International Conference On Applied Economics (ICOAE) 2014

Volatility skews of indexes of BRICS securities exchanges

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Abstract

The shapes of volatility skews of an index on a securities exchange can describe the volatility and liquidity of a local market. However the volatility skews of various exchanges are not made public and as a result alternative means need to be employed to compare indexes on different securities exchanges. In this paper we use a method used to obtain the volatility skew of an index which only requires the return time series of the index and the country’s central bank rate. The BRICS countries which consist of Brazil, Russia, India, China and South Africa are similar in certain attributes, i.e. political and trade aims. They do however differ in macro-economic factors like gross domestic product (GDP) and inflation. We compare the volatility skews of the major indexes of BRICS securities exchanges. South Africa, Brazil and Turkey are three emerging economies. A comparison of the volatility skews of indexes on their securities exchanges is also made.

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Selection and/or peer-review under responsibility of the Organizing Committee of ICOAE 2014

Keywords: Borsa 100 index; BRICS; CSI 300; FTSE/JSE Top 40; index; IBrX; INDEXCF; JSE; option pricing; securities exchange; S&P BSE SENSEX; volatility skew.

1. Introduction

Option pricing models depend on various parameters to yield a price for an option. These parameters may vary for different kinds of options and different types of models, but the key parameter in pricing options is the volatility of the underlying security.

Volatility is a measure of the uncertainty of the return realized on an asset, while implied volatility is the volatility of the price of the underlying security that is implied by the market price of the option-based-on-an-option pricing model. The curve that is obtained when plotting implied volatility of an option versus the strike price of that option is called a volatility skew. According to the classic theory of Black, Scholes and Merton (BSM), the BSM implied volatility of an option is independent of its strike, yielding a flat volatility skew.

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Volatility skews became more prominent after the financial crash of October 1987. The crash prompted a greater demand for risk protection. This resulted in an increase in the price of put options (and in particular for deep out-the-money put options), and an increase in selling of deep out-the-money call options. Consequently, there was an increased demand for out-the-money put options and an increased supply of out-the-money call options, which lead to more prominent volatility skews.

Volatility skews are indicators of supply and demand for in-the-money and out-the-money options. They provide information on whether fund managers prefer to write call or put options. Volatility skews are also used for pricing of options.

Exchanges use different methods to determine their volatility skews. The Johannesburg Securities Exchange (JSE), for example, uses a market-to-model approach and generate volatility surfaces using models. If the implied volatility is plotted as a function of both strike price and time to maturity, the resulting three-dimensional surface is called the volatility surface. When a fixed maturity is considered, the volatility skew pertaining to that maturity, may then be obtained from the volatility surface. However, there are other exchanges that use a mark-to-market approach and do not use a model to obtain a volatility surfaces, but use market makers (see Kotzé et al. [10]).

The volatility skew data that is used by the JSE, is published on the website of the JSE (see [8]). This transparency makes it convenient for academics, financial institutions and other stakeholders to use this information to (for example) generate prices of options and derivatives pertaining to the South African markets. The exact method that the JSE uses to construct the FTSE/JSE Top 40 volatility skew is not made public.

There are other securities exchanges in the world where the volatility skew data used by those exchanges is not published. The volatility skew data is therefore not freely available and stakeholders need to resort to models to generate volatility skews pertaining to an underlying on such exchanges. An example such a securities exchange is the Stock Exchange of Thailand (SET).

In view of the fact that different exchanges use different methods to obtain their volatility skew data and some exchanges publish this data, while others do not, the question that arises is: how does one compare the volatility skews of (for example) an index on an exchange with the volatility skew of an index on another exchange? Addressing this question is the aim of this paper.

