New Photoelectric System on the Basis of Cascade Homogeneous Photoconverters and Solar Radiation Concentrators

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Abstract

Ultimate characteristics of the cascade solar cell based on a homogeneous semiconductor with tunneling transitions between the individual elements are researched. The advantage of the use of such structures is the reduction of internal losses, which is especially important when converting concentrated radiation. Ultimate theoretical and physical characteristics of the photocurrent, open circuit voltage and efficiency are found depending on the concentration of radiation and the number of individual elements in the cascade.

Keywords: Solar Cell; Cascade Solar Cell; Radiation Concentration; Upper Limit Characteristics; Spectral Responce

1. Introduction

In this paper, we study theoretical and physical possibilities of manufacturing high efficiency cascade solar photoelectric converters (PC) on the basis of new high-voltage multilayer tunnel-type $n^{+}-p-p^{+}(t)n^{+}-p-p^{+}(t)...n^{+}-p-p^{+}$ structures with the quantum-mechanical charge carriers tunneling effect in $p^{+}(t)n^{+}$ junction in homogeneous semiconductor [1, 2] used in photovoltaic systems with solar radiation concentrators.

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2. General theory of optimization semiconductor structure with cascaded converters

Research was carried out based on the theory of linear photoresponse corresponding to the conventional mechanism of photovoltaic effect in semiconductors which limits the range of applicable solar radiation concentration ratios $K$ by the condition of relatively low charge carriers injection level. It means that the photogenerated charge carriers density shall be low compared to that of majority charge carriers in the semiconductor.

The photovoltaic characteristics of cascade PCs are considered within the model that ignores recombination of charge carriers in the low-doped p-layers structure since the optimal value of p-layers' thickness, as it will be shown, is small compared to the diffusion length of charge carriers in them, while the surface recombination can be eliminated by creating the structure of high potential barriers at the junctions of the base layer with p-type heavily doped $p^+$ and $n^+$ layers. At the same time, in this model, the recombination rate in the tunnel layer ($t$) on the contacts of heavily doped $p^+$- and $n^+$- layers is considered to be very high, so that they are "dead" (i.e. completely nonphotoactiv) despite that they absorb radiation.

The density of photocurrent $J_{\phi i}$ generated in the $i$-th single PC ($i = 1, 2,..., N$) of cascade structure comprising $N$ series-connected PCs under radiation with a spectral quanta flux density distribution of $d\Phi/d\omega$, where $\omega$ is frequency, and a radiation concentration ratio of $K$, with account of the light absorption in the upper layers, is equal to:

$$J_{\phi i} = q \cdot K \cdot \int_0^\infty \frac{d\Phi}{d\omega} \cdot S_i(\omega) \cdot d\omega; \quad i = 1, 2,..., N,$$

(1)

where $q$ is the electron charge, $S_i(\omega)$ is the spectral photoresponse of the $i$-th PC of the cascade that represents the fraction of the incident monochromatic radiation converted into photocurrent, the radiation absorption in the upper layers and on the back junctions of the cascade having been taken into account:

$$S_1(\omega) = e^{-\alpha(\omega) \left( \sum_{i=2}^{N} d_i + 2\delta(N-1) \right)} \cdot Q(\omega);$$

(2)

$$S_i(\omega) = e^{-\alpha(\omega) \left( \sum_{i=1}^{i} d_i + 2\delta(N-i)+\delta \right)} \cdot (1 - e^{-\alpha(\omega)d_i}); \quad i = 2,..., N - 1;$$

(3)

$$S_N(\omega) = e^{-\alpha(\omega)\delta} \cdot (1 - e^{-\alpha(\omega)d_N}).$$

(4)

$\alpha(\omega)$ is the spectral absorption coefficient of photons, $d_i$ is the thickness of photoactive layer of the $i$-th PC, $\delta$ is the thickness of heavily doped ("dead") $p^+$- or $n^+$-layer which is assumed to be the same for all particular PCs in the photovoltaic structure manufactured using a unified technology, $Q(\omega)$ is the spectral charge carriers collection efficiency of the 1-st (basic) PC, dependent on its diffusion, recombination and geometrical parameters. Functions $\alpha(\omega)$ and $Q(\omega)$ have a threshold at a frequency $\omega = \omega_0 = \frac{E_g}{h}$, where $E_g$ is the band gap of the semiconductor and $h$ is Planck constant.

