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A hidden symmetry in the Standard Model

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Abstract

We found an additional symmetry hidden in the fermion and Higgs sectors of the Standard Model. It is connected to the centers of the $SU(3)$ and $SU(2)$ subgroups of the gauge group. A lattice regularization of the whole Standard Model is constructed that possesses this symmetry.

1. Introduction

It is well known that to put a quantum field theory onto a lattice one should keep as much symmetries of the original model as possible. That is why, for example, any lattice gauge model is made to preserve the gauge symmetry [1] while it is possible, in principle, to construct a lattice model that comes as a discretization of a gauge fixed continuum theory. Other examples of this kind are the attempts to put fermions on a lattice both avoiding doubling and keeping the chiral symmetry [2].

It is the conventional point of view that all the symmetries of the Standard Model (SM), which must be used when dealing with its discretization, are known. In this Letter we demonstrate (in the framework of lattice regularization) that an additional symmetry is hidden within the fermion and Higgs sectors of the SM. It is connected to the centers of the $SU(3)$ and $SU(2)$ subgroups. It turns out possible to redefine the gauge sector of the lattice realization of the SM in such a way that it has the same naive continuum limit as the conventional one, while keeping the additional symmetry.

The Standard Model contains the following variables:

1. The gauge field $\mathcal{U} = (\Gamma, U, \theta)$, where

$$\Gamma \in SU(3), \quad U \in SU(2), \quad e^{i\theta} \in U(1), \tag{1}$$

realized as link variables on the lattice.
2. A scalar doublet

$$\Phi^{\alpha}, \quad \alpha = 1, 2. \tag{2}$$

3. Anticommuting spinor variables, representing leptons and quarks:

$$\begin{pmatrix} \nu_e & \nu_\mu & \nu_\tau \\ e & \mu & \tau \end{pmatrix}, \quad \begin{pmatrix} u & c & t \\ d & s & b \end{pmatrix}. \tag{3}$$

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The action has the form

$$S = S_g + S_H + S_f, \tag{4}$$

where we denote the fermion part of the action by S_f, the pure gauge part is denoted by S_g, and the scalar part of the action by S_H.

In *any* lattice realization of S_H and S_f both these terms depend upon link variables $\mathcal{U}$ considered in the representations corresponding to quarks, leptons, and the Higgs scalar field, respectively. Therefore, $\mathcal{U}$ appears in the combinations shown in Table 1. Our observation is that *all* the listed combinations are invariant under the following transformations:

$$\begin{aligned} &U \to U e^{-i\pi N}, \\ &\theta \to \theta + \pi N, \\ &\Gamma \to \Gamma e^{(2\pi i/3)N}, \end{aligned} \tag{5}$$

where N is an arbitrary integer link variable. It represents a three-dimensional hypersurface on the dual lattice. Both S_H and S_f (in *any* realization) are invariant under the simultaneous transformations (5). This symmetry reveals the correspondence between the centers of the $SU(2)$ and $SU(3)$ subgroups of the gauge group.

After integrating out fermion and scalar degrees of freedom any physical variable should depend upon gauge-invariant quantities only. Those are the Wilson loops: $\omega_{SU(3)}(\mathcal{C}) = \mathrm{Tr}\prod_{\mathrm{link}\in\mathcal{C}} \Gamma_{\mathrm{link}}$, $\omega_{SU(2)}(\mathcal{C}) = \mathrm{Tr}\prod_{\mathrm{link}\in\mathcal{C}} U_{\mathrm{link}}$, and $\omega_{U(1)}(\mathcal{C}) = \prod_{\mathrm{link}\in\mathcal{C}} \exp(\frac{i}{3}\theta_{\mathrm{link}})$. Here $\mathcal{C}$ is an arbitrary closed contour on the lattice (with self-intersections allowed). These Wilson loops are trivially invariant under the transformation (5) with the field N representing a *closed* three-dimensional hypersurface on the dual lattice. Therefore, the non-trivial part of the symmetry (5) corresponds to a closed two-dimensional surface on the dual lattice that is the boundary of the hypersurface represented by N. Then in terms of the gauge-invariant quantities ω the transformation (5) acquires the form:

$$\begin{aligned} &\omega_{U(1)}(\mathcal{C}) \to \exp\bigl(-i\tfrac{1}{3}\pi \mathbf{L}(\mathcal{C}, \Sigma)\bigr)\omega_{U(1)}(\mathcal{C}), \\ &\omega_{SU(2)}(\mathcal{C}) \to \exp\bigl(i\pi \mathbf{L}(\mathcal{C}, \Sigma)\bigr)\omega_{SU(2)}(\mathcal{C}), \\ &\omega_{SU(3)}(\mathcal{C}) \to \exp\bigl(i\tfrac{2}{3}\pi \mathbf{L}(\mathcal{C}, \Sigma)\bigr)\omega_{SU(3)}(\mathcal{C}). \end{aligned} \tag{6}$$