The only way that this could be addressed in a meaningful manner, is if we generate the volatility skew on each exchange using the same model. We opt for a non-parametric model that has its origins in Stutzer [11], Zou & Derman [12], Duan [5] and Araújo & Maré [4]. In this model, an empirical distribution is constructed from a sample of the underlying asset prices. Then a risk-neutral distribution is obtained from the constructed empirical distribution, subject to the condition that the expected return equals the risk-free rate, by using the relative entropy principle. The risk-neutral distribution is then used to determine the implied volatility and the volatility skews are generated.

This method is implemented to generate the volatility skew of the FTSE/JSE Top 40 index of the JSE. This volatility skew is benchmarked against that produced by using the volatility skew data published by the JSE.

BRICS is an acronym for an association of five developing or newly industrialised countries. These countries are Brazil, Russia, India, China and South Africa. The latter joined the association in 2010, which was known as BRIC prior to 2010. The BRICS countries are considered to become a powerful economic block and it is expected that the BRICS countries will become the world’s dominant suppliers of goods, services and raw materials. In 2013 the BRICS countries represented nearly half the population of the world and includes the Chinese economy which is the second most productive economy in the world and is the fastest growing consumer market.

In this paper we compare the following indexes of the BRICS securities exchanges:

- IBrX index - top 100 stocks traded on the Bovespa, Brazil.
- INDEXCF index - 50 most liquid Russian stocks on the Moscow exchange.
- S&P BSE Sensex index - is a cap weighted index on the Bombay Stock Exchange, India.
- CSI 300 index - which consists of 300 A-shares stocks listed on the Shanghai or Shenzhen Stock Exchange, China.
- FTSE/JSE Top 40 index - it includes the 40 largest companies by market capitalization on the JSE, South Africa.

Table 3 tabulates the central bank rates and the current index level for the various BRICS indexes on the 27 March 2014 (see [7, 8]).

Financial turbulence continues to surround the emerging markets. A further comparison is made of three emerging market economies which are Brazil, Turkey and South Africa.
Table 1. The central bank rate for the various BRICS countries (Source: [7, 8])

<table>
<thead>
<tr>
<th>BRICS nation</th>
<th>Central bank rate</th>
<th>Index</th>
<th>Current index level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>11%</td>
<td>IBrX</td>
<td>20587</td>
</tr>
<tr>
<td>Russia</td>
<td>7%</td>
<td>INDEXCF</td>
<td>1331</td>
</tr>
<tr>
<td>India</td>
<td>8%</td>
<td>S&amp;P BSE Sensex</td>
<td>22214</td>
</tr>
<tr>
<td>China</td>
<td>6%</td>
<td>CSI 300</td>
<td>2155</td>
</tr>
<tr>
<td>South Africa</td>
<td>5.5%</td>
<td>FTSE/JSE Top 40</td>
<td>43060</td>
</tr>
</tbody>
</table>

The paper is organised as follows. In section 2, the model that is used to construct the volatility skews, is presented. We implement the model, in section 3, by using observed data for indexes including the FTSE/JSE Top 40 index. This skew is benchmarked against the corresponding volatility skews published by the JSE. In section 4, we use the model to generate volatility skews for the main indexes on the exchanges of the BRICS countries and compare these volatility skews to the FTSE/JSE Top 40, the main index of the JSE. In section 5, we compare the volatility skews of the IBrX index, the FTSE/JSE Top 40 index and the BIST 100 index, which is a capitalization-weighted index of the Turkish stock exchange composed of national market companies except investment trusts. Section 6 concludes.

2. The model used

First, we determine a distribution from suitable historical data, which is our real-world distribution:
- Get the daily index history for relevant option maturity $T$.
- Choose $M$ daily index prices for calculations to be made.
- Find the historical returns, using
  \[
  \text{return}_i = \frac{\text{index}_{i+T}}{\text{index}_i} \quad \text{for } i = 1, \ldots, M - T.
  \]
- Construct the historical return distribution:
  - Split the returns into equal bins and calculate the cumulative sum of the number of observations that lie in each bin.
  - Divide by the total number of observations in the sample and obtain the cumulative distribution function (CDF).
  - From the CDF, obtain the probability density function (pdf), which is the historical return distribution $P$.