Since for an arbitrary set of values of the thickness of PC-cells in the cascade their spectral photoresponse $S_i(\omega)$ and the integral of the photocurrent $J_{\phi i}$ generally differ the photocurrent $J_{\phi i}$ of the entire cascade shall be equal to the smallest value of photocurrents among all individual PCs ($\min J_{\phi i}, i = 1...N$).

In the optimal operation conditions of the cascade, each cell under illumination shall operate in the optimal point of its current-voltage characteristics. Because the value of the optimum current in modern high-performance PCs is close to that of photocurrent the condition of the maximum power generation is reduced to the equality of all photocurrents within the cascade PC structure [6]. Therefore, the problem of the semiconductor structure optimization with respect to thickness $d_i$ of all cascade PC’s active layers deposited on its base PC (for a fixed $\delta$ value) can be reduced to solving the system of $N$-1 nonlinear equations in $d_i$: 
\[ J_{\phi i} = J_{\phi 1}; \quad i = 2,...,N. \] (5)

Thus, the optimal parameters of semiconductor structures do not depend on the radiation concentration ratio and are determined from the following equations provided that condition \( d_{N+1} = 0 \) is introduced:

\[ \int_{\omega_0}^{\infty} \frac{d\Phi}{d\omega} \cdot S_1(\omega) \cdot d\omega = \int_{\omega_0}^{\infty} \frac{d\Phi}{d\omega} \cdot e^{-\alpha(\omega) \left( \sum_{k=1}^{N} d_k + 2\delta(N-1) \right)} \cdot Q(\omega) \cdot d\omega; \] (6)

\[ \int_{\omega_0}^{\infty} \frac{d\Phi}{d\omega} \cdot e^{-\alpha(\omega) \left( \sum_{k=2}^{N} d_k + 2\delta(N-1) \right)} \cdot \left( e^{-\alpha(\omega) \sum_{k=1}^{N} d_k} - e^{-\alpha(\omega) \sum_{k=1}^{N} d_k} \right) \cdot d\omega = \int_{\omega_0}^{\infty} \frac{d\Phi}{d\omega} \cdot e^{-\alpha(\omega) \left( \sum_{k=2}^{N} d_k + 2\delta(N-1) \right)} \cdot Q(\omega) \cdot d\omega; \] (7)

The solution of these nonlinear equations can be done numerically. It gives the required optimal thickness values of the deposited PCs \( d_i \) \((i = 2, 3,...,N)\), the integral photoresponse \( S \) of the entire optimised cascade:

\[ S = \int_{\omega_0}^{\infty} \frac{d\Phi}{d\omega} \cdot e^{-\alpha(\omega) \left( \sum_{k=2}^{N} d_k + 2\delta(N-1) \right)} \cdot Q(\omega) \cdot d\omega / \int_{\omega_0}^{\infty} \frac{d\Phi}{d\omega} \cdot d\omega, \] (10)

and the photocurrent of cascade PC under concentrated radiation:
where $\Phi_0(E_g)$ is the radiation quanta flux density in the semiconductor’s intrinsic absorption range $\omega \geq \omega_0$.

These expressions present the final general solution of the problem of optimization of parameters of idealized cascade PC based on tunnel-type homogeneous semiconductor structures for arbitrary values of converted radiation spectral density $d\Phi/d\omega$, radiation concentration ratio $K$, spectral absorption coefficient $\alpha(\omega)$ of semiconductor material, spectral charge carriers collection efficiency in the base $Q(\omega)$ and dead layer thickness $\delta$ at the contacts between the PC of the cascade which is a function of electrophysical characteristics of these semiconductor structures.