Here Σ is an arbitrary closed surface (on the dual lattice) and $\mathbf{L}(\mathcal{C}, \Sigma)$ is the integer linking number of this surface and the closed contour $\mathcal{C}$.

It is worth mentioning that after integrating out fermion degrees of freedom as well as the Higgs scalar the Standard Model in its continuum formulation becomes a theory defined in a loop space [3], i.e., any physical variable depends upon gauge fields only through the $SU(3)$, $SU(2)$ an $U(1)$ Wilson loops. If we again denote them as $\omega_{SU(3)}$, $\omega_{SU(2)}$, and $\omega_{U(1)}$ (where $\omega_{U(1)}$ corresponds to the worldline of a particle of $U(1)$ charge $\frac{1}{3}$ while $\omega_{SU(2)}$ and $\omega_{SU(3)}$ are the Wilson loops considered in the fundamental representations of $SU(2)$ and $SU(3)$, respectively), the symmetry (6) understood in the continuum notation would appear if we neglect the pure gauge-field part of the action. It is obvious that the latter in its conventional continuum formulation (or, say, in lattice Wilson formulation) is not invariant under (6). However, the lattice realization of the pure gauge field term of the action can be constructed in such a way that it also preserves the mentioned symmetry. For example, we can consider the following expression for S_g:

$$\begin{aligned} S_g = \sum_{\text{plaquettes}} \Bigl\{ &\beta_1\bigl(1 - \tfrac{1}{2}\,\mathrm{Tr}\, U_p \cos\theta_p\bigr) \\ &+ \beta_2(1 - \cos 2\theta_p) \\ &+ \beta_3\bigl(1 - \tfrac{1}{6}\,\mathrm{Re}\,\mathrm{Tr}\,\Gamma_p\,\mathrm{Tr}\, U_p \exp(i\theta_p/3)\bigr) \\ &+ \beta_4\bigl(1 - \tfrac{1}{3}\,\mathrm{Re}\,\mathrm{Tr}\,\Gamma_p \exp(-2i\theta_p/3)\bigr) \\ &+ \beta_5\bigl(1 - \tfrac{1}{3}\,\mathrm{Re}\,\mathrm{Tr}\,\Gamma_p \exp(4i\theta_p/3)\bigr)\Bigr\}, \end{aligned} \tag{7}$$

where the sum runs over the elementary plaquettes of the lattice. Each term of the action Eq. (7) corresponds to a parallel transporter along the boundary of a plaquette considered in one of the representations listed above. The coefficients β_i $(i = 1, \ldots, 5)$ must be chosen in such a way as to give rise to the correct value of the Weinberg angle.

Naively Eq. (7) has the same continuum limit (with the appropriate choice of β_i) as, say the following

Table 1

$Ue^{-i\theta}$	left-handed leptons
$e^{-2i\theta}$	right-handed leptons
$\Gamma U e^{\frac{i}{3}\theta}$	left-handed quarks
$\Gamma e^{-\frac{2i}{3}\theta}$	right-handed d-, s-, and b-quarks
$\Gamma e^{\frac{4i}{3}\theta}$	right-handed u-, c-, and t-quarks
$Ue^{i\theta}$	the Higgs scalar field

conventional action:

$$S_g^0 = \sum_{\text{plaquettes}} \{\beta_1^0(1 - \tfrac{1}{2}\operatorname{Tr} U_p) + \beta_2^0(1 - \cos\theta_p) + \beta_3^0(1 - \tfrac{1}{3}\operatorname{Re}\operatorname{Tr}\Gamma_p)\}. \quad (8)$$

However, (7) possesses the additional symmetry (6) while (8) does not. If the symmetry (6) does occur in nature, a regularization that does not maintain it would be inappropriate. The situation here could be similar to that of an attempt to construct a lattice gauge model while not keeping the gauge invariance: the corresponding lattice model may describe physics improperly.

