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Anomalous $U(1)$'s masses in non-supersymmetric open string vacua

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Abstract

Anomalous $U(1)$'s are omnipresent in realizations of the Standard Model using D-branes. Such models are typically non-supersymmetric, and the anomalous $U(1)$ masses are potentially relevant for experiment. In this Letter, the string calculation of anomalous $U(1)$ masses [hep-th/0204153] is extended to non-supersymmetric orientifolds.

1. Introduction

Recently, many attempts have been made in order to embed the Standard Model in open string theory, with partial success [1–7]. In such a context the Standard Model particles are open string states attached on (different) stacks of D-branes. N coincident D-branes away from an orientifold plane typically generate a unitary group $U(N)$. Therefore, every U-factor in the gauge group supplies the model with extra Abelian gauge fields.[1]

Such $U(1)$ fields have generically 4D anomalies. The anomalies are cancelled via the Green–Schwarz mechanism [8–10] where a scalar axionic field (zero-form, or its dual two-form) is responsible for the anomaly cancellation. This mechanism gives a mass to the anomalous $U(1)$ fields and breaks the associated gauge symmetry. The masses of the anomalous $U(1)$'s are typically of order of the string scale but in open string theory they can be also much lighter [11,12]. If the string scale is around a few TeV, observation of such anomalous $U(1)$ gauge bosons becomes a realistic possibility [13].

As it has been shown in [12], we can compute the general mass formulae of the anomalous $U(1)$'s in supersymmetric models by evaluating the ultraviolet tadpole of the one-loop open string diagram with the insertion of two gauge bosons on different boundaries. In this limit, the diagrams of the annulus with both gauge bosons in the same boundary and the Möbius strip do not contribute when vacua have cancelled tadpoles. Mass formulae were provided for $N=1$ and $N=2$ supersymmetric orientifolds.

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[1] There are cases where we can also have $SO(n)$ or $Sp(n)$ gauge factors. However, $SU(3)$ can be minimally embedded only in $U(3)$ and in non-minimal cases (bigger gauge groups that are then broken by projections to those of the Standard Model), they leave also other potentially anomalous $U(1)$'s.

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In this Letter we are interested in the masses of the anomalous $U(1)$'s in non-supersymmetric models since such are the models that will eventually represent the low energy physics of the Standard Model. In particular, intersecting-brane realizations of the Standard Model are generically non-supersymmetric. We calculate the mass formulae using the "background field method" and find that they are the same as the supersymmetric ones when we have cancellation of all tadpoles. In cases where NSNS tadpoles do not vanish, there are extra contributions proportional to the non-vanishing tadpole terms.

The formulae are valid even if we add Wilson lines that move the branes away from the fixed points. The Wilson lines generically break the gauge group and they will affect the masses of the anomalous $U(1)$'s through the traces of the model dependent γ matrices.

The formulae, are applied to a Z_2 non-supersymmetric orientifold model, with RR and NSNS tadpoles to be cancelled, where supersymmetry is broken by a Scherk–Schwarz deformation [14].

This ultraviolet mass is not the only source for the mass of anomalous $U(1)$'s. In Standard Model realizations, the Higgs is necessarily charged under one of the anomalous $U(1)$'s. As it was described in [15], the Higgs contribution to the mass of these $U(1)$'s is $g_A\sqrt{M^2+e_H^2\langle H\rangle^2}$ where g_A the gauge coupling of the anomalous $U(1)$ and e_H the $U(1)$ charge of the Higgs. The Higgs contribution to the $U(1)$ mass can be obtained from the effective field theory unlike the ultraviolet mass we calculate here which can only calculated in string theory.

The Letter is organized as follows. In Section 2, we evaluate the general mass of the anomalous $U(1)$'s using the background-field method. In Section 3, we review the non-supersymmetric Z_2 orientifold with a Scherk–Schwartz deformation, and we use the results of the previous section to calculate the anomalous $U(1)$ masses.