2.1. Volatility skew for an index

Next, we seek the risk-neutral historical probability distribution (RNHD) $Q$. From the work of Stutzer [11], Zou & Derman [12], Duan [5] and Araújo & Marê [4] already mentioned, it is known that, by using the relative entropy principle, there is a distribution $Q$ such that $E_Q(S_T) = S_0 \exp(rT)$ and

\[
E_Q \left[ \log \left( \frac{Q(x)}{P(x)} \right) \right] = \min \left\{ E_R \left[ \log \left( \frac{R(x)}{P(x)} \right) \right] : R \text{ is a distribution and } E_R(S_T) = S_0 \exp(rT) \right\},
\]

where $S_0$ is the spot price of the index, $S_T$ is the price at maturity $T$ and $r$ is the risk-free rate. In fact, from the theory of Lagrange multipliers, it is known that (see Cover and Thomas [3]) that there exists $\lambda$ (which depends on the constant $S_0 \exp(rT)$) such that

\[
Q(x) = \frac{P(x) \exp(\lambda x)}{\int_{-\infty}^{\infty} P(x) \exp(\lambda x) dx}.
\]

- The $\lambda$ parameter can be determined numerically.
- Using the $Q$-measure obtained from the RNHD model, we find the price of an option by discounting the expected future payoff, under the $Q$-measure. If $C_t$ is the price at time $t$ of an option with maturity $T$, then
  \[
  C_0 = e^{-rT} E_Q \left[ C_T \right],
  \]
which, in the case of a call option with strike \( K \), can be written as

\[
C_0 = \sum_x Q(x) \max\{S_0 e^{rT} \text{return}_x - K, 0\}.
\]

- From the option prices, we determine the implied volatility by finding the volatility term in the BSM formula,

\[
C_0 = \alpha \left[ S_0 N(\alpha d_1) - Ke^{-rT} N(\alpha d_2) \right],
\]

where

\[
d_1 = \frac{\log \frac{S_0}{K} + \left( r + \frac{1}{2} \sigma^2 \right) T}{\sigma \sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma \sqrt{T},
\]

\( N() \) is the cumulative distribution function (CDF) of a standard normal distribution, and

\[
\alpha = \begin{cases} 
1 & \text{for a call} \\
-1 & \text{for a put} 
\end{cases}
\]

- Applying this process to a series of strike prices allows us to compute the implied volatility skew under the \( Q \)-measure. We refer to the model obtained in this way as the RNHD model.

3. Comparing the RNHD volatility skew to the market implied volatility skew

In this section and all subsequent sections, we consider a three month option that starts on 27 March 2014 and matures on 19 June 2014.

We benchmark the generated volatility skew, using the model presented in section 2, against that obtained from the volatility skew data published by the JSE. In this analysis we show the plausibility of the model proposed by this paper. We use the RNHD to price the same set of options on the FTSE/JSE Top 40 index, over a range of different strike prices and compare this to the option prices obtained from the volatility skew obtained from the JSE.

![Market implied- versus RNHD implied volatility skew](image)

Fig. 1. The volatility skew published by the JSE versus the volatility skew implied by the RNHD, plotted against the moneyness for options on the FTSE/JSE Top 40 index from 27 March 2014 with expiry 19 June 2014, using 1853 historical return dates for the RNHD model ranging from 01 March 2009 to 27 March 2014, a spot price of 43060 and interest rates of 5.5%.
In Fig. 1 we plot the RNHD implied volatility against the Market implied volatility published by the JSE. The moneyness is calculated by dividing the strike price by the current future price. As the moneyness decreases or increases, our approximation of the implied volatility skew becomes less accurate. Options in the South African market rarely trade below 85% or above 115% moneyness, so that we could use the RNHD skew as a good approximation to the actual market skew.

