Propagation of liquid surface waves over finite graphene structured arrays of cylinders

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Abstract Based on the multiple scattering method, this paper investigates a benchmark problem of the propagation of liquid surface waves over finite graphene (or honeycomb) structured arrays of cylinders. Comparing the graphene structured array with the square structured and with triangle structured arrays, it finds that the finite graphene structure can produce more complete band gaps than the other finite structures, and the finite graphene structure has less localized ability than the other finite structures.

Keywords propagation, liquid surface wave, graphene structured array, multiple scattering method

When a light wave or acoustic wave propagates inside a periodic structure, it is modulated with a periodic structure. The corresponding lattice structure is called a photonic crystal or a sonic crystal. Similarly, once a liquid surface wave propagates over a periodic structure, due to multiple Bragg scatterings, it is also modulated by the introduced periodicity. As a result, many interesting phenomena found in photonic crystals or sonic crystals may also exist in liquid surface waves. However, a unique advantage of using liquid surface waves lies in the fact that many interesting phenomena can be observed visually. Because of the above reasons, liquid surface waves propagating over periodically and randomly structured bottoms are both investigated by theories and experiments. Some important phenomena on liquid surface waves, such as the complete band gap, negative refraction, and localization are found.

Graphene, a two dimensional honeycomb structured material, was discovered by Geim’s team in 2004. Later, it is found that when electronic waves propagating inside graphene an unusual propagation pattern appears. For a liquid surface wave being one kind of classical waves, what is the propagation pattern when it propagates inside a graphene (or honeycomb) structure? In 2002, Ha et al. investigated the propagation of water surface waves through a photonic crystal or sonic crystal. Comparing the graphene structured array with the square structured and with triangle structured arrays, it finds that the finite graphene structure can produce more complete band gaps than the other finite structures, and the finite graphene structure has less localized ability than the other finite structures.

Making the calculational results experimentally testable, we pick a specific liquid which possesses capillary length $b = 0.93$ mm and is often used in experiments. We take the depth of liquid $b = 2.5$ mm, the radius of each cylinder $a = 0.875$ mm, and the filling fraction of three kinds of arrays $f_s = 0.296$ mm. The lattice constant of arrays $d$ has different values in different structures, and can be acquired by the radius of cylinder and the filling fraction (e.g. the lattice constant of graphene structure $d = 2.5$ mm can be gotten from $f_s = 4\pi a^2/(3\sqrt{3}d^2) = 0.296$ and $a = 0.875$ mm).

Before calculation, let us present some conclusive equations which are used in our calculation. These conclusive equations have been derived rigorously in Ref. 6 from the multiple scattering theory. If $x$-$y$ is set in the calm liquid surface and $z$ is set to the vertical and upward axis, the displacement of liquid surface waves at any point $\eta(r)$ can be expressed as

$$\eta(r) = i\pi H_0^{(1)}(k|r-r_s|) + \sum_{i=1}^{N} \sum_{n=-\infty}^{\infty} i\pi A_n^{i} H_n^{(1)}(k|r-r_s|)e^{i\phi_{r-r_s}},$$

where $n$ and $i$ are the indices of order and cylinders respectively, $N$ is the number of all cylinders, $r_s$ denotes the place of the stimulating source, $\phi_{r-r_s}$ is the azimuthal angle of the vector $r-r_s$ relative to the positive $x$ axis, $H_0^{(1)}$ and $H_n^{(1)}$ are the zeroth order and the

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nth order Hankel functions of the first kind respectively, \( k \) denotes the wave number of liquid surface waves over the bottom and satisfies

\[
\omega^2 = gk \left( 1 + \frac{\sigma}{\rho g} k^2 \right) \tanh(kh),
\]

where \( \omega \) is the angular frequency, \( g \) is the gravity acceleration, \( \sigma \) is the liquid surface tension, \( \rho \) is the liquid density, \( h \) is the depth of liquid over the bottom, \( \sqrt{\sigma/\rho g} \) denotes the capillary length of liquid \( b \). Equation (2) is also suitable for the wave number over the \( i \)th cylindrical step \( k_i \) and the depth of liquid over the \( i \)th cylindrical step \( h_i \). In Eq. (1), the coefficient \( A_n^i \) can be obtained from the matrix equation