2. Computing with the background-field method

Our purpose is to evaluate the bare masses of the anomalous $U(1)$ which appear in the one-loop amplitudes with boundaries where two gauge fields are inserted [12]. Here we will use another technique which is based on turning on a magnetic field on the D-branes and pick out the second order terms to this magnetic field. This method is called "the background-field method" [16]. We turn on different magnetic fields B_a in every stack of branes, longitudinal to x^1, a non-compact dimension,

$$F_{23}^a = B_a Q_a, \tag{1}$$

where Q_a are the $U(1)_a$ generators from every stack of branes. The effect of the magnetic field on the open-string spectrum is to shift the oscillator frequencies of the string non-compact x^2+ix^3 coordinate by an amount ϵ_a:

$$\epsilon_a = \frac{1}{\pi}\left[\arctan\left(\pi q_i^a B_a\right)+\arctan\left(\pi q_j^a B_a\right)\right], \tag{2}$$

where q_i^a, q_j^a are the $U(1)_a$ charges of the i,j endpoints. The Chan–Paton states λ_{ij} that describe the endpoint i,j of the open string, are the generators of gauge group that remains after the orientifold construction. Diagonalizing these matrices, we can replace the Q_i with λ_{ii}.

The expansion of the one-loop vacuum energy is

$$\begin{aligned}\Lambda(B) &= \frac{1}{2}\left(\mathcal{T}+\mathcal{K}+\mathcal{A}(B)+\mathcal{M}(B)\right)\\ &= \Lambda_0+\frac{1}{2}\left(\frac{B}{2\pi}\right)^2\Lambda_2+\cdots,\end{aligned} \tag{3}$$

where B one of the different magnetic fields. Generically, it appears a linear to B term that is a pour tadpole and it is coming from the RR sector. This term vanishes when we have tadpole cancellation. The quadratic term in the background field contains a lot of information. In the IR limit, we have a logarithmic divergence whose coefficient is the β-function. The UV limit provides the mass-term of the anomalous gauge bosons. The finite part of this term is the threshold correction in the gauge couplings [16]. The annulus amplitude in the Z_N type I orientifolds (without the magnetic field) can be written as

$$\mathcal{A}^{ab} = -\frac{1}{2N}\sum_{k=1}^{N-1}\int_0^\infty \frac{dt}{t}\mathcal{A}_k^{ab}(q), \tag{4}$$

where a,b the different kind of D-branes at the ends of the open strings. The $\mathcal{A}_k^{ab}$ is the contribution of the

kth sector:

$$\mathcal{A}_k^{ab} = \frac{1}{4\pi^4 t^2} \operatorname{Tr}\big[\gamma_a^k\big] \operatorname{Tr}\big[\gamma_b^k\big] \times \sum_{\alpha,\beta=0,1} \eta^{\alpha\beta} \frac{\vartheta\big[{}^{\alpha}_{\beta}\big]}{\eta^3} Z^{ab}_{\mathrm{int},k}\big[{}^{\alpha}_{\beta}\big]\bigg|_{\mathcal{A}}. \tag{5}$$

Similarly, we can exchange $\mathcal{A}$ with $\mathcal{M}$ in (4) to have an analogous expression for the Möbius strip. The $\mathcal{M}_k^a$ is given by

$$\mathcal{M}_k^{a} = -\frac{1}{4\pi^4 t^2} \operatorname{Tr}\big[\gamma_a^{2k}\big] \times \sum_{\alpha,\beta=0,1} \eta^{\alpha\beta} \frac{\vartheta\big[{}^{\alpha}_{\beta}\big]}{\eta^3} Z^{a}_{\mathrm{int},k}\big[{}^{\alpha}_{\beta}\big]\bigg|_{\mathcal{M}}. \tag{6}$$

