we consider the case of three right-handed neutrino with an exact $\mu-\tau$ symmetry in the Dirac mass matrix as well as the right-handed neutrino mass matrix. In this case, the right-handed neutrino mass matrix $M_R$ and the Dirac–Yukawa coupling $Y_{\nu}$ can be written, respectively, as:

$$M_R = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{12} & M_{22} & M_{23} \\ M_{13} & M_{23} & M_{33} \end{pmatrix},$$

$$Y_{\nu} = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix},$$

where $M_{ij}$ and $h_{ij}$ are all complex. An important property of these two matrices is that they can be cast into a block diagonal form by the same transformation matrix $U_{23}(\pi/4) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\theta} P(\beta) R(\theta) P(\gamma) \\ 0 & 0 & 1 \end{pmatrix}$ on the $v$'s and $N$'s. Let us denote the block diagonal forms by a tilde, i.e., $\tilde{Y}_{\nu}$ and $\tilde{M}_R$. We then go to a basis where the $\tilde{M}_R$ is subsequently diagonalized by the most general $2 \times 2$ unitary matrix as follows:

$$V_T (2 \times 2) U_{23}^T (\pi/4) M_R U_{23} (\pi/4) V (2 \times 2) = M_R^d,$$

where $V(2 \times 2) = \begin{pmatrix} V & 0 \\ 0 & 1 \end{pmatrix}$ where $V$ is the most general $2 \times 2$ unitary matrix given by $V = e^{i\alpha} P(\beta) R(\theta) P(\gamma)$. The $3 \times 3$ case therefore reduces to a $2 \times 2$ problem. The third mass eigenstate in both the light and the heavy sectors play no role in the leptogenesis as well as generation of solar mixing angle [12]. Note also that we have $\theta_{13} = 0$. The see-saw formula in the 1–2 subsector has exactly the same form except that all matrices in the left- and right-hand side of Eq. (9) are $2 \times 2$ matrices. The formula for the Dirac–Yukawa coupling in this case can be inverted to the form:

$$\tilde{Y}_{\nu}(2 \times 2) = i M_R^{d_1/2} (2 \times 2) R(z_{12}) \left(M_R^d\right)^{-1/2} (2 \times 2) \tilde{U}^\dagger,$$

where $U = U_{23}(\pi/4) \begin{pmatrix} 0 & U \end{pmatrix}$. Using this, we can cast $\epsilon_l$ in the form:

$$\epsilon_l = \frac{3}{8\pi} \frac{M_1}{v^2} \frac{\Im(\cos^2 z_{12}) \Delta m_2^2}{(|\cos z_{12}|^2 m_1 + |\sin z_{12}|^2 m_2)}.$$  

This could also have been seen from Eq. (12) by realizing that for the case of exact $\mu-\tau$ symmetry, we have $z_{13} = 0$ and $z_{23} = \pi/4$.

The above result reproduces the direct proportionality between $\epsilon_l$ and solar mass difference square found in Ref. [12]. To simplify this expression further, let us note that out of equilibrium condition for the decay of the lightest right-handed neutrino leads to the condition:

$$M_R^2 \left| m_1 \cos z_{12}^2 + m_2 \sin z_{12}^2 \right| \leq 14 \frac{M_1^2}{M_{Pl}} ,$$

which implies that

$$|m_1 \cos z_{12}^2 + m_2 \sin z_{12}^2| \leq 2 \times 10^{-3} \text{ eV}.$$  

Since solar neutrino data require that in a hierarchical neutrino mass picture $m_2 \simeq 0.9 \times 10^{-2} \text{ eV}$, in Eq. (19), we must have $|\sin z_{12}|^2 \sim 0.2$. If we parameterize $\cos^2 z_{12} = \rho e^{i\eta}$, we recover the conclusions of Ref. [12]. This provides a different way to arrive at the conclusions of Ref. [12].

4. Lepton asymmetry and $\mu-\tau$ symmetry breaking

In this section, we consider the effect of breaking of $\mu-\tau$ symmetry on lepton asymmetry. Within the seesaw framework, this breaking can arise either from the Dirac mass matrix for the neutrinos or from the right-handed neutrino sector or both. We focus on the case, when the symmetry is broken in the right handed sector only. Such a situation is easy to realize in see-saw models where the theory obeys exact $\mu-\tau$ symmetry at high scale (above the see-saw scale) prior to $B-L$ symmetry breaking as we show in a subsequent section. We will also show that in this case there is a simple generalization of the lepton asymmetry formula that we derived in the exact $\mu-\tau$ symmetric case [12].