3. Optimal structure of idealised cascade PC

In the absence of dead layer in the tunnel-type junction condition $\delta = 0$ shall be applied to equations (9), (10). By summing the both sides of equation (9) for $i$ from 2 to $N$ we obtain closed nonlinear equation to determine the optimal total thickness of the deposited PCs:

$$d_t = \sum_{k=2}^{N} d_k$$

(12)

and the corresponding value of the integral photoresponse $S$ of the entire optimised cascade:

$$S = f(\sum_{k=2}^{N} d_k^{(0)}) = f(x_2^{(0)}) = \frac{1}{N}.$$  

(13)

It means that the photoresponse of ideal, highly effective, cascade PC does not dependent on the radiation spectrum to be converted and is equal to the inverse number of PCs in the cascade.

The theoretical top limit of photocurrent for a PC with operating under concentrated radiation can be therefore determined from expression:

$$J_{ph} = q \cdot K \cdot S \cdot \Phi_0(E_g) = q \cdot \frac{K}{N} \cdot \Phi_0(E_g).$$

(14)

i.e. it equals to the upper theoretical value of conventional single-junction planar PC’s photocurrent multiplied by concentration ratio $K$ and divided by the number $N$ of PCs in the cascade.

There are currently no reliable data on existence of dead layers and their thickness $\delta$ in tunnel-type $p^+ (t)n^+$ junctions, as well as on whether or not they depend on homogeneous tunnel-type structures manufacture technologies providing tools for their ionization. It is therefore advisable to use the term of physically limited
cascade structures with finite $\delta$ values and an ideal base PC corresponding to $Q(\omega) = 1$ at any frequency within the operation range of incident radiation. In this case, photoresponse is equal to:

$$S = f \left( \sum_{k=2}^{N} d_k + 2\delta(N-1) \right).$$  \hfill (15)

The solution of this equation system (9) makes it possible to analyze the dependence of the cascade structure’s photovoltaic characteristics on imperfections of electrophysical and optical properties of real tunnel-type junctions.

The most important requirement to implement an effective cascade PC based on real tunnel-type junctions is that of low value of dead layer thickness $\delta$ in order to provide for sufficiently high level of PC’s photoresponse i.e. conditions $\delta << d_i$ ($i = 2, 3, ..., N$) have to be met. The problem of these structures’ parameters optimisation can be reduced to the solution of the common system of equations linearized in relation to $\delta$. Introducing function

$$g(x) = -\frac{df(x)}{dx} = \int_{\epsilon_b}^{\infty} \frac{d\Phi(\omega)}{d\omega} \cdot \alpha(\omega) \cdot e^{-\alpha(\omega)x} \ d\omega \mbox{,}$$

that represents the charge carriers generation rate in the bulk of semiconductor at a distance $x$ from the illuminated surface of the structure per unit of photoactive radiation quanta flux density in the spectral range $\omega \geq \omega_b$, we obtain the resulting expression for calculating the first-order corrections of photoresponse $\Delta S$:

$$\Delta S = -\frac{\delta}{N} \left( g(0) + g(x_2^{(0)}) + 2\sum_{i=3}^{N} g(x_i^{(0)}) \right); \quad N > 2.$$ \hfill (17)

$$\Delta S = -\frac{\delta}{2} \left( g(0) + g(x_2^{(0)}) \right); \quad N = 2.$$ \hfill (18)

$$x_i^{(0)} = \sum_{k=i}^{N} d_k^{(0)}; \quad i = 2, 3, ..., N; \quad x_{N+1}^{(0)} = 0.$$ \hfill (19)

and index (0) indicates values corresponding to $\delta = 0$.

This general expression makes it possible to determine the optimal parameters of the structure of physically limited homogeneous PC cascade of highest efficiency manufactured from real semiconductor materials using different technological methods, as well as their photocurrents under concentrated radiation of any spectral composition.