\[
\Gamma_n^i A_n^i - \sum_{j=1,j\neq i}^{N} \sum_{i=-\infty}^{\infty} G_{i,n}^{i,j} A_j^i = T_n^i,
\]

where

\[
\Gamma_n = \frac{H_n^{(1)}(ka) J_n^{(1)}(ka) - \beta H_n^{(1)}(ka) J_n^{(1)}(ka)}{\beta J_n^{(1)}(ka) J_n^{(1)}(ka) - J_n(ka) J_n^{(1)}(ka)},
\]

and

\[
T_n^i = H_n^{(1)}(k|r_i - r_s|) e^{-i n \phi_{r_i,r_s}},
\]

and

\[
G_{i,n}^{i,j} = H_n^{(1)}(k|r_i - r_j|) e^{i (l-n) \phi_{r_i,r_j}}, i \neq j.
\]

In Eqs. (3)–(6), \( \beta = \tanh(kh)/\tanh(k_i h_i) \), \( i \) and \( j \) are also the indices of order and cylinders, respectively, \( a \) is the radius of each cylinder, \( J_n(ka) \) is the Bessel’s function of the first kind, the prime refers to the derivative.

Here, according to the conclusion equations above, we calculate the transmission spectra of the finite graphene structured, square structured and triangle structured arrays. Our calculation is along the \( \Gamma M \) and \( \Gamma K \) directions. The \( \Gamma M \) and \( \Gamma K \) directions of the graphene, square and triangle structures are shown in Fig. 1. Based on the transmission spectra along the \( \Gamma M \) and \( \Gamma K \) directions, we can show the different band gaps of three kinds of structures.

We arrange the cylinders in an about 6.3 cm \( \times \) 6.5 cm rectangular area which has the graphene structure or the square structure or the triangle structure, and set the \( \Gamma M \) or \( \Gamma K \) direction of the structure to parallel the short side of rectangle. The stimulating source is set in the centric place about one lattice constant away from one long side of rectangle, whereas the detector is placed in the centric place about one lattice constant away from the other long side of rectangle.

The numerical results are shown in Fig. 2. In these numerical results, the transmission spectra are normalized, and expressed as \( \ln |T|^2 = \ln |\eta_1/\eta_2|^2 \), \( \eta_0 = i\pi H_0^{(1)}(k|r - r_s|) \). In Fig. 2, when the depth of liquid over the steps \( h_i \) equals 0 mm, that is to say, the top of the steps is on a level with liquid surface (see the left column in Fig. 2), for the finite graphene structured array, the normalized transmission spectra along the \( \Gamma M \) and \( \Gamma K \) directions both have two troughs which can inoscillate well, which indicates that the liquid surface waves have two complete band gaps. The two complete band gaps locate at the region of frequency about between 15 Hz and 20 Hz and the region about between 32.5 Hz and 36 Hz, respectively (see Fig. 2(a1)). While for the finite square structured and triangle structured arrays, the normalized transmission spectra along the \( \Gamma M \) and \( \Gamma K \) directions do not have the trough, which means that the liquid surface waves do not have the complete band gap (see Figs. 2(b1) and 2(c1)).

When \( h_i \) equals 0.01 mm (see the middle column in Fig. 2), for the finite graphene structured array, the transmission spectra along the \( \Gamma M \) and \( \Gamma K \) directions both have three troughs inoscillating very well, which means that the liquid surface waves have three complete band gaps. The three complete band gaps locate the region of frequency about between 11.5 Hz and 14 Hz, the region about between 14.4 Hz and 20.4 Hz and the region about between 22.5 Hz and 25.5 Hz, respectively (see Fig. 2(a2)). As for the finite square and triangle structured arrays, the normalized transmission spectra show only one complete band gap. What is more, the complete band gaps in the finite square and triangle structured arrays locate the same region about between 11.4 Hz and 14.5 Hz (see Figs. 2(b2) and 2(c2)), and the region nearly coincides with first gap region in the finite graphene structured array.

When \( h_i \) equals 0.1 mm (see the right column in Fig. 2), for the finite graphene, square and triangle structured arrays, the liquid surface waves all do not have the complete band gap, this fact illustrates that the steps no longer affect well the propagation of liquid surface waves. Besides the complete band gaps, we also can study the partial band gaps of liquid surface waves according to the transmission spectra in Fig. 2.
In Fig. 2, all transmission spectra do not have the obvious deviation between the $\Gamma M$ direction and the $\Gamma K$ direction, this indicates that the partial band gaps over three kinds of finite structured arrays are inconspicuous.