In the presence of the background magnetic field B_a, the above amplitudes become

$$\mathcal{A}_k^{ab}(B) = \frac{i}{4\pi^3 t} \times \operatorname{Tr}\Bigg[\big(B_a\lambda_a\gamma_a^k \otimes \gamma_b^k + \gamma_a^k \otimes B_b\lambda_b\gamma_b^k\big) \times \sum_{\alpha\beta} \eta^{\alpha\beta} \frac{\vartheta\big[{}^{\alpha}_{\beta}\big]\big(\frac{i\epsilon t}{2}\big)}{\vartheta\big[{}^{1}_{1}\big]\big(\frac{i\epsilon t}{2}\big)}\Bigg] Z^{ab}_{\mathrm{int},k}\big[{}^{\alpha}_{\beta}\big]\bigg|_{\mathcal{A}},$$

$$\mathcal{M}_k^{a}(B) = -\frac{i}{2\pi^3 t} \operatorname{Tr}\Bigg[B_a\lambda_a\gamma_a^{2k} \sum_{\alpha\beta} \eta^{\alpha\beta} \frac{\vartheta\big[{}^{\alpha}_{\beta}\big]\big(\frac{i\epsilon t}{2}\big)}{\vartheta\big[{}^{1}_{1}\big]\big(\frac{i\epsilon t}{2}\big)}\Bigg] \times Z^{a}_{\mathrm{int},k}\big[{}^{\alpha}_{\beta}\big]\bigg|_{\mathcal{M}}. \tag{7}$$

Notice that the only differences from (5), (6) are in the contribution of the non-compact part of the partition functions. This is expected since the presence of the magnetic fields affect only the x^2, x^3 coordinates. Therefore, the expressions (7) are valid for all kinds of orientifold models.

Since we are interested in the quadratic B^2 terms of the above amplitudes, we expand the above formulae to quadratic order in the background field,[2] using the following Taylor expansions:

$$\epsilon \simeq \begin{cases} B_a\lambda_a \otimes 1 + 1 \otimes B_b\lambda_b & \text{in } \mathcal{A}^{ab}, \\ 2B_a\lambda_a & \text{in } \mathcal{M}^a. \end{cases} \tag{8}$$

The zero-order B terms are the amplitudes in the absence of the magnetic field (5), (6). These expressions give the tadpole cancellation conditions in virtue of the UV divergences. The linear to B terms appear from the $a = b = 1$ sector in (7). This is a pour tadpole and vanishes when we have tadpole cancellation. Therefore, it does not affect higher order in B amplitudes. The second order-terms on B are

$$\mathcal{A}_{2,k}^{ab} = \pi^2 i\big[\operatorname{Tr}\big[\lambda_a^2\gamma_a^k\big]\operatorname{Tr}\big[\gamma_b^k\big] + \operatorname{Tr}\big[\gamma_a^k\big]\operatorname{Tr}\big[\lambda_b^2\gamma_b^k\big] + 2\operatorname{Tr}\big[\lambda_a\gamma_a^k\big]\operatorname{Tr}\big[\lambda_b\gamma_b^k\big]\big] F_k^{ab}\big|_{\mathcal{A}}, \tag{9}$$

$$\mathcal{M}_{2,k}^{a} = -4\pi^2 i \operatorname{Tr}\big[\lambda_a^2\gamma_a^{2k}\big] F_k^{aa}\big|_{\mathcal{M}} \tag{10}$$

defining F_k^{ab} as a term which contains all the spin-structure and the orbifold information

$$F_k^{ab}\big|_\sigma = \frac{1}{4\pi^4} \sum_{\alpha\beta} \eta_{\alpha\beta}\pi i\,\partial_\tau \left[\log \frac{\vartheta\big[{}^{\alpha}_{\beta}\big](0|\tau)}{\eta(\tau)}\right] \times \frac{\vartheta\big[{}^{\alpha}_{\beta}\big](0|\tau)}{\eta^3(\tau)} Z^{ab}_{\mathrm{int},k}\big[{}^{a}_{b}\big]\bigg|_\sigma \tag{11}$$

for both surfaces (the choice of τ define the surface σ). Note that the $a = b = 1$ sector is not contained in the (11). This term can be formally written as the supertrace over states from the open ab k-orbifold sector

$$F_k^{ab}\big|_\sigma = \frac{|G|}{(2\pi)^2} \operatorname{Str}^{ab}_{k,\mathrm{open}}\left[\frac{1}{12} - s^2\right] e^{-tM^2/2}\bigg|_\sigma \tag{12}$$

where the s is the 4D helicity.