4. Optimal parameters of the structure and photocurrent of cascade converter under concentrated solar radiation

While studying solar radiation converters the spectrum of black body at temperature $T_S=6000K$ is commonly used as a model one that adequately represents the solar radiation quantum flux density distribution over frequency $\omega$:

$$\frac{d\Phi}{d\omega} = W \cdot \frac{15}{\pi^3} \cdot \frac{\hbar^3}{(kT_s)^4} \cdot \frac{\omega^2}{e^\frac{\hbar\omega}{kT_s} - 1},$$ \hfill (20)

where $W$ is the energy flux density.
Functions \( f(x) \) and \( g(x) \), \( (\mu m^{-1}) \) for various semiconductor materials with spectral absorption coefficients \( \alpha(\omega) \) in the photoactive radiation range \( (\omega \geq \omega_0) \) can be calculated using numerical methods. For semiconductors with indirect optical transitions such as silicon and germanium the following approximation is well applicable for a fairly wide range of solar spectrum:

\[
\alpha(\omega) \approx a(\hbar\omega - \hbar\omega_0)^2,
\]

where \( a = \text{const} \), which yields correct theoretical values near the edge of intrinsic absorption \( (\omega \approx \omega_0) \). For silicon, the value of (21) for \( a \approx 0.5 \, \mu m^{-1} \cdot eV^2 \), \( \hbar\omega_0 = E_g = 1.1 \, eV \) is close to the empirical dependence of \( \alpha \) on radiation wavelength in the range of 0.4 \( \mu m \) to 1.1 \( \mu m \) [9].

Functions \( f(x) \) and \( g(x) \) calculated with account to (20) and (21) are shown in Fig. 1.

As can be seen, \( f(x) \) decreases sharply with growing \( x \) at for small values of \( x < 10 \mu m \) and changes slightly for \( x > 10 \mu m \) at distances of the order of several hundreds of microns. Function \( g(x) \) has the value of 0.6444 \( \mu m^{-1} \) for \( x = 0 \) and decreases with growing \( x \) more sharply than function \( f(x) \), tending to zero for \( x > 50 \mu m \) (to be correct, \( g(x) < 0.0014 \mu m^{-1} \)).

The optimal values of photoresponse \( S^{(0)} \) and those of thickness for individual PCs in ideal cascade silicon structures \( d_i^{(0)} \) (\( \mu m \)) for \( N = 2 \) to 5 calculated for solar radiation spectra using function \( f(x) \) are presented in table 1.

<table>
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<th>( N )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
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<tr>
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<td>1/3</td>
<td>1/4</td>
<td>1/5</td>
</tr>
<tr>
<td>( d_2^{(0)} )</td>
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<td>7.87</td>
<td>14.68</td>
<td>23.45</td>
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<td>( d_4^{(0)} )</td>
<td>0.70</td>
<td>1.26</td>
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<tr>
<td>( d_5^{(0)} )</td>
<td>0.49</td>
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</table>

The theoretical upper limit of photocurrent for cascade PC corresponds to general expression (14) given the above, wherein the radiation quanta flux workspace \( \Phi_0 \) \( (cm^2 \cdot s^{-1}) \) in the silicon operating range is linked with the total energy flow \( W \) \( (W \cdot cm^{-2}) \) by ratio: \( \Phi_0 = 2.268 \cdot 10^{18} \cdot W \).
5. Structure parameters optimisation of cascade PC’s physically limited in implementation

The values of photoresponse $S$ for solar radiation calculated using function $g(x)$, in $\mu$m-1, for cascade silicon PCs comprising $N$ ($N = 2$ to $5$) solar cells of optimized structure and having the highest possible, physically limited, efficiency are presented in Fig. 2 for different values of dead layer thickness $\delta$ at tunnel-type junctions $p^+(t)n^+$. 