In a word, the finite graphene structure can produce more complete band gaps than the finite square and triangle structures, which is another important phenomenon besides the phenomenon that the graphene structure has the complete band gap at a lower filling fraction than other structures.\(^7\)

In order to verify the band gaps based on the transmission spectra, we calculate the normalized intensity field $|\eta/\eta_0|^2$ and phase vector field over the finite graphene structured array in the localized state and in the diffused state. We take the depth of liquid over the steps $h_i = 0.01$ mm and the size of the finite graphene structured array about 5.0 cm $\times$ 5.4 cm, set the stimulating source in the center of the array. As for the localized state, we take the frequency $f = 12$ Hz inside the complete band gaps. While for the diffused state, we take the frequencies $f = 30.5$ Hz and $f = 41.5$ Hz outside the complete band gaps. The phase vector is defined as $\mathbf{u} = \hat{e}_x \cos \theta + \hat{e}_y \sin \theta$, where $\theta$ comes from the definition $\eta(r) = |\eta(r)|e^{i\theta(r)}$. $\hat{e}_x$ and $\hat{e}_y$ are the unit vectors of $x$ axis and $y$ axis respectively. The phase vector method is often used to analyze the localized and diffused states of liquid surface waves.\(^{10}\)

The spatial distributions of the normalized intensity field and phase vector field are shown in Fig. 3. In Fig. 3, for the frequency $f = 12$ Hz, the normalized intensity field is confined in the center of the array (see Fig. 3(a1)), the distribution of the phase vector field is an ordering distribution that all the phase vectors either point to the same direction or the opposite direction (see Fig. 3(b1)). Figures 3(a1) and 3(b1) indicate clearly that the liquid surface waves in the localized state are confined in the center of the array. As for $f = 30.5$ Hz, the normalized intensity field diffuses mainly along the $\Gamma K$ direction although other directions also have the intensity distribution, and the distribution of the phase vectors is unordered (see Figs. 3(a2) and 3(b2)). As for $f = 41.5$ Hz, the normalized intensity field diffuses mainly along the $\Gamma M$ direction, the phase vector field is similar to that for $f = 30.5$ Hz (see Figs. 3(a3) and 3(b3)). The diffused intensity field and the unordered phase vector field both indicate that the liquid surface waves in the diffused state diffuse from the stimulating source to the outside. The $\Gamma K$ or $\Gamma M$ direction in the diffused state originates from the inconspicuous partial band gaps in Fig. 2(a2).

Here, in order to compare the localized ability of the finite graphene structure with other structures, using the least square method, we calculate the localized normalized transmissions $\ln |T|^2$ of the finite graphene,
Fig. 3. The spatial distributions of the normalized intensity field $|\eta/\eta_0|^2$ (left column) and phase vector field (right column) of liquid surface waves over the finite graphene structured array of cylinders with $h_i = 0.01$ mm. The top row denotes the case of the frequency $f = 12$ Hz, the middle and bottom rows denote the cases of the frequencies $f = 30.5$ Hz and $f = 41.5$ Hz, respectively.

Fig. 4. Normalized transmissions $\ln |T|^2$ of the localized liquid surface waves over the finite graphene, square and triangle structured arrays of cylinders versus the dimensionless distance. The least square method is used in calculation.

the absolute slope of the finite graphene structured array (about 1.4) is less than the absolute slopes of the finite square (about 2.2) and triangle (about 2.6) structured arrays. It indicates that the finite graphene structure has longer localized radius than the square and triangle structures, that is to say, the finite graphene structure has less localized ability than the finite square and triangle structures.

In summary, we calculated and discussed the normalized transmission spectra of the finite graphene, square and triangle structured arrays of cylinders with a uniform height on a level with liquid surface and below it, and found that the finite graphene structure can produce more complete band gaps than the finite square and triangle structures. Then, according to the normalized intensity field and phase vector field, we verified the band gaps based on the transmission spectra. Finally, we compared the localized transmission of the graphene structured array versus the distance with other structured arrays, and found the finite graphene structure has less localized ability than the finite square and triangle structures.

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