Thus, for *large* τ_2 we have

$$\lim_{\tau_2\to\infty} F_k^{ab} = C^{ab}_{k,\mathrm{IR}} + \mathcal{O}\big[e^{-2\pi\tau_2}\big] \tag{13}$$

with

$$C^{ab}_{k,\mathrm{IR}} = \frac{|G|}{(2\pi)^2} \operatorname{Str}_k\left[\frac{1}{12} - s^2\right]_{\mathrm{open}}. \tag{14}$$

For *small* τ_2 we have

$$\lim_{\tau_2\to 0} F_k^{ab} = \frac{1}{\tau_2}\big[C^{ab}_{k,\mathrm{UV}} + \mathcal{O}\big[e^{-\frac{\pi}{2\tau_2}}\big]\big], \tag{15}$$

where

$$C^{ab}_{k,\mathrm{UV}} = \frac{|G|}{(2\pi)^2} \operatorname{Str}_k\left[\frac{1}{12} - s^2\right]_{\mathrm{closed}}. \tag{16}$$

The helicity supertrace is now in the closed-string k-sector mapped from the open k-sector by a modular transformation.

[2] Where the normalized expansion is $\mathcal{A} \equiv \mathcal{A}_0 + \frac{B}{2\pi}\mathcal{A}_1 + \big(\frac{B}{2\pi}\big)^2\mathcal{A}_2 + \cdots$. Similarly for $\mathcal{M}$.

Notice that in the annulus amplitude (9), the two first terms are proportional to the square of the B field. This cases are proportional to annulus amplitudes $\mathcal{A}_2$, where two vertex-operators (VOs) are on the same boundary. In the last component of (9), the B fields are coming from the opposite D-branes and is proportional to $\mathcal{A}_{11}$, with the VOs on different boundaries. The (10) is proportional to a Möbius strip amplitude with the insertion of two VOs.

The IR limit $t \to \infty$ can be found easily using the (13). We regularize the integral by $\mu \to 1/t^2$ and we find the β-function

$$\begin{aligned} b &= -\frac{2}{N}\sum_{k=1}^{N-1}\lim_{t\to\infty}\big(\mathcal{A}^{ab}_{2,k}(t)+\mathcal{M}^{a}_{2,k}(t)\big) \\ &= -\frac{2\pi^2 i}{N}\sum_{k=1}^{N-1}\big[\big(\mathrm{Tr}\big[\lambda_a^2\gamma_a^k\big]\,\mathrm{Tr}\big[\gamma_b^k\big] \\ &\qquad + \mathrm{Tr}\big[\gamma_a^k\big]\,\mathrm{Tr}\big[\lambda_b^2\gamma_b^k\big] \\ &\qquad + 2\,\mathrm{Tr}\big[\lambda_a\gamma_a^k\big]\,\mathrm{Tr}\big[\lambda_b\gamma_b^k\big]\big)C^{ab}_{k,\mathrm{IR}}\big|_{\mathcal{A}} \\ &\qquad - 4\,\mathrm{Tr}\big[\lambda_a^2\gamma_a^{2k}\big]C^{a}_{k,\mathrm{IR}}\big|_{\mathcal{M}}\big]. \end{aligned} \tag{17}$$

For the UV limit $t \to 0$, we use the (15) and we regularize the integral by $\mu \leqslant t$. The A_2 and M together are giving terms proportional to the tadpole cancellation conditions.[3] Therefore, when we have vanishing of RR and NSNS tadpoles, the masses of the anomalous gauge bosons are given by $\mathcal{A}_{11}$:

$$\frac{1}{2}M^2_{aa} = \frac{\pi^2 i}{N}\sum_{k=1}^{N-1}\mathrm{Tr}\big[\lambda_a\gamma_a^k\big]^2 C^{ab}_{k,\mathrm{UV}}\big|_{\mathcal{A}}, \tag{18}$$

$$\frac{1}{2}M^2_{59} = \frac{\pi^2 i}{2N}\sum_{k=1}^{N-1}\mathrm{Tr}\big[\lambda_5\gamma_5^k\big]\,\mathrm{Tr}\big[\lambda_9\gamma_9^k\big]C^{59}_{k,\mathrm{UV}}\big|_{\mathcal{A}}, \tag{19}$$

where $\alpha = 5, 9$. When we have non-vanishing NSNS tadpoles there is an extra contribution to the mass formulas, proportional to the non-vanishing tadpole.