1 Leptogenesis in a specific $\mu-\tau$ symmetric model where the Dirac Yukawa coupling has the form $Y_{\nu} = \text{diag}(a, b, b)$ has been discussed in Ref. [13]. Our discussion applies more generally.
In this case the neutrino Yukawa matrix is given in the mass eigenstates basis of the right-handed neutrinos by
\[
\tilde{Y}_\nu = V^+_{1/2} V^+_{2/3} Y_v, \tag{20}
\]
where \(Y_v\) is the neutrino Dirac matrix in the flavor basis; The notation \(V^+_{ij}\) denotes a unitary \(2 \times 2\) matrix in the \((i, j)\) subspace. In the above equation, \(V_{2/3} = V_{2/3}(\pi/4)\). Now if we substitute for \(\tilde{Y}_\nu\) the expression in Eq. (3) and use maximal mixing for the atmospheric neutrino we obtain
\[
\begin{bmatrix}
\tilde{\nu}_{2\times2} & 0 \\
0 & \tilde{y}_3
\end{bmatrix} = V_{1/3} M_R^{1/2} R_{1/2} R_{1/3} m_v^{1/2} U^+_{1/2} U^+_{1/3}.
\tag{21}
\]
Since the \(\mu - \tau\) symmetry breaking is assumed to be small and from reactor neutrino experiments \(\theta_{13} \ll 1\) we will expand the mixing matrices in the 1–3 subspace to first order in mixing parameter:
\[
(V, R, U)_{1/3} \approx 1 + (\epsilon, z, \theta)_{13} E, \tag{22}
\]
where
\[
E = \begin{bmatrix}
0 & 0 & 1 \\
0 & 0 & 0 \\
-1 & 0 & 0
\end{bmatrix}. \tag{23}
\]
To first order in \(\epsilon_{13}, z_{13}\) and \(\theta_{13}\) we have
\[
z_{13} M_R^{1/2} R_{1/2} E m_v^{1/2} U^+_{1/2} + \epsilon_{13} E M_R^{1/2} R_{1/2} m_v^{1/2} U^+_{1/2}\]
\[- \theta_{13} M_R^{1/2} R_{1/2} m_v^{1/2} U^+_{1/2} E = 0. \tag{24}\]
It is straightforward to show that the perturbation parameters should satisfy the following equations
\[
\epsilon_{13} M_{R_1} m_3 + z_{13} M_{R_1} m_3 R_{11} - \theta_{13} e^{-i\delta} M_{R_1} c_0(m_1 R_{11} - m_2 R_{12}) \simeq 0,
\]
\[
\epsilon_{13} M_{R_2} (m_2 R_{12} s_0 - m_1 R_{11} c_0) - z_{13} M_{R_1} m_1 c_0 - \theta_{13} e^{-i\delta} M_{R_1} m_3 \simeq 0,
\]
\[
\epsilon_{13} M_{R_4} (m_1 R_{11} s_0 + m_2 R_{12} c_0) + z_{13} M_{R_1} m_1 s_0 \simeq 0,
\]
\[
z_{13} M_{R_1} m_2 R_{21} - \theta_{13} e^{-i\delta} M_{R_1} c_0 (m_1 R_{21} - m_2 R_{22}) \simeq 0. \tag{25}\]
Where \(R_{ij}\) are the matrix elements of \(R_{1/2}\) and \(c_0\) and \(s_0\) are the sine and cosine of the solar neutrino mixing angle. Hence one can see that the parameter \(z_{13}\) is proportional to the \(\theta_{13}\) neutrino mixing angle and is given to first order by
\[
z_{13} = \left(\frac{m_1}{m_3}\right) R_{21} - \left(\frac{m_2}{m_3}\right) R_{22} \theta_{13} e^{-i\delta} c_0. \tag{26}\]
This proves that the matrix element \(R_{13}\) that goes into the leptogenesis formula is directly proportional to the physically observable parameter \(\theta_{13}\). This enables us to write \(\epsilon_{i} = a \Delta m^2_{32} + b \Delta m^2_{21} \theta_{13}^2\). A consequence of this is that if the coefficient of proportionality is chosen to be of order one, then as experimental upper limit goes down, unlike the generic type I see-saw case in Section 2, the solar mass difference square starts to dominate for the LMA solution to the solar neutrino problem.