A particularly interesting question is how the symmetry (5) emerges in lattice discretizations of unified models. Namely, (5) may naturally appear after the breakdown $G \to SU(3)\otimes SU(2)\otimes U(1)$. The simplest example of the unified model of such type is the conventional $SU(5)$ theory [4]. If we consider its lattice definition with the Wilson action, the low energy limit would coincide with Eq. (7) for the following choice of couplings:

$$\beta_1 = \frac{2\beta}{5}, \quad \beta_4 = \frac{3\beta}{5}, \quad \beta_2 = \beta_3 = \beta_5 = 0. \quad (9)$$

Relation (5) itself appears to be the trivial consequence of expressing $SU(5)$ link matrices in terms of Γ, U and θ in the low energy approximation:

$$\begin{pmatrix} \Gamma e^{-\frac{2i\theta}{3}} & 0 \\ 0 & U e^{i\theta} \end{pmatrix}. \quad (10)$$

The same picture emerges in any unified theory, if its gauge group G contains $SU(5)$ and the symmetry breakdown pattern is $G \to \cdots \to SU(5) \to SU(3) \otimes SU(2) \otimes U(1)$.

The other unified models may be transferred to the lattice either violating or preserving (5). As an example, let us consider the $SU(2)_L \otimes SU(2)_R \otimes SU(4)'_{L+R}$ Pati–Salam unified model [5]. We arrange the fermions (of the first generation) as the elements of 2×4 matrices $\mathcal{F}^{ab}_{L,R}$ (the $SU(2)_{L,R}$ subgroups act on the first index, the $SU(4)'$ subgroup acts on the second index):

$$\mathcal{F}_{L,R} = \begin{pmatrix} u^1 & u^2 & u^3 & \nu \\ d'^1 & d'^2 & d'^3 & e \end{pmatrix}_{L,R}. \quad (11)$$

Leptons and quarks of the other generations are arranged in a similar way.

Let us construct the Higgs sector in such a way that it provides link matrices which have the form (at low energies, after the breakdown $SU(2)_L \otimes SU(2)_R \otimes SU(4) \to SU(3) \otimes SU(2)_L \otimes U(1)$):

$$U \otimes \begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix} \otimes \begin{pmatrix} \Gamma e^{\frac{i\theta}{3}} & 0 \\ 0 & e^{-i\theta} \end{pmatrix}. \quad (12)$$

We can define the pure gauge field action, say, in the following two ways:

1. Let $\mathcal{V} = \mathcal{Y}^L \otimes \mathcal{Y}^R \otimes \mathcal{Z} \in SU(2)_L \otimes SU(2)_R \otimes SU(4)$ be the $SU(2)_L \otimes SU(2)_R \otimes SU(4)$ link matrix (here $\mathcal{Y}^{L,R} \in SU(2)$, $\mathcal{Z} \in SU(4)$). Then let us consider the action of the form:
$$S = \beta \sum_{\text{plaq}} \{(1 - \tfrac{1}{2}\operatorname{Re}\operatorname{Tr}\mathcal{Y}^L_{\text{plaq}}) + (1 - \tfrac{1}{2}\operatorname{Re}\operatorname{Tr}\mathcal{Y}^R_{\text{plaq}}) + (1 - \tfrac{1}{4}\operatorname{Re}\operatorname{Tr}\mathcal{Z}_{\text{plaq}})\}. \quad (13)$$
The lattice model defined in this way obviously violates (5) in the low energy limit.
2. With the above definition of the link variable let us now consider the lattice model with another action
$$S = \beta \sum_{\text{plaq}} [1 - \tfrac{1}{16}\operatorname{Re}(\operatorname{Tr}\mathcal{Y}^L_{\text{plaq}} + \operatorname{Tr}\mathcal{Y}^R_{\text{plaq}})\operatorname{Tr}\mathcal{Z}_{\text{plaq}}] \sim \beta \sum_{\text{plaq}} \{1 - \tfrac{1}{16}\operatorname{Re}[\operatorname{Tr} U_{\text{plaq}} + 2\cos(\theta_{\text{plaq}})] \times (\operatorname{Tr}\Gamma_{\text{plaq}} e^{\frac{i}{3}\theta_{\text{plaq}}} + e^{-i\theta_{\text{plaq}}})\}. \quad (14)$$
This is exactly the action (7) with the following choice of couplings:
$$\beta_1 = \frac{\beta}{8}, \quad \beta_2 = \frac{\beta}{16}, \quad \beta_3 = \frac{3\beta}{8}, \quad \beta_4 = \beta_5 = \frac{3\beta}{16}. \quad (15)$$
Therefore, the full unified model preserves our additional symmetry after the breakdown $SU(2)_L \otimes SU(2)_R \otimes SU(4) \to SU(3) \otimes SU(2)_L \otimes U(1)$.