Table 2 compares the volatility skew published by the JSE to the volatility skew obtained through the RNHD model, over a range of strike prices.

Table 2. The implied volatility of the JSE and the implied volatility of the RNHD model, with spot price 43060, for options on the FTSE/JSE Top 40 index from 27-March-2014 with maturity 19 June 2014, using 1853 historical data points ranging from 01 March 2009 to 27 March 2014, using an interest rate of 5.5%.

<table>
<thead>
<tr>
<th>Strike price</th>
<th>JSE implied volatility</th>
<th>RNHD implied volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>39000</td>
<td>0.2031</td>
<td>0.3049</td>
</tr>
<tr>
<td>39722</td>
<td>0.1969</td>
<td>0.2724</td>
</tr>
<tr>
<td>40444</td>
<td>0.1907</td>
<td>0.2401</td>
</tr>
<tr>
<td>41167</td>
<td>0.1846</td>
<td>0.2105</td>
</tr>
<tr>
<td>41889</td>
<td>0.1788</td>
<td>0.1858</td>
</tr>
<tr>
<td>42611</td>
<td>0.1730</td>
<td>0.1651</td>
</tr>
<tr>
<td>43333</td>
<td>0.1673</td>
<td>0.1483</td>
</tr>
<tr>
<td>44056</td>
<td>0.1619</td>
<td>0.1348</td>
</tr>
<tr>
<td>44778</td>
<td>0.1566</td>
<td>0.1261</td>
</tr>
<tr>
<td>45500</td>
<td>0.1512</td>
<td>0.1197</td>
</tr>
</tbody>
</table>

Volatility skews are generally used to price derivatives on the underlying. In order to test how accurate or plausible the RNHD volatility skew is to be used as an estimate for the Market implied volatility, we price a call option on the underlying index. In order to simplify matters, we do not consider dividends paid by various stocks within the index.

Volatility skews are generally used to price derivatives on the underlying. In order to test how accurate or plausible the RNHD volatility skew is to be used as an estimate for the Market implied volatility, we price a call option on the underlying index. In order to simplify matters, we do not consider dividends paid by various stocks within the index.

![RNHD implied volatility versus Market implied price](image_url)

Fig. 2. Option prices calculated using the volatility skew published by the JSE versus the volatility skew implied by the RNHD, plotted against the moneyness for call options on the FTSE/JSE Top 40 index from 27 March 2014, with expiry 19 June 2014, using 1853 historical return dates for the RNHD model ranging from 01 March 2009 to 27 March 2014, and an interest rate of 5.5%.

In Fig. 2 we price the same European call option using the two different volatility skews. Fig. 2 conveys similar
information to Fig. 1, however in Fig. 2 the difference between the two volatility skews are represented in the difference in the price of an option. The prices of the two options are similar at a moneyness ratio of 1 and differs the most at far in-the-money moneyness levels.

The results are similar to those of De Araújo and Maré [4].

The RNHD volatility skew was generated from historical data and may contain information regarding market behaviour that is not reflected in the volatility skew of the JSE, which is obtained using a different technique. The volatility skews and the application to pricing are expected to be different, but similar in shape, which shows that the RNHD skew is a good approximation to the market implied skew. This is confirmed by the findings.

The interest rate used in this comparison is the South African Reserve Bank (SARB) repo rate. The repo rate is the rate at which a central bank repurchases government securities from the commercial banks, depending on the level of money supply it decides to maintain in the country’s monetary system.

4. Comparing the volatility skews of the major indexes of the BRICS nations’ Securities Exchanges

In this section we compare the major indexes of the five BRICS nations volatility skews obtained using the RNHD model. Using the model presented in section 2, we plot in Fig. 3 the volatility skews of the various indexes, as mentioned in section 1, of the BRICS countries. The central bank rate of a given country is used as a proxy for the risk-free rate pertaining to that country when implementing the model. The data used was from 01 March 2009 to 27 March 2014. This period does not contain the peak of the 2008 financial crisis, i.e. the default of Lehman Brothers.