5. Lepton asymmetry for two right-handed neutrinos

In this section, we consider the case of two right-handed neutrinos which transform into one another under \(\mu - \tau\) symmetry. The leptogenesis in this model with exact \(\mu - \tau\) symmetry was discussed in [12] and was shown that it vanishes. In this model therefore, a vanishing or very tiny \(\theta_{13}\) would not provide a viable model for leptogenesis. Turning this argument around, enough leptogenesis should provide a lower limit on the value of \(\theta_{13}\).

To set the stage for our discussion, let us first review the argument for the exact \(\mu - \tau\) symmetry case [12]. The symmetry under which \((N_\mu \leftrightarrow N_\tau)\) and \(L_\mu \leftrightarrow L_\tau\) whereas the \(m_\mu \neq m_\tau\) constrains the general structure of \(Y_v\) and \(M_R\) as follows:
\[
M_R = \begin{pmatrix}
M_{22} & M_{23} \\
M_{23} & M_{22}
\end{pmatrix},
\]
\[
Y_v = \begin{pmatrix}
h_{11} & h_{22} & h_{23} \\
h_{11} & h_{23} & h_{22}
\end{pmatrix}. \tag{27}\]
In order to calculate the lepton asymmetry using Eq. (7), we first diagonalize the right-handed neutrino mass matrix and change the \(Y_v\) to \(\tilde{Y}_v\). Since \(M_R\) is a symmetric complex \(2 \times 2\) matrix, it can be diagonalized by a transformation matrix \(U(\pi/4) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\), i.e., \(U(\pi/4)M_R U^T(\pi/4) = \text{diag}(M_{11}, M_{22})\) where \(M_{12}\) are complex numbers. In this basis we have \(\tilde{Y}_v = U(\pi/4)Y_v\). We can therefore rewrite the
formula for \( nL \) as

\[
e_l \propto \sum_j \text{Im}[U(\pi/4)Y_\nu Y_\nu^T (\pi/4)]^2 (M_1/M_2).
\]  

(28)

Now note that \( Y_\nu, Y_\nu^\dagger \) has the form \((A \ B; B \ A)\) which can be diagonalized by the matrix \( U(\pi/4) \). Therefore it follows that \( \epsilon_l = 0 \).

Let us now introduce \( \mu-\tau \) symmetry breaking. If we introduce a small amount of \( \mu-\tau \) breaking in the right-handed neutrino sector as follows: we keep the \( Y_\nu \) symmetric but choose the right-handed neutrino mass matrix as:

\[
M_r = \begin{pmatrix} M_{22} & M_{23} \\ M_{23} & M_{22}(1 + \beta) \end{pmatrix}.
\]  

(29)

After the right-handed neutrino mass matrix is diagonalized, the \( 3 \times 2 \ Y'_\nu \) takes the form (for \( \theta_{13} \ll 1 \) and in the basis where the light neutrino masses are diagonal):

\[
\begin{bmatrix} A & B \\ x\theta_{13} & y\theta_{13} \end{bmatrix} w\theta_{13}.
\]  

Here \( B, D, x, y, w \) are of order one and \( \theta_{13} \propto \beta \).

To first order in the small mixing \( \theta_{13} \), the complex parameters \( A, B, D \) satisfy the constraint

\[
A B = m_2 M_1, \quad D e^\gamma = m_3 M_2.
\]

(31)

Using these order of magnitude values, we now find that

\[
e_l \simeq \frac{3}{8\pi} \frac{M_1 \sin \theta [m_2 \theta_{13}^2 \xi]}{v^2}.
\]  

(32)

where \( \xi \) is a function of order one. It is clear that very small values for \( \theta_{13} \) will lead to unacceptably small \( \epsilon_l \).

In Fig. 1, we have plotted \( \eta_B \) against \( \theta_{13} \) for values of the parameters in the model that fit the oscillation data and find a lower bound on \( \theta_{13} \geq 0.1–0.15 \) for two different values of the CP phases (Fig. 1). In this figure, we have chosen, \( M_1 \simeq 7 \times 10^{13} \) GeV. For higher values of \( M_1 \) the allowed range \( \theta_{13} \) moves to the lower range and goes down like \( M_1^{-1/2} \). It must however be noted that in Ref. [23] an upper bound on \( M_1 \) of about \( 10^{14} \) GeV has been derived from the constraint \( m_1 \geq 10^{-5} \) eV that follows from the requirement that there must be a large enough density of the lightest right-handed neutrinos to lead to sufficient lepton asymmetry. If we take this upper bound, then we get an absolute lower bound on \( \theta_{13} \geq 0.015–0.008 \). Also we note that for values of \( M_1 < 7 \times 10^{13} \) GeV, the baryon asymmetry becomes lower than the observed value.