![Fig. 2. Top limit photoresponse of cascaded silicon PC with dead layers in tunnel-type junctions under radiation from blackbody with surface temperature equal to that of the Sun (6000K).](image)

The upper curve ($\delta = 0$) corresponds to the value of photoresponse ($S^0 = 1/N$) theoretical limit for all radiation spectra. The other curves correspond to the highest physically attainable values in conditions when the recombination of charge carriers in PC cells of the cascade is entirely determined by the presence of thin non-photoactive layers directly at the tunnel-type contacts within the cells. These values are attainable using standard semiconductor technology being essentially dependent on particular radiation spectrum, which, in this case, is the solar radiation.

It should be noted that linear correction calculated for cell number $N = 2$ to $5$ is approximately equal to $\Delta S \approx -\delta/3$, where $\delta$ is in microns, and decreases slowly with increasing $N$ (within 20% over this range). Therefore, the linear approximation is only applicable for the range $\delta \ll 3/N$ ($\delta$ is in $\mu$m).

This limitation condition becomes more stringent with growing PC number in the cascade. For the cascades considered in this paper (for $N = 2$ to $5$), this condition is practically met when $\delta \leq 0.3\mu$m.

6. An upper limit efficiency of cascade converters of concentrated radiation

6.1. General theory of upper limit efficiency

The most important energy parameter of concentrated radiation converters is their upper limit efficiency. In the previous sections, we considered the theoretical upper limit efficiency of cascade PC that is determined by only fundamental laws of physics and the physically limited efficiency that takes the reduction of PC performance into account due to the inevitable technologically conditioned losses of generated charge carriers at the contacts of cascade PC structures.

Thus, the best realisable voltage-current characteristic (dependence of current density $J$ on total voltage $U$ in the system) of a cascade PC represents the performance of $N$ series-connected PC cells with optimized cascade semiconductor structure, the ideal current-voltage characteristic of each $p-n$ junction, obtained upper limit...
photocurrent $J_\phi$ in the cell and the lowest possible value of reverse saturation dark current $J_{0r}$ determined by the fundamental mechanism of its generation, i.e. charge carriers diffusion exposed to only radiative recombination [6]:

$$U = \frac{N \cdot kT}{q} \ln \left( \frac{J_\phi - J}{J_{0r} + 1} \right).$$  \hspace{1cm} (22)

The upper limit value of photocurrent $J_\phi$ for optimized cascade PC is given by (11) while the upper limit value of photovoltage of cascade PC equals to:

$$U_{oc} = \frac{N \cdot kT}{q} \ln \left( K \cdot S \cdot \frac{q \cdot \Phi_0(E_g)}{J_{0r}} + 1 \right).$$  \hspace{1cm} (23)

It grows logarithmically with increasing radiation concentration ratio $K$ and, weaker than linearly, with the increasing number $N$ of PCs in the cascade.

Assuming that $K \cdot S \cdot q \cdot \Phi_0(E_g) >> J_{0r}$ we obtain three members of the asymptotic series in the expression for the efficiency of cascade PC (see [6]):

$$\eta_{ct} = kT \cdot \frac{\Phi_0(E_g)}{W} \cdot \left\{ \ln \left( K \cdot S \cdot \frac{q \cdot \Phi_0(E_g)}{J_{0r}} \right) - \ln \left( K \cdot S \cdot \frac{q \cdot \Phi_0(E_g)}{J_{0r}} - 1 \right) \right\}. \hspace{1cm} (24)$$

If we use the expressions for the theoretical efficiency limit of conventional planar solar radiation converter with $p$-$n$ junction $\eta [6]$ and that for the upper limit share of solar energy utilization in single $p$-$n$ junction PC $\eta_0 [7]$:

$$\eta_0 = \frac{E_g \Phi_0(E_g)}{W},$$  \hspace{1cm} (25)

factually corresponding to the correct solution of the original problem for the upper theoretic efficiency limit for $K \to \infty$, obtained expression (24) for the upper limit efficiency of the cascade PC for various concentrations, in the approximation linear photoresponse, can be written in form:

$$\eta_{ct} = \eta_c + \eta_0 \frac{kT}{E_g} \cdot \left\{ \ln(K \cdot S) - \ln \left( \frac{\eta_0 kT}{\eta_c E_g} \ln(K \cdot S + 1) \right) \right\}. \hspace{1cm} (26)$$