Fig. 3 shows that the volatility skews of the Russian INDEXCF and the Chinese CSI 300 indexes are almost identical. These indexes also have the flattest skew, which could indicate greater liquidity in the equity market compared to the equity markets of the other BRICS countries.

The Brazilian IBrX, the Indian S&P BSE SENSEX and the South African FTSE/JSE Top 40 skews lie almost perfectly parallel. They noticeably have a steeper skew than the Russian or Chinese indexes, which could possibly indicate that South Africa, Brazil and India have the least liquid equity market of all of the BRICS nations.

Fig. 3. The BRICS countries’ volatility skews obtained from the RNHD model, on the IBrX (Brazil, $S_0 = 20587.8$, $r = 0.11$), INDEXCF (Russia, $S_0 = 1331.9$, $r = 0.07$), S&P BSE SENSEX (India, $S_0 = 22214.4$, $r = 0.08$), CSI 300 (China, $S_0 = 2155.7$, $r = 0.06$) and FTSE/JSE Top 40 (South Africa, $S_0 = 43060$, $r = 0.055$) indexes for options on the 27 March 2014 with expiry 19 June 2014, using 1853 historical return dates for the RNHD model ranging from 01 March 2009 to 27 March 2014.

Since options trade on volatility, and a higher volatility implies a higher price for the option, a steeper skew means
that the option prices are more expensive when traded further out-of- and in-the-money indicates that the supply for such option is less, and therefore the equity market appears to be less liquid.

Upon further investigation we noticed that the RNHD volatility skew is highly sensitive to the underlying interest rate. Fig. 3 can not be further interpreted without considering the underlying interest rate of each of the countries.

![CSI 300 index volatility skew over different interest rates](image)

Fig. 4. The RNHD skew for options on the CSI 300 index (China, \(S_0 = 2155.7\), where current interest rates lie at 6%), for different interest rates. The option begins on 27 March 2014 with expiry 19 June 2014, using 1853 historical return dates for the RNHD model ranging from 01 March 2009 to 27 March 2014.

We show how a change in the interest rate will shift the volatility skew. Fig. 4 shows the effect of different interest rates on the CSI 300 index, China.

As the interest rates increase (decrease) the volatility skews shift upwards (downwards) in an almost parallel fashion. Table 3 indicates the sensitivity of the ‘at-the-money’ RNHD implied volatility as interest rates change. An increase (decrease) in the current interest rate increases (decreases) the ‘at-the-money’ RNHD implied volatility.

Table 3. The change in the ‘at-the-money’ RNHD implied volatility of the Chinese CSI 300 index as interest rates change, for options from beginning 27 March 2014 with maturity 19-June-2014, using 1853 historical data points ranging from 01 March 2009 to 27 March 2014. Current Chinese interest rates lie at 6%.

<table>
<thead>
<tr>
<th>Interest rate</th>
<th>‘At-the-money’ RNHD implied volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>0.1797</td>
</tr>
<tr>
<td>4%</td>
<td>0.2042</td>
</tr>
<tr>
<td>7%</td>
<td>0.2331</td>
</tr>
<tr>
<td>10%</td>
<td>0.2665</td>
</tr>
</tbody>
</table>

Brazil currently has the highest interest rate from all the BRICS members, hence its volatility skew generally lies above those of the other countries. The shift in India’s, Brazil’s and South Africa’s almost perfectly parallel volatility skews can be explained by the difference in interest rates. The skews also indicate that Russia’s and China’s interest rates are very similar. India’s interest rate is the second highest interest rate from all the BRICS countries, but generally has the lowest volatility skew. This could indicate that India has a less volatile equity market than the other BRICS countries.