6. A model for \( \mu-\tau \) symmetry for neutrinos

In this section, we present a simple extension of the minimal supersymmetric standard model (MSSM) by adding to it specific high scale physics that at low energies can exhibit \( \mu-\tau \) symmetry in the neutrino sector as well as real Dirac masses for neutrinos.

First we recall that MSSM needs to be extended by the addition of a set of right-handed neutrinos (either two or three) to implement the see-saw mechanism for neutrino masses [1]. We will accordingly add three right-handed neutrinos \((N_e, N_\mu, N_\tau)\) to MSSM. We then assume that at high scale, the theory has \( \mu-\tau \) symmetry under which \( N_\pm \equiv (N_\mu \pm N_\tau) \) are even and odd combinations; similarly, we have for leptonic doublet superfields \( L_\pm \equiv (L_\mu \pm L_\tau) \) and lepton singlet ones \( \ell_\pm \equiv (\ell e^\pm \tau^c) \); two pairs of Higgs doublets \((\phi_{u, d} \pm)\), and a singlet superfields \( S_{d} \). Other superfields of MSSM such as \( N_e, L_e, e^c \) as well as quarks are even under the \( \mu-\tau \) symmetry. Now suppose that we write the superpotential involving the \( S \) fields as follows:

\[
W_S = \lambda_1 \phi_{u, -} \phi_{d, +} S_- + \lambda_2 \phi_{u, -} \phi_{d, +} S_+.
\]  

(33)
then when we give high scale vevs to \( S_L \) = \( M_\pm \), then below the high scale there are only the usual MSSM Higgs pair \( H_u \equiv \phi_{u,+} \) and \( H_d \equiv (c \phi_{d,+} + s \phi_{d,-}) \) that survive whereas the other pair becomes superheavy and decouple from the low energy Lagrangian. The effective coupling at the MSSM level is then given by:

\[
W = h_e L_e H_d e^c + h_1 L_e H_d e^c_+ + h_2 L_e H_d m^c \\
+ h_3 L_d H_u e^c + h_4 L_d H_d e^c + h_5 L_d H_u m^c \\
+ h_6 L_d H_u m^c_+ + h_7 L_d H_d m^c_+ \\
+ f_1 L_e H_u, N_e + f_2 L_d H_u, N_e \\
+ f_3 L_d H_u, N_+ + f_4 L_d H_u, N_.
\]

(34)

Note that the \( \mu-\tau \) symmetry is present in the Dirac neutrino mass matrix whereas it is not in the charged lepton sector as would be required to.

We show below that it is possible to have a high scale supersymmetric theory which would lead to real Dirac–Yukawa couplings \( f_i \) if we require the high scale theory to be left–right symmetric. To show how this comes about, consider the gauge group to be \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \) with quarks and leptons assigned to left- and right-handed doublets as usual [22], i.e., \( Q, Q', L_c, L_c', (1, 2, -1/3); L(2, 1, -1) \) and \( L_c(1, 2, +1) \); Higgs fields \( \Phi(2, 2, 0); \chi(2, 1, +1); \bar{\chi}(2, 1, -1); \chi^c(1, 2, -1) \) and \( \bar{\chi}^c(1, 2, 1) \). The new point specific to our model is that we have two sets of the Higgs fields with the above quantum numbers, one even and the other odd under the \( \mu-\tau \) \( S_2 \) permutation symmetry, i.e., \( \Phi_{\pm}, \chi_{\pm}, \bar{\chi}_\pm, \bar{\chi}^c_\pm \) (plus for fields even under \( S_2 \) and ‘+’ for fields odd under \( S_2 \)). Furthermore, we will impose the parity symmetry under which \( Q \leftrightarrow Q^c, L_c \leftrightarrow L_c^c, (\chi, \bar{\chi} \leftrightarrow \chi^c, \bar{\chi}^c), \Phi \leftrightarrow \Phi^c \).