7. **Theoretical top limit efficiency of cascade PC**

The theoretical top limit efficiency of cascade PC, $\eta_{ct}^{(0)}$, corresponding to $S = S^{(0)} = 1/N$, grows approximately logarithmically with increasing solar radiation concentration ratio $K$. At the same time, it decreases logarithmically with increasing number N of PC in the cascade, i.e. with the growing PC’s total voltage. Particularly, for the single solar flux density ($K = 1$) it is lower than the maximum theoretical efficiency of conventional PC having only one $p$-$n$ junction $\eta_c$.

For silicon PCs: $\eta_c = 0.26$, $\eta_0 = 0.44$ and $kT = 25$ mV the dependence of the theoretical upper limit efficiency of the cascade under solar radiation with concentration ratio $K$ for various values of the number of PC cells $N$ in the cascade (for $N = 1$ to 5) is presented in Fig. 3 (a).
As can be seen, for the highest solar radiation concentration ratio values (about 100) practically applicable to silicon PCs the theoretical upper limit efficiency of cascade exceeds the that of conventional PC under normal solar radiation ($\eta_t = 26.0\%$ for $N = 1$) by approx. 4.4%, 3.4% and 2.9% (absolute values) for $N = 1$, $N = 3$ and $N = 5$, respectively. At the same time, the highest possible photocurrent is $N$ times lower compared to that of conventional planar PC which meets the condition of low light-induced charge carriers injection level and proves the validity of linear photoresponse model applicability to cascade PCs operating under concentrated solar radiation.

7.1. Physical top limit efficiency of cascade PC

The physical upper limit efficiency of silicon cascade PC depending on the solar radiation concentration ratio $K$ is presented in Fig. 3 (b) for various values of the number $N$ of PC cells in a cascade ($N = 2$ to 5) and for different values of the dead layer thickness $\delta$ (in $\mu$m) arising in tunnel-type junctions of the PC semiconductor structure, in the linear approximation over $\delta$. It grows logarithmically with the increasing solar radiation concentration ratio $K$ and decreases logarithmically with the increasing number $N$ of PCs in the cascade.

It is clear from the data presented in Fig. 3 that the decrease in maximum efficiency of cascade with the dead layer thickness by $\Delta\delta \sim 0.1\mu$m is $\Delta\eta_t \approx 0.1\%$ to $0.2\%$ in absolute units (percentage), while for high concentration ratios ($K > 10$) the efficiency does not depend on the concentration ratio and increases weakly with the number of PC cells in the cascade.

8. Conclusion

The use concentrated radiation is an effective tool to improve such parameters as photocurrent, open circuit voltage and efficiency of cascade homogeneous PCs.

The linearity range of photoresponse of cascade structures related to the radiation concentration ratio $K$ tends to an approx. $N$ times expansion with the number $N$ of individual PCs in the cascade compared to conventional single-junction PC.

Cascade PC comprising a large number of PC cells are more advantageous compared to conventional planar PCs, especially when concentrated radiation is used to reduce the internal electric power loss in PC.
The optimisation of cascade PC semiconductor structure can be performed using a universal function representing the normalized distribution of the integral of the radiation quanta flux over the distance from the illuminated surface of the structure.

The open circuit voltage grows weaker than linearly with $N$ while the upper limit efficiency monotonically decreases, nearly proportionally to $\ln N$.

The highest attainable values of open circuit voltage and efficiency of cascade PCs monotonically grow, approximately proportionally to $\ln K$, with radiation concentration ratio $K$ for fixed PC number in the cascade $N$.

In the linear approximation of photoresponse, the optimal values of individual PCs thickness in the cascade do not depend on the radiation concentration ratio, and for silicon structures having $N \leq 5$ under normal solar radiation spectrum they are of the order of 1$\mu$m to 10$\mu$m which proves the applicability of the presented model of semiconductor PC structures, as well as the possibility of such structures manufacture using modern technological methods.

References