Finally, we consider a unified model with *arbitrary* gauge group G and the arrangement of fermions such that there exist representations $\alpha, \gamma, \ldots$ of G that are *completely* composed of the full set of Standard Model fermions. Let again $\mathcal{V} \in G$ be the link variable. We

choose the action

$$S = \beta_\alpha \sum_{\text{plaq}} \bigl(1 - \operatorname{Re} \chi_\alpha(\mathcal{V}_{\text{plaq}})\bigr) + \beta_\gamma \sum_{\text{plaq}} \bigl(1 - \operatorname{Re} \chi_\gamma(\mathcal{V}_{\text{plaq}})\bigr) + \cdots, \tag{16}$$

where χ_α is the character of the representation α and the sum is over the mentioned representations. The resulting model preserves (5) after the breakdown $G \to SU(3) \otimes SU(2) \otimes U(1)$. We like to mention here that Eq. (7) with the couplings given by Eq. (15) (the $SU(2) \otimes SU(2) \otimes SU(4)$ model) would appear also in the low energy limit of the $SU(5)$ unified model if the action of the latter is chosen as the sum of (16)-like terms corresponding to both representations, in which the fermions are arranged. This happens because in both cases the action (16) involves all the representations that exhaust the full set of the Standard Model fermions.

So, the symmetry (5) being confirmed (or rejected) would give a criterion for the choice of a unified model. The dynamical consequence of (5) could appear due to the fact, that it ties the centers of the $SU(3)$ and $SU(2)$ subgroups of the gauge group. It is well known that the center elements of the color subgroup of the gauge group play an important role in the description of the confinement of color [6–9]. Therefore, one might expect that in the model with the pure gauge field action (7) it may not be possible to investigate color dynamics alone (without taking into account the $SU(2)$ or $U(1)$ subgroups of the gauge group) and the confinement picture may be different from the one found within the framework of the conventional discretization.

On the other hand, the topological excitations corresponding to the center of the $SU(2)$ subgroup may play an important role in the finite temperature nonperturbative electroweak phenomena [10]. Therefore, due to the mentioned ties, the description of, say, the finite temperature electroweak phase transition may also be different for the lattice models which do or do not maintain the additional symmetry.

A comparison of the two approaches in these respects may be important for understanding whether it is necessary or not to take into account the additional symmetry considered, while constructing the lattice approximation to the Standard Model.

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References

[1] M. Creutz, Quarks, Gluons and Lattices, Cambridge Univ. Press, Cambridge, 1985.
[2] N.B. Nielsen, M. Ninomiya, Nucl. Phys. B 185 (1981) 20; N.B. Nielsen, M. Ninomiya, Nucl. Phys. B 185 (1981) 173; M. Lüscher, Phys. Lett. B 428 (1998) 342; H. Neuberger, Phys. Lett. B 417 (1998) 141.
[3] Y.M. Makeenko, A.A. Migdal, Nucl. Phys. B 188 (1981) 269; A.A. Migdal, Phys. Rep. 102 (1983) 199.
[4] L.B. Okun, Leptons and Quarks, North-Holland, Amsterdam, 1982.
[5] J.C. Pati, S. Radjpoot, A. Salam, Phys. Rev. D 17 (1978) 131.
[6] B.L.G. Bakker, A.I. Veselov, M.A. Zubkov, Phys. Lett. B 502 (2001) 338, hep-lat/0011062.
[7] B.L.G. Bakker, A.I. Veselov, M.A. Zubkov, Phys. Lett. B 544 (2002) 374.
[8] M. Faber, J. Greensite, S. Olejnik, in: Confinement, Topology, and other Non-Perturbative Aspects of QCD, NATO Advanced Research Workshop, Stara Lesna, Slovakia, 2002.
[9] J. Greensite, Prog. Part. Nucl. Phys. 51 (2003) 1.
[10] M. Gurtler, E.M. Ilgenfritz, A. Schiller, Phys. Rev. D 56 (1997) 3888; M.N. Chernodub, F.V. Gubarev, E.M. Ilgenfritz, A. Schiller, Phys. Lett. B 443 (1998) 244.