The formulae (18), (19) still hold even if we add Wilson lines. Generically, adding a Wilson line we shift the windings or the momenta in a coordinate with Newmann or Dirichlet boundary conditions, respectively. This breaks the gauge group. In the transverse (closed) channel the shifts appears as phases $e^{2\pi i n\theta}$ where θ the shift and n the momenta or windings, respectively, to the above. Since only the massless states contribute in the UV limit, the effect of the Wilson line will appear only in the traces of the γ matrices.

The threshold correction [18] is the finite part of (9) and (10). Generically we have

$$\begin{aligned} \frac{16\pi^2}{g^2} &= \frac{16\pi^2}{g_0^2} - \frac{1}{2N}\sum_{k=1}^{N-1}\int_{\mu}^{1/\mu^2}\frac{dt}{t}\big(\mathcal{A}_2^{ab}+\mathcal{M}_2^{a}\big) \\ &\qquad - b\log\frac{\mu^2}{M^2} - \frac{1}{2}M^2_{ab}\frac{1}{\mu}, \end{aligned} \tag{20}$$

where we separate the divergences from the quadratic terms to B. The above formulae for the β-function, the corrections to the gauge couplings and the masses of the anomalous $U(1)$'s are the same to the supersymmetric ones found in [12,16]. Next, we will apply the above formulae to a non-supersymmetric model that has been constructed by Scherk–Schwarz deformation [14].

3. A 4D non-supersymmetric orientifold example

In this section we will evaluate the masses of the anomalous $U(1)$'s in a Z_2 orientifold model where supersymmetry is broken by a Scherk–Schwarz deformation [20–26] and where RR and NSNS tadpoles cancel locally [14]. To start with, we give a review of this model defining some useful quantities. Consider the $\mathcal{N}=1$ orbifold of type IIB string theory in 4 dimensions, $\mathbb{R}^4 \times T^2 \times (T^4/Z_2)$. The elements of this orbifold are $\{1, g\}$, acting only on the T^4 [17]. In addition, we can act with a freely-acting Z_2 orbifold with elements $\{1, (-1)^F\delta\}$. We denote by h the non-trivial element of this group. This orbifold is known as a Scherk–Schwarz deformation. The $F = F_L + F_R$ is the space–time fermion number and δ is the ele-

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[3] The UV limit of $\partial_\tau \log \frac{\vartheta[^{\alpha}_{\beta}]}{\eta}$ in (11) is generically of order τ_2^{-2}. Terms that in the closed sector appear as $\vartheta[^1_0]$ are contributions from the RR part. These terms have limits $2\pi i/3t^2$ and $\pi i/6t^2$ coming from the annulus and Möbius strip, respectively. Terms that in the closed string sector appear as $\vartheta[^0_\alpha]$ are the NSNS sectors which have contribution only from the $\partial_\tau \log\eta$. The UV limits are $-\pi/3t^2$ and $-\pi/12t^2$ from the annulus and Möbius strip, respectively. Therefore (9), (10) have the same form as (5), (6) that provides the tadpole conditions. It is important that both, R and NS sectors contribute to the mass formulas of the anomalous $U(1)$'s.