The Yukawa couplings of this theory invariant under the gauge group as well as parity are given by the superpotential:

\[
W = h_{11} L_c^T \Phi^c L_c \\
+ h_{-+} L_c^T \Phi^c L_c h_{--} L_c^T \Phi^c L_c h_{++} L_c^T \Phi^c L_c \\
+ h_{+} L_c^T \Phi^c L_c + h_{-} L_c^T \Phi^c L_c \\
+ h_{+} L_c^T \Phi^c L_c + h_{-} L_c^T \Phi^c L_c \\
+ h_{+} L_c^T \Phi^c L_c + h_{-} L_c^T \Phi^c L_c \\
+ h_{+} L_c^T \Phi^c L_c + h_{-} L_c^T \Phi^c L_c
\]

(35)

where \( h_{11}, h_{++}, h_{--} \) are real.

The Higgs sector of the low energy superpotential is determined from this theory after left–right gauge group is broken down to the Standard Model gauge group by the vevs of \( \chi^c \). The phenomenon of doublet–doublet splittings leaves only two Higgs doublets out of the four in \( \Phi_{\pm} \) and is determined by a generic superpotential of type

\[
W_{DD} = \sum_{i,j,k} \lambda_{ijk} \Phi_i \bar{\Phi}_j \bar{\Phi}_k \\
+ M \left( \chi_{\pm} \bar{\chi}_{\pm} + \chi^c_{\pm} \bar{\chi}^c_{\pm} \right).
\]

(36)

where \( i, j, k \) go over ‘+’ and ‘-’ for even and odd and only even terms are allowed by \( \mu-\tau \) invariance, e.g., \( \lambda_{+++}, \lambda_{+-+}, \ldots \) are nonzero. Now suppose that \( \langle \chi^c_+ \rangle = 0 \) but \( \langle \chi^c_- \rangle \neq 0 \) and \( \langle \bar{\chi}^c_+ \rangle \neq 0 \). These vevs break the left–right group to the Standard Model gauge group. It is then easy to see that below the \( \langle \chi^c \rangle \) scale, there are only one Higgs pair with \( H_u = \phi_{u,+} \) and \( H_d = \sum_{i=+, -3, 4} \phi_{d,i} \). Here we have denoted the \( \Phi = (\phi_u, \phi_d) \) and \( \Phi_{d,3,4} = \chi_{\pm} \). The upshot of all these discussions is that the right-handed neutrino Yukawa couplings are \( \mu-\tau \) even and therefore have the form:

\[
Y_v = \begin{pmatrix}
    h_{11} & h_{++} & 0 \\
    h_{-+} & h_{++} & 0 \\
    0 & 0 & h_{--}
\end{pmatrix}.
\]

(37)

It is easy to see that redefining the fields appropriately, we can make \( Y_v \) real. So the only source of complex phase in this model is in the RH neutrino mass matrix, which in this model are generated by higher dimensional couplings of the form \( L^c L^c \bar{\chi}^c \bar{\chi}^c \) as we discuss now.

The most general nonrenormalizable interactions that can give rise to right-handed neutrino masses are of the form:

\[
W_{NR} = \frac{1}{M} \left[ (L^c \bar{\phi}_{d+})^2 + (L^c \bar{\phi}_{d-})^2 \right. \\
+ (L^c \bar{\phi}_{d+})^2 (L^c \bar{\phi}_{d-})^2 + (L^c \bar{\phi}_{d-})^2 \\
\left. + (L^c \bar{\phi}_{d+})^2 (L^c \bar{\phi}_{d-})^2 \right].
\]

(38)

Note that since both \( \bar{\chi}_{\pm} \) acquire vevs, the last term in the above expression will give rise to \( \mu-\tau \) breaking in the RH neutrino sector while preserving it in the
$Y_{\nu}$. The associated couplings in the above equations are in general complex. This leads to a realistic three generation model with approximate $\mu$–$\tau$ symmetry as analyzed in the previous sections.

In summary, we have studied the implications for leptogenesis in models where neutrino masses arise from the type I see-saw mechanism and where the near maximal atmospheric mixing angle owes its origin to an approximate $\mu$–$\tau$ symmetry. We derive a relation of the form \[ \epsilon_l = (a/D\Delta m^2_{13} + b/D\Delta m^2_{13}\theta_{13}^2) \]
for the case of three right-handed neutrinos, which directly connects the neutrino oscillation parameters with the origin of matter. We also show that if $\theta_{13}$ is very small or zero, only the LMA solution to the solar neutrino puzzle would provide an explanation of the origin of matter within this framework. Finally for the case of two right-handed neutrinos with approximate $\mu$–$\tau$ symmetry, we predict values for $\theta_{13}$ in the range 0.1–0.15 for specific choices of the high energy phase between $\pi/4$ and $\pi/3$.

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References