Both the current interest rates and the selection of historical data influence the shape of the RNHD skew. In Fig. 5, we illustrate how the historical data selection influences the shape of the RNHD skew. For the skews in Fig. 5, we included data from the market crash in 2008. The skews are generally higher than the skews observed in Fig. 3, which
we obtained by excluding the market crash historical data. Because options trade on volatility, and a higher volatility implies a higher option price, we observe that the RNHD skew implies higher option prices when we include data from the 2008 crash. This is an accurate reflection of the market situation in 2008, but care has to be taken when selecting the historical data.

The 2008 market crash affected all the indexes under consideration. Equity markets had higher volatility for in-the-money calls/out-the-money puts. This is a result of market makers not writing out-the-money puts/in-the-money calls. As a result, there was an increase in the price of those options. It appears that Russia was affected the most of the BRICS countries. The volatility skews for certain indexes are now more conventional smiles. This may be attributed to a lower demand for in-the-money calls and out-the-money puts.

![BRICS volatility skews](image)

Fig. 5. The BRICS countries’ volatility skew obtained from the RNHD method for the same countries and parameters as in Fig. 3, but including historical data from the market crash in 2008. The historical data ranges from 15 March 2008 to 27 March 2014. We leave the South African skew as it was in Fig. 3 for comparison.

5. Comparing the volatility skews of the Brazilian IBrX index, the South African FTSE/JSE Top 40 index and the Turkish Borsa 100 index

The economies of Brazil, South Africa and Turkey, which are emerging economies, are considered to be similar in nature. In 2013 the announcement of the Federal reserve tapering off of their Quantitative Easing (QE) led to emerging market countries subsequently experiencing a sharp reversals of capital inflows, resulting in sizable currency depreciation (see [6]). Also as a result of this tapering, emerging markets experienced an increase in volatility in their financial markets. The data set used in this study includes this financial turbulence experienced by emerging markets.

In Fig. 6 we compare the volatility skews of the major indexes of these three emerging economies.

The volatility skews have similar shapes, but Turkey’s skew is flatter than the other two. The parallel shift between the South African volatility skew and that of Brazil and Turkey can be explained by the significantly lower interest rate in South Africa relative to the other countries.
Emerging markets volatility skew

Fig. 6. The volatility skews obtained through the RNHD from emerging markets represented by the IBrX (Brazil, $S_0 = 20587.8$, $r = 0.11$), Borsa 100 (Turkey, $S_0 = 67048.4$, $r = 0.1$), and FTSE/JSE Top 40 (South Africa, $S_0 = 43060.9$, $r = 0.055$) indexes, for options beginning on 27 March 2014 with expiry 19 June 2014, using 1853 historical return dates for the RNHD ranging from 01 March 2009 to 27 March 2014.

6. Conclusion

We used a risk-neutral historic data distribution model to generate volatility skews for certain indexes. This model is non-parametric and suitable to compare volatility skews across different securities exchanges. We compared the generated volatility skew of the FTSE/JSE Top 40 index with that generated using published data from the JSE and priced options with the two different skews. The prices were approximately the same for both skews, which shows that the RNHD method is a viable method to construct volatility skews. We compared volatility skews of the major indexes on the BRICS exchanges, as well as the emerging countries of South Africa, Brazil and Turkey, using the RNHD model, which allowed us to compare economies across different countries. The RNHD model is sensitive to both the current interest rates and the historical data selection, which might be a drawback of this method. However, the model provides a convenient and meaningful tool to compare different economies for which we cannot obtain current volatility skews.

Disclaimer

This paper represents the views of Chadd Hunzinger alone and not the views of Rand Merchant Bank.

Acknowledgements

The authors would like to thank Antonie Kotzé, Eben Maré, Niël Oberholzer and Theresa Offwood-Le Roux for many interesting discussions and helpful remarks.

The research contribution of the second named author was funded by the National Research Foundation, Grant Number 87502.

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