Table 1
The massless spectrum for the two inequivalent solutions $\gamma_h^2 = \pm 1$ of the Z_2 accompanied with a transverse SS deformation. The gauge group in both cases is $U(8)_9 \times U(8)_{9'} \times U(8)_5 \times U(8)_{5'}$. The spectrum is non-chiral and consequently anomaly-free

$\gamma_h^2 = -1$ Gauge group: $U(8)_9^2 \times U(8)_5^2$	Scalars	Fermions
(99)/(55) matter	$(8,8) + (\bar{8},\bar{8})$	$(28,1) + (\overline{28},1) + (1,28)$ $(1,\overline{28}) + 2\times(8;\bar{8}) + 2\times(\bar{8};8)$
(59) matter	$(8,1;\bar{8},1) + (\bar{8},1;8,1)$ $(1,8;1,\bar{8}) + (1,\bar{8};1,8)$	$(8,1;1,\bar{8}) + (\bar{8},1;1,8)$ $(1,8;\bar{8},1) + (1,\bar{8};8,1)$
$\gamma_h^2 = +1$ Gauge group: $U(8)_9^2 \times U(8)_5^2$	Scalars	Fermions
(99)/(55) matter	$(28,1) + (\overline{28},1) + (1,28) + (1,\overline{28})$	$(8,8) + (\bar{8},\bar{8})$ $2\times(8;\bar{8}) + 2\times(\bar{8};8)$
(59) matter	$(8,1;\bar{8},1) + (\bar{8},1;8,1)$ $(1,8;1,\bar{8}) + (1,\bar{8};1,8)$	$(8,1;1,\bar{8}) + (\bar{8},1;1,8)$ $(1,8;\bar{8},1) + (1,\bar{8};8,1)$

ment $(-1)^{m_4}$ (which geometrically corresponds to the shift $x_4 \to x_4 + \pi R_4$ of a compact dimension). As it was shown in [14], the tadpole cancellation provides two different solutions that depend on the inequivalent choices of $\gamma_h^2 = \pm 1$ where γ_h the action of h on the Chan–Paton matrices. The 16-dimensional 'shift' vector of the Z_2 orientifold is [10,19]

$$V_g^9 = V_g^5 = \frac{1}{4}(1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1). \tag{21}$$

The 'shift' vector of the SS deformation generically is

$$V_h^9 = V_h^5 = \frac{1}{4}\begin{cases} (1_a, -1_b) & \text{for } \gamma_h^2 = -1, \\ (2_a, 0_b) & \text{for } \gamma_h^2 = +1, \end{cases} \tag{22}$$

where the index referred to the number of the same components in the vector. In both cases $a + b = 16$, however we implement for simplicity $a = b = 8$. The massless spectrums are provided in Table 1. The gauge group in both cases is the same. The only difference appears in the exchange of the antisymmetric reps with the bi-fundamental $(8,8) + (\bar{8},\bar{8})$ in the (99)/(55) matter sector. The spectrum is anomaly-free in 4D since it is non-chiral. The internal annulus partition functions for 99, 55 and 59 strings are

$$Z_{\mathrm{int},k}^{99,55}\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right] = -\sum_{s,r=0}^{1} (-1)^{\alpha s + \beta r}\left[(-1)^{s\cdot m_4} P_{m_4} P_{m_5}\right] \times \frac{\vartheta\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right](0|\tau)}{\eta(\tau)}\left(2\sin\frac{\pi k}{2}\right)^2 \times \prod_{j=1}^{2}\frac{\vartheta\left[\begin{smallmatrix}\alpha\\ \beta+2v_j k\end{smallmatrix}\right](0|\tau)}{\vartheta\left[\begin{smallmatrix}1\\ 1+2v_j k\end{smallmatrix}\right](0|\tau)},$$

$$Z_{\mathrm{int},k}^{59}\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right] = 2\sum_{s,r=0}^{1} (-1)^{\alpha s + \beta r}\left[(-1)^{s\cdot m_4} P_{m_4} P_{m_5}\right] \times \frac{\vartheta\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right](0|\tau)}{\eta(\tau)} \prod_{j=1}^{2}\frac{\vartheta\left[\begin{smallmatrix}\alpha+1\\ \beta+2v_j k\end{smallmatrix}\right](0|\tau)}{\vartheta\left[\begin{smallmatrix}0\\ 1+2v_j k\end{smallmatrix}\right](0|\tau)}. \tag{23}$$

For $s = r = 0$, we have the internal partition function of a $T^2 \times K^3/Z_2$ orientifold. s denotes the direct action of the SS deformation and r the twisted sector. The $(-1)^{s\cdot m_4} P_{m_4} P_{m_5}$ is the lattice sum over momenta along the first torus T^2:

$$(-1)^{s\cdot m_i} P_{m_i}(i\tau_2/2) = \frac{1}{\eta(i\tau_2/2)}\sum_{m_i}(-1)^{s\cdot m_i} q^{\frac{\alpha'}{4}\left(\frac{m_i}{R_i}\right)^2}. \tag{24}$$

For $s = 1$ we have the SS deformation that shifts the m_4 momenta. As we mention before, $r = 0, 1$ denotes the h untwisted and twisted sectors, respectively. However we will neglect the twisted sector since it requires the insertion of anti-D-branes [14].

To evaluate the masses of the anomalous bosons, we insert (23) and (11) in the mass formulae.[4] After

[4] This model has local vanishing of RR and NSNS tadpoles, and there will not be contributions from $\mathcal{A}_2$ and $\mathcal{M}$.

some 'thetacology' we find for $\alpha = 5, 9$:

$$F^{\alpha\alpha}_{k=1} = \frac{\eta^2}{2\pi^2}\big\{\mathrm{Tr}\big[\lambda^a\gamma_g\big]\,\mathrm{Tr}\big[\lambda^a\gamma_g\big] + \mathrm{Tr}\big[\lambda^a\gamma_{gh}\big]\,\mathrm{Tr}\big[\lambda^a\gamma_{gh}\big](-1)^{m_4}\big\} \times P_{m_4}P_{m_5}, \tag{25}$$

$$F^{59}_{k=1} = -\frac{\eta^2}{2\pi^2}\left\{\frac{1}{2}\,\mathrm{Tr}\big[\lambda^5\gamma_g\big]\,\mathrm{Tr}\big[\lambda^9\gamma_g\big] + \frac{i}{2\pi}\frac{\vartheta_2^2\vartheta_4^2}{\eta^6\vartheta_3^2}\partial_\tau \log\frac{\vartheta_2\vartheta_4}{\eta^2} \times \mathrm{Tr}\big[\lambda^5\gamma_{gh}\big]\,\mathrm{Tr}\big[\lambda^9\gamma_{gh}\big](-1)^{m_4}\right\} \times P_{m_4}P_{m_5}. \tag{26}$$

The γ-matrices point out the sector that each term is coming from. In the UV region, only the first terms in both formulae contribute to the mass of the anomalous $U(1)$'s. The second terms (that contains the SS action h) after the Poisson re-summation become proportional to $W_{v_4+1/2}$ and does not contribute to the $C^{99,55,59}_{\mathrm{UV}}$. Since SS deformation does not contribute to the mass terms of the anomalous $U(1)$'s, we can directly evaluate their masses for both two inequivalent solutions ($\gamma_h^2 = \pm 1$):

$$\frac{1}{2}M^2_{\alpha\alpha,ij} = -\frac{4\pi^2}{4}\,\mathrm{Tr}\big[\lambda^a_i\gamma_g\big]\,\mathrm{Tr}\big[\lambda^a_j\gamma_g\big]\frac{\mathcal{V}_1}{\pi^2\alpha'} = -\frac{\mathcal{V}_1}{\alpha'}\left(-\frac{i}{\sqrt{8}}\sin\big[2\pi V^a_i\big]\right) \times \left(-\frac{i}{\sqrt{8}}\sin\big[2\pi V^a_j\big]\right) = \frac{\mathcal{V}_1}{8\alpha'}, \tag{27}$$

$$\frac{1}{2}M^2_{59,ij} = \frac{4\pi^2}{2\times 4}\,\mathrm{Tr}\big[\lambda^5_i\gamma_g\big]\,\mathrm{Tr}\big[\lambda^9_j\gamma_g\big]\frac{\mathcal{V}_1}{2\pi^2\alpha'} = -\frac{\mathcal{V}_1}{32\alpha'}, \tag{28}$$

where $\alpha = 5, 9$. The mass-matrix has two massless gauge bosons $-\tilde{A}_1 + \tilde{A}_2$, $-A_1 + A_2$ and two massive $A_1 + A_2 + \tilde{A}_1 + \tilde{A}_2$, $-A_1 - A_2 + \tilde{A}_1 + \tilde{A}_2$ with masses $3\mathcal{V}_1/32\alpha'$, $5\mathcal{V}_1/32\alpha'$, respectively.

There are no anomalous $U(1)$'s in these models since the spectrum is non-chiral. However, the existence of the two massive gauge bosons are the consequence of 6D anomalies [2,11,12,19]. The decompactification limit of the first torus (where the SS deformation acts) leads to the $N = 1$ 6D Z_2 orientifolds that contains two anomalous $U(1)$'s that become massive via the Green–Schwarz mechanism. Therefore, axions that participate in the anomaly cancellation in the 6D model, contribute to the 4D masses of the anomalous $U(1)$'s by volume dependent terms. The ratio of the masses found in [19] for the Z_2 supersymmetric orientifold are the same to the above.

4. Conclusion

In this Letter we evaluated the general mass formula for the anomalous $U(1)$'s in non-supersymmetric orientifolds. We have shown that the supersymmetric formulae of [12] are also valid in non-supersymmetric orientifolds provided that the tadpoles cancel.

Our analysis has direct implications for model building, both in string theory and field theory orbifolds. It provides a necessary and sufficient condition for a non-anomalous $U(1)$ to remain massless (the hypercharge for example). The masses of the anomalous $U(1)$'s are always as heavy or lighter than the string scale. Therefore, production of these new gauge bosons in particle accelerators provides both constrains on model building and new potential signals at colliders, if the string scale is around a few TeV.

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Appendix A. Definitions and identities

The Dedekind function is defined by the usual product formula (with $q = e^{2\pi i\tau}$)

$$\eta(\tau) = q^{\frac{1}{24}}\prod_{n=1}^{\infty}\big(1 - q^n\big). \tag{A.1}$$

The Jacobi ϑ-functions with general characteristic and arguments are

$$\vartheta\left[{\alpha \atop \beta}\right](z|\tau) = \sum_{n\in Z} e^{i\pi\tau(n-\alpha/2)^2} e^{2\pi i(z-\beta/2)(n-\alpha/2)}. \tag{A.2}$$

We define: $\vartheta_1(z|\tau) = \vartheta\left[{1 \atop 1}\right](z|\tau)$, $\vartheta_2(z|\tau) = \vartheta\left[{1 \atop 0}\right](z|\tau)$, $\vartheta_3(z|\tau) = \vartheta\left[{0 \atop 0}\right](z|\tau)$, $\vartheta_4(z|\tau) = \vartheta\left[{0 \atop 1}\right](z|\tau)$. The modular properties of these functions are

$$\begin{aligned}
&\eta(\tau+1) = e^{i\pi/12}\eta(\tau),\\
&\vartheta\left[{\alpha \atop \beta}\right](z|\tau+1) = e^{-\frac{i\pi}{4}\alpha(\alpha-2)}\vartheta\left[{\alpha \atop \alpha+\beta-1}\right](z|\tau),\\
&\eta(-1/\tau) = \sqrt{-i\tau}\,\eta(\tau),\\
&\vartheta\left[{\alpha \atop \beta}\right]\left(\frac{z}{\tau}\middle|\frac{-1}{\tau}\right) = \sqrt{-i\tau}\,e^{i\pi(\frac{\alpha\beta}{2}+\frac{z^2}{\tau})}\vartheta\left[{\beta \atop -\alpha}\right](z|\tau).
\end{aligned} \tag{A.3}$$

A very useful identity that is valid for $\sum h_i = \sum g_i = 0$ is

$$\begin{aligned}
&\sum_{\alpha,\beta=0,1} \eta_{\alpha\beta}\vartheta\left[{\alpha \atop \beta}\right](v)\prod_{i=1}^{3}\vartheta\left[{\alpha+h_i \atop \beta+g_i}\right](0)\\
&\quad = \vartheta_1(-v/2)\prod_{i=1}^{3}\vartheta\left[{1-h_i \atop 1-g_i}\right](v/2).
\end{aligned} \tag{A.4